Numerical Analysis of Soliton Pulse in Birefringence Coupler

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Abstract—A simple analytical approach based on a variational formalism is applied to an analysis of soliton switching in a birefringent nonlinear coupler. It is achieved by controlling the polarization angle one can have nearly 100% transmission with excellent switching characteristics. It can also be shown that soliton remains stable during its propagation inside the coupler. However after observing the high birefringent coupler exhibits relatively better soliton stability. In this connection we have determined the parameters coupler namely, the coupling length and core-to-core separation as well as the first-order coupling constant dispersion coefficient. We show that the coupler could be used as a soliton switch even at an input peak power less than the critical power. The power at which equal power sharing takes place between the two cores, just by a astute choice of the polarization angle.

Index Terms—Non-Linear Directional Coupler, Birefringent Coupler, Soliton Switching.

1. INTRODUCTION

A. Soliton

A fascinating manifestation of the fiber nonlinearity occurs through optical solitons, formed as a result of the interplay between the dispersive and nonlinear effects. The word soliton refers to special kinds of wave packets that can propagate undistorted over long distances. Soliton have been discovered in many branches of physics. In the context of optical fibers, not only are solitons of fundamental interest but they have also found practical applications in the field of fiber-optic communications. Optical couplers provide a means for controlling light by light. Significant progress has been made in controlling light beams due to non linear waveguides with both linear and non linear coupling. Non linear couplers offer excellent possibilities in making switching and memory elements for optical devices.

Non linear directional couplers (NLDC) are the building blocks of all optical communication systems and signal processing devices. Couplers can be constructed using optical fibers and certain organic polymers with high third order nonlinearities based on Kerr effect. The Nonlinear directional couplers exchange energy periodically between the guides similar to the linear couplers for low intensities. For high intensities they also trap the energy from the guide into which it has been launched initially.

B. Birefringence coupler

Birefringence is the optical property of a material having a refractive index that depends on the polarization and propagation direction of light. These birefringent are said to be optically anisotropic materials. Within the material, the birefringence is often quantified by the maximum difference in refractive index. Birefringence can also be termed as a pseudonym for double refraction since a ray of light its decomposed into two rays when it passes through a birefringent material.

Birefringence and related optical effects can be measured by measuring the changes in the polarization of light passing through the material, and measured as polarimetry. Birefringence of lipid bilayers can be measured using dual polarization interferometry. This provides the degree of order within these fluid layers and is disrupted when the layers interacts with other biomolecules. optical microscopes features is a pair of crossed polarizing filters and Against a dark background a brightness appear between the crossed polarizers in a birefringent sample because polarization of a light ray is rotated after passing through a birefringent material and the amount of rotation is proportional to wavelength. This hence method can be termed as photo-elasticity used for analyzing stress distribution in solids is based on the same principle.

Fig 1 Displacement of light rays with perpendicular polarization through a birefringent material.
In polarization-maintaining fibers, the built-in birefringence is made much larger than random changes occurring due to stress and core-shape variations. As a result, such fibers exhibit nearly constant birefringence along their entire length. This kind of birefringence is called linear birefringence. When the nonlinear effects in optical fibers become important, a sufficiently intense optical field can induce nonlinear birefringence whose magnitude is intensity dependent. In this section, we discuss the origin of nonlinear birefringence and develop mathematical tools that are needed for studying the polarization effects in optical fibers assuming a constant modal birefringence.

II. PROCEDURE

A. Solving NLSE using Split Step Fourier Method

Origin of the Split-Step Error

The nonlinear Schrödinger equation involved is

$$\frac{\partial u}{\partial z} - \beta \frac{\partial^2 u}{\partial t^2} + \gamma |u|^2 u = 0$$

(1)

To estimate the local and global errors in the split-step Fourier method it is convenient to represent

$$\frac{\partial u(z,t)}{\partial z} = (\hat{D} + \delta |u|)u(z,t)$$

(2)

Although the following discussion is for the nonlinear Schrödinger equation (1), the arguments and conclusions also apply to the modified versions of (1) that model realistic optical fiber transmission systems and to general reaction-diffusion equations. In the symmetric split-step scheme, the solution to (2) is approximated by

$$u(z+h,t) \approx \exp(\int_{0}^{h} \hat{D}) \exp(hN[u(z+\frac{h}{2},t)]) \exp(\int_{h}^{0} \hat{D})$$

(3)

Since the dispersion and nonlinear operators do not commute in general, the solution (3) is only an approximation to the exact solution. An argument based on the Baker-Campbell-Hausdorff formula shows that the local error, which is the error incurred in a single step of the symmetric split-step scheme, has a leading order term which is of third order in the step size $h$. Finding an optimal step size distribution depends on the particular optical transmission system. We will review several criteria for choosing the step size in the split-step Fourier method, and we will introduce a new criterion based on a measure of the local error.

B. Nonlinear Phase Rotation Method

The nonlinear phase rotation method is a variable step size method that is designed for systems in which nonlinearity plays a major role. For a step of size $h$, the effect of the nonlinear operator $\hat{N}$ is to increment

$$h \leq \frac{\phi_{\text{NL}}}{\gamma |u|^2}$$

(4)

This criterion for selecting the step size was originally applied to simulate soliton propagation and is widely used in optical fiber transmission simulators. An improper distribution of the step sizes may lead not only to a general reduction of accuracy, but also to numerical artifacts. The power of the four-wave mixing products can be greatly overestimated by a constant step size method, since four-wave mixing is a resonance effect. A logarithmic distribution of the step sizes to keep the spurious four-wave mixing components below a certain level. For a fiber span of length $L$ and loss coefficient, the step size of $n$-th step is given by

$$h_n = \frac{-1}{2\gamma} \ln \left[ \frac{1-na}{1-(n-1)a} \right]$$

(5)

In many optical fiber communications systems chromatic dispersion is the dominant effect, and nonlinearity only plays a secondary role, particularly in multi-channel systems in which the wave-6 length channels cover a broad spectrum. In this case it can be reasonable to use the walk-off method, in which the step size is determined by the largest group velocity difference between channels.

$$H = \frac{C}{\Delta V g}$$

(6)

The walk-off method can be applied to single-channel as well as multi-channel systems by choosing $\lambda_1$ and $\lambda_2$ at the two edges of the signal spectrum.

C. Constant Step Size Method

The simplest way to implement the split-step Fourier method is to use a constant step size along the whole transmission path. The global accuracy can be improved only by increasing the total number of steps. Note that the walk-off and constant step size methods are identical in systems with only one type of fiber. We have implemented a scheme based on bounding the error in each step using the technique of step doubling and local extrapolation.

$$u_c = u_1 + k(2h)^3 + O(h^4)$$

(7)

Where the true solution $z$ is the exact solution at $z + h$ obtained from the given solution at $z$. Next, we return to $z$ and compute the fine solution $u_f$ at the same distance $z + h$ using two steps of size $h$. As above, the fine solution is related to the true solution by

$$u_f = u_1 + 2kh^3 + O(h^4)$$

(8)

By taking an appropriate linear combination of the fine and coarse solutions we can obtain an approximate solution at $z + h$ for which the leading order error term is of fourth order in the step size $h$. From (7) and (8) it follows that this higher-order solution is given by
\( u_4 = \frac{4}{3} u_1 - \frac{1}{3} u_2 = u_1 + O(h^4) \) (9)

In the local error method the step size is adaptively chosen so that the local error incurred from \( z \) to \( z+h \) is bounded within a specified range. Now the relative local error \( \delta_4 \) of the higher-order solution is defined by

\[
\delta_4 = \left| \frac{u_4 - u_i}{u_4 + u_i} \right| \times \left| \frac{u_4 + u_i}{u_4} \right| = \left| \frac{u_4 - u_i}{u_4} \right| \] (10)

However, since we cannot compute the true solution \( u \) in practice, we cannot compute the local error using (10). Instead, we define the relative local error of a step to be the local error in the coarse solution relative to the fine solution:

\[
i \left( \frac{\partial u_1}{\partial \tau} + k_1 \frac{\partial u_2}{\partial \tau} \right) + \frac{1}{2} \beta_2 \frac{\partial^2 u_1}{\partial \tau^2} - i \delta \frac{\partial^3 u_1}{\partial \tau^3}
\]

\[
+ |u_1|^2 u_1 + i \frac{\partial}{\partial \tau} (|u_1|^2 u_2) - \tau_1 u_1 \frac{\partial |u_1|^2}{\partial \tau} + k_0 u_2 = 0, \] (11)

\[
i \left( \frac{\partial u_2}{\partial \tau} + k_1 \frac{\partial u_1}{\partial \tau} \right) + \frac{1}{2} \beta_2 \frac{\partial^2 u_2}{\partial \tau^2} - i \delta \frac{\partial^3 u_2}{\partial \tau^3}
\]

\[
+ |u_2|^2 u_2 + i \frac{\partial}{\partial \tau} (|u_2|^2 u_2) - \tau_2 u_2 \frac{\partial |u_2|^2}{\partial \tau} + k_0 u_1 = 0, \] (12)

Since we do not make any assumptions about the physical properties of the system, such as the amount of nonlinearity or dispersion, we expect the local error method to work well in an arbitrary system. In order to simulate a system with optimal efficiency, one first needs to investigate it to ascertain the major sources of the split-step error. Assuming that the system is dominated by one source of error, one can select an appropriate criterion for choosing the step sizes. The local error method allows us to deal with general systems when the major source of error is unknown or may even change during the propagation, or when performing a series of simulations in which the system parameters are varied. The method can be applied to a variety of systems without sacrificing too much computational efficiency.

**III. NUMERICAL RESULTS AND DISCUSSIONS**

As the set of coupled equations (11) and (12) is not analytically solvable, we solve them numerically by the so-called split-step Fourier method. The linear dispersive part is solved by the fast Fourier transform method and the nonlinear part is solved by the fourth-order Runge–Kutta method with auto-control of the step size for a given accuracy of the results. We calculate the fractional output energy in core 1 to represent according to the formula the transmission coefficient \( T \) is calculated using the formulae

\[
T = \frac{\int_{-\infty}^{\infty} |u_1(\epsilon, \tau)|^2 d\tau}{\int_{-\infty}^{\infty} (|u_1(\epsilon, \tau)|^2 + |u_2(\epsilon, \tau)|^2) d\tau} \] (13)

In this work we have calculated the transmission coefficient \( T \) at end of one coupling length of the coupler as defined in §3. Some typical calculations are provided in table 2. To analyse the switching process we consider the following initial conditions:

\[
u_1(0, \tau) = \sqrt{p_0} \sec h (\tau), \quad u_2(0, \tau) = 0 \] (14)

In this work the results are presented for \( \kappa_0 \) equal to 1.0, 0.5 and 0.1. They correspond to strong coupling moderate coupling and weak coupling respectively. The corresponding coupler parameters are taken from table 2.

To study how IMD may affect the switching characteristics, in figure 3 we have plotted the transmission coefficient as a function of the normalized input peakpower, \( \rho_0 \), for a soliton of pulse width 10 fs and \( \kappa_0 = 0.1 \). The solid curve (a) shows \( T \) as a function of \( \rho_0 \) for the case when no perturbative effects (including IMD) are present, while the dotted curve (b) shows the same for the case when only IMD is taken into account.

It is clearly seen that the lower part of the transmission curve is essentially modified by IMD (intermodal dispersion). It is observed that the transmission coefficient \( T \) starts at zero for curve (a), while \( T \) starts at a nonzero value for curve (b). In fact, this result has already been reported and agrees well with what we have obtained. But they have not provided any physical explanation of their results. It may be interpreted as follows: In the absence of IMD the phase-matching condition is completely achieved even at low input peak powers, which results in coupling of all the soliton energy to the cross state of the coupler after propagation over one coupling length. On the contrary, the presence of IMD destroys the phase-matching condition even at low peak powers and small fractions of soliton energy remain in the parallel state after one coupling length.

As the input peak power \( \rho_0 \) of the soliton is increased to a certain value of \( \rho_0 \) an equal energy sharing between the parallel and the cross states are observed. The corresponding value \( \rho_c \) corresponding to \( \rho_0 \) is called the critical power for switching. If the input peak power of the soliton is increased beyond \( \rho_c \), more and more of the soliton energy appears in the parallel state, implying \( T \rightarrow 1 \) and, we say that soliton is getting switched. As seen in figure 3, the critical power for switching is not affected by IMD.

**Figure 3.** Plot of the transmission coefficient as a function of the normalized input peak power for soliton of pulse width 10 fs for \( \kappa_0 = 0.1 \). Curve (a) corresponds to the case without any perturbative effects including IMD and curve (b) corresponds to the case when only IMD is taken into account.

In figures 4–6, we plot the transmission coefficient as a function of the normalized input peak power for solitons of various pulse widths \( T_0 \) and for \( \kappa_0 = 0.1, 0.5 \) and 1.0 respectively. The pulse duration is actually varied via the non-dimensionalization. Changing \( T_0 \) changes the non-dimensionalized reduced time \( \tau \).

Here we consider the simultaneous presence of all the perturbative effects noted from these transmission curves that for a given \( \kappa_0 \), the pulse width \( T_0 \) of the input soliton decreases when the critical power of switching increases. It may be implied from the fact that for a given \( \kappa_0 \) as the pulse width decreases IMD coefficient increases and these results in increase in the coupling coefficient \( C' (\omega) \). As the critical
power of switching is given by $P_c = 4C/\gamma$ in real units, it is quite evident that increase in $C$ results increase in $P_c$. It is clear from the transmission curves that the effect of the perturbative terms gets more pronounced with the decrease in the pulse width, that the corresponding intensities of the perturbative terms like TOD, self-steepening and Raman effect increase with decrease in the input pulse width.

![Figure 4](image1.png)

**Figure 4.** Plot of the transmission coefficient as a function of the normalized input peak power for solitons of various pulse widths $T_0$. Here the coupling coefficient $\kappa_0 = 0.1$.

![Figure 5](image2.png)

**Figure 5.** Plot of the transmission coefficient as a function of the normalized input peak power for solitons of various pulse widths $T_0$. Here the coupling coefficient $\kappa_0 = 0.5$.

![Figure 6](image3.png)

**Figure 6.** Plot of the transmission coefficient as a function of the normalized input peak power for solitons of various pulse widths $T_0$. Here the coupling coefficient $\kappa_0 = 1.0$.

Also, we observe that as the coupling coefficient $\kappa_0$ increases the influence of the perturbative terms also increases. These perturbative effects adversely affect the switching characteristics of the coupler. These effects become progressively dominant for increasing $\kappa_0$ and we observe that for $\kappa_0 = 0.5$ and 1.0 it is not possible to switch a 10 fs soliton pulse. With $\kappa_0 = 1.0$ it is not possible to switch even a 20 fs soliton pulse. It may be expected because with increase in $\kappa_0$ the core-to-core separation $a$ decreases and the absolute value of the first-order coupling constant dispersion $\kappa_1$ increases. In other words IMD does not allow the phase-matching condition to be fulfilled at the input powers considered here. In order to get an idea as to which perturbation plays a major role in the switching performance, the perturbation terms are included separately and compared their respective transmission coefficients. As an example, in figure 7, we plot the transmission coefficient as a function of the normalized input peak power for $T_0 = 30$ fs and $\kappa_0 = 0.1$.

![Figure 7](image4.png)

**Figure 7.** Plot of the transmission coefficient as a function of the normalized input peak power for soliton of pulse width 30 fs with $\kappa_0 = 0.1$. Curve (a) corresponds to the case without any perturbative effects including IMD. Curves (b), (c) and (d) represent respectively the cases when only the TOD, the Raman and the self-steepening effects are taken into account.

Curve (a) represents the transmission characteristics without any perturbative effects. From curves (b), (c) and (d) corresponding to the TOD, the Raman effect and the self-steepening effect respectively. The Raman effect is the most dominant one. In the context of switching the TOD effect is completely negligible. It is because of the very small length scale involved. In fact, it is observed that the upper part of the transmission curve is modified mainly due to the Raman Effect. Before going further, let us see the order of magnitudes of physical parameters that are required for practical implementation of a soliton switch based on our study.

In table 1 we have presented some typical values based on our study of switching characteristics. From table 1, a particular applications’ necessity depends on trade-off between the coupler and various parameters of the switching characteristics has to be reached. For example, (1) if we take $\kappa_0 = 0.1$ and $T_0 = 20$ fs, the required coupler length will be $L_c = 31.45$ cm and for a core radius of 5 $\mu$m the core-to-core separation turns out to be $a = 28.45$ $\mu$m.

For this case we conclude from figure 4 that around 95% switching is possible for $\rho_0 = 1.125$ which, in real units, corresponds to 10 kW of switching power. On the other hand, (2) if we take $\kappa_0 = 1.0$ and $T_0 = 50$ fs, we can have a lesser
switching power (=7.6 kW) and a smaller coupler length (=19.62 cm) but then the transmittivity also reduces to about 85%. Therefore the choice, as stated above, depends on a particular requirement, as the case may be. Finally, in order to have an idea about the behavior and stability of a soliton pulse during propagation inside the coupler, in figure 8, we have depicted the spatiotemporal evolution of the 20 fs soliton pulse of example 1 above. Figure 8a shows soliton evolution in core 1 while figure 8b shows the evolution of a small soliton, with some radiation attached leaking into the second core. It can clearly be seen that the soliton is preserved during evolution inside core 1. However, it gets shifted in the temporal domain along the positive time axis. This is a typical behavior, characteristics of soliton evolution under higher-order perturbations, mainly due to the split step fourier method.

IV. CONCLUSION

This method is used to overcome the transmission characteristics of the coupler which could be manipulated by varying the polarization angle. The coupler could be used as a soliton switch even at an input peak power less than the critical power of switching. A detailed numerical analysis of the effect of the simultaneous presence of all the perturbative effects on soliton switching in a twin-core fiber coupler. The coupler parameters such as the coupling length and core-to-core separation is determined in this connection as well as the first-order coupling constant dispersion coefficient. The calculations prove that for useful switching characteristics it is preferable to use shorter pulses and a compromise between switching power and coupling length can be made depending on our requirement as shown in table 2. As discussed, with astute choice of the pulse and coupler parameters one can have transmission up to 95%. Soliton can propagate with an unchanging pulse envelope and its spectrum over long distance.

REFERENCES