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Reduced SER for Maximum Likelihood Decoding of the Golden Code Using Modified Golden Ratio

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***ABSTRACT*-In the IEEE 802.16 (Wi-MAX) standard the Golden code is incorporated as a full-rate full-diversity space-time code. The worst case will dominate average decoding complexity on any channel with a significant line of sight component in this paper, we present Golden code that can be employed by mobile terminals with either one or two receive antennas, that is resilient to near singularity of the channel matrix, and that gives maximum likelihood (ML) performance. We have modified the golden ratio for low symbol error rate. The golden code is best for 2×2 system.**

***Index Terms*— Golden code, low complexity decoding, maximum likelihood decoding, space-time codes.**

I. INTRODUCTION

Space time codes that are constructed by multiplexing blocks with a different mathematical structure. One good example is the Golden code [1]–[3], which is constructed from cyclic division algebra, and employs two antennas to transmit four complex QAM symbols over two time slots. The Golden code achieves a tradeoff between rate and reliability for both one and two receive antennas that is best possible in terms of the diversity-multiplexing bound derived by Zheng and Tse [4 This code is incorporated in the IEEE 802.16 standard and]. A drawback of the Golden code has been the high ML decoding complexity of , where is the underlying QAM constellation, [5], [6]. Although the sphere decoder can be $O(N^4)$ implemented to reduce the complexity significantly, the worst case complexity defaults to $O(N^4)$ However, if a square QAM constellation is used, the complexity of an ML decoder reduces to using conditional optimization [8]. In this paper we propose a simple fast decoding algorithm for the Golden code with low symbol error rate which has essentially ML performance. In this new algorithm, the likelihood function is maximized with respect to one of the pair of signal points conditioned on the other pair. This choice of which pair to condition on is crucial to the performance of the algorithm and is made by comparing the determinants of two covariance matrices, and the underlying geometry of the Golden code guarantees that one of these choices is good with high probability. We have also compared proposed code with alamouti code and golden code given by[1].

II.SYSTEM MODEL

We have considered Rayleigh quasi-static flat-fading MIMO channel with full channel state information (CSI) at the receiver but not at the transmitter. For $n_t \times n_r$ MIMO transmission, we have

$$R = HX+ N \qquad (1)$$

where $X \in C\,(n_t{\times}T)$ is the codeword matrix, transmitted over T channel uses, $N \in C(n_r{\times}T)$ is a complex white Gaussian noise matrix with independent and identically distributed (i.i.d.) entries $\sim \mathcal{N}_C(0,N_0)$ and $H \in \in C(n_r\times n_t)$ is the channel matrix with the entries assumed to be i .i.d circularly symmetric Gaussian random variables $\sim \mathcal{N}_C(0,1)$. $R \in C\,(n_r{\times}T)$ is the received matrix.

*Definition 1: (***Code rate***):* , The code rate is defined to be k/T complex symbols per channel use If there are k independent complex information symbols in the codeword which are transmitted over T channel uses, then. For instance, for the Alamouticode k=2, and T=2 . So, its code rate is 1 complex symbol per channel use.

*Definition 2: (***Full-rate code***):* A code is *full rate* if it transmits at the rate of n_{min} complex symbols per channel use, where $n_{min} = \min(n_t,n_r)$. So, the Alamouti code can be considered to be full-rate for the 2× 1 MIMO system alone, while the Golden code is full-rate for $n_r \geq 2$,Considering ML-decoding, the decoding metric that is to be minimized over all possible values of code words is X given by

$$M(X) = ||R\text{-}HX||^2 \qquad (2)$$

Design criteria:
1) ***Rank criterion***: To achieve maximum diversity, the nonzero codeword difference matrix (X-X') must have full-rank for all possible code word pairs (X,X') and the diversity gain is n_t,n_r . If full-rank is not achievable, then, the diversity gain is given by $n_r r$, where r is the minimum

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rank of the codeword difference matrix over all possible codeword pairs.

2) ***Determinant criterion***: For a STBC to be full-ranked, the minimum determinant $\boldsymbol{\delta}_{\min}$, defined as

$$\boldsymbol{\delta}_{\min} = \min \det [(X-X')(X-X')^H]$$

should be maximized. The coding gain is given by $(\boldsymbol{\delta}_{\min})^{1/nt}$, with n_t being the number of transmit antennas. If the STBC is non full-diversity and is the minimum rank of the codeword difference matrix over all possible codeword pairs, then, the coding gain $\boldsymbol{\delta}$ is given by

$$\boldsymbol{\delta} = \min_{(X-X')}(\Pi^{r}_{i=1})^{1/r}$$

where λ_i, i=1,2.....r, are the nonzero eigen values of the matrix $(X-X')(X-X')^H$. It should be noted that for high SNR values at each receive antenna, the dominant parameter is the diversity gain which defines the slope of the CER curve. This implies that it is important to first ensure full-diversity of the STBC and then try to maximize the coding gain.

III.THE GOLDEN CODE

The Golden code is a 2×2 block space-time code that employs 2 transmit and 2 receive antennas and encodes four complex symbols over two time slots yet achieves full diversity ([1], [2],[3]). The Golden code codeword takes the form

$$X = (1/\sqrt{5})\begin{pmatrix} \alpha & 0 \\ 0 & \alpha \end{pmatrix}\begin{pmatrix} a_1 + a_2\theta & a_3 + a_4\theta \\ i(a_3+a_4\mu) & a_1 + a_2\mu \end{pmatrix} \quad (1)$$

Where a_j,j = 1,…4 ε C ⊂ Z[i] are the transmitted symbols, and C is a signal constellation taken to be 2^m-QAM with in-phase and quadrature components equal to ±1, ±3…and m bits per symbol. The parameters θ and μ are the real roots of the polynomial x^2-x-1, that is, the Golden ratio θ= (1+√5)/2 and its algebraic conjugate μ= 1- θ = (1-√5)/2 , which is the negative of the inverse of the Golden ratio. The diagonal matrix diag[α,α'] where α= 1+iμ ,and its algebraic conjugate serves to equalize transmitted signal power across the two transmit antennas. The entries of Golden space-time code words are drawn from Z[i][√5]⊂ Q[i,√5] . where Z[i][√5] is the ring of elements of the form $(n_1+in_2)+(n_3+in_4)\sqrt{5}n_i$⊂ Z[i]and is the field of elements of the form $(a_1+ia_2)+(a_3+a_4)\sqrt{5}$, a_i ⊂Q. Here Z,Q denote the integers and rationales, respectively Following [9] we rewrite (1) as

$$X = (1/\sqrt{5})\begin{pmatrix} \alpha & 0 \\ 0 & \alpha' \end{pmatrix}\begin{pmatrix} a_1 + a_3 \\ ia_3+a_1 \end{pmatrix} + \begin{pmatrix} \theta & 0 \\ 0 & \mu \end{pmatrix}\begin{pmatrix} a_2 + a_4 \\ ia_4+a_2 \end{pmatrix} \quad (2)$$

Let r be the received signal from the receive antenna at time slot and h be the channel gains from the transmit antenna to the receive antenna . Here we assume quasi-static fading channel. The received signal vectors can be written as

$$(r_{11}\ r_{12}) = 1/\sqrt{5}(\alpha h_{11,}\alpha' h_{12}) \begin{pmatrix} a_1 + a_3 \\ ia_3 + a_1 \end{pmatrix} + 1/\sqrt{5}(\theta h_{11,}\ \mu h_{12})\begin{pmatrix} a_2 + x_4 \\ ia_4+ a_2 \end{pmatrix} + (n_{11},n_{12}) \quad (3)$$

$$(r_{21}\ r_{22}) = 1/\sqrt{5}(\alpha h_{12,}\alpha' h_{22})\begin{pmatrix} a_1 + a_3 \\ ia_3 + a_1 \end{pmatrix} + 1/\sqrt{5}(\theta h_{21,}\ \mu' h_{22})\begin{pmatrix} a_2 + a_4 \\ ia_4+ a_2 \end{pmatrix} + (n_{21},n_{22}) \quad (4)$$

Where n_{ij} are complex Gaussian random variables with zero mean and variance $2\sigma^2$. Rewriting (3) and (4) as

$$r = sH+ cG + n \quad (5)$$

$$r = (r_{11},r_{12},r_{21},r_{22})$$

$$n = (n_{11},n_{12},n_{21},n_{22})$$

$$s= (a_1 + a_3) ,\ c = (a_2 + a_4)$$

$$H = 1/\sqrt{5}\begin{pmatrix} \alpha h_{11} & \alpha' h_{21} & \alpha h_1 & \alpha' h_{22} \\ i\alpha' h_{21} & {}_{,}\alpha h_{11} & {}_{i}\alpha' h_{22} & \alpha h_{12} \end{pmatrix}$$

$$G = 1/\sqrt{5}\begin{pmatrix} \alpha\,\theta h_{11} & \alpha'\mu h_{21} & \alpha\,\theta h_{12} & \alpha'\mu\, h_{22} \\ i\alpha'\mu h_{21} & \alpha\,\theta h_{11} & i\alpha'\mu h_{22} & \alpha\,\theta h_{12} \end{pmatrix} \quad (6)$$

the likelihood function of code words and given the received signal is given by

$$p(r|s,c)\ \alpha\ \exp(-1/2\sigma^2 \| r\text{-}sH\text{-}cG \|^2) \quad (7)$$

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Given that the channel gains are known at the receiver and each symbol is transmitted with equal probability, i.e., taking the prior distribution of the symbols s and c to be uniform on the constellation C, we obtain the ML solution

$$(s', c') = \arg\max p(r|s,c) \quad (8)$$

IV.PROPOSED GOLDEN CODE USING MODIFIED GOLDEN RATIO (GC-MGR)

The Golden code and its codeword format are explained in the aforementioned section. In this section, the proposed code and its properties satisfying golden codes are explained further. The GC-MGR is a scaled version of golden code which is used to normalize the energy and in turn, yields the performance improvement in the system. A golden code- modified golden ratio (GC-MGR) is obtained by choosing a minimum polynomial equation in such a way that, it obtains the modified golden ratio without losing golden codes key properties. The minimum polynomial equation considered to obtain the modified golden ratio is

$$x^2-x-1.5=0$$

$\theta= (1+\sqrt{7})/2$, Where, 'θ' is the golden ratio and $\mu= 1- \theta = (1-\sqrt{7})/2$,using this golden ratio we get reduced symbol error rate.A codeword X belonging to the Golden Code has the form

$$X = (1/\sqrt{5}) \begin{pmatrix} \alpha & 0 \\ 0 & \alpha \end{pmatrix} \begin{pmatrix} a_1 + a_2\theta & a_3 + a_4\,\theta(1) \\ i(a_3+a_4\mu) & a_1 + a_2\mu \end{pmatrix}$$

wherea_1, a_2 ,a_3 ,a_4 are QAM symbols,now using $\theta = (1+\sqrt{7})/2$, $\sigma(\theta) = 1- \theta = (1-\sqrt{7})/2$, $\alpha = 1+i -i\theta=1+ \sigma(\theta)$,$\sigma(\alpha) = 1+i-\sigma(\theta) =1+i\theta$,The code is full rate since it contains 4 information symbols a_1,a_2 ,a_3 ,a_4. Let us now compute the minimum determinant of the infinite code. Since $\alpha\sigma(\alpha) = 2.5 + i$, we have det(X)

$$=(1/5)\{\alpha\sigma(\alpha)[a_1+a_2\theta][a_1+a_2a_4\sigma(\theta)]-i\alpha\sigma(\alpha)[a_3+a_4\theta][c+a_4\sigma(\theta)]\}$$

$$= ((2.5+i)/5)\{[a_1+ a_2\theta][a_1+ a_2a_4\sigma(\theta)]- i[a_3+ a_4\theta][a_3+ a_4\sigma(\theta)]\}$$

$$=(1.05/(2.5-i))\{[a_1+a_2\theta][a_1+a_2a_4\sigma(\theta)]- i[a_3+ a_4][a_3+ a_4\sigma(\theta)]\}$$

as definition of a_1, a_2 ,a_3 ,$a_4$we have that the minimum of $|a_1^2 + a_1 a_2 - a_2^2-i(c^2 + a_3 a_4 - a_4^2)|^2$ is 1thus

$$\underline{\delta_{min}}(C_\infty) = min_{0\neq X\in C}|\det(X)|^2 =0.1521$$

Thus the minimum determinant of the infinite code is away from zero, as required. For the diagonal layer consider the code. It can be written

$$1/\sqrt{5} \begin{pmatrix} \alpha & \alpha\theta \\ \alpha\sigma(\alpha) & \sigma(\theta) \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$$

To satisfy cubic shaping the matrix can be checked to be unitary. The factor i in code word in the second row of the code word, which confirm uniform average transmitted energy since $|i|^2 = 1$ .This code has designed to match all the required properties. Its generation from cyclic division algebra, and the shaping is confirmed by interpreting the signals on each layer as points in a lattice.

V. SIMULATION RESULTS

The elements of the channel matrix are modeled as samples of independent complex Gaussian random variable with variance 0.5 and zero mean per real dimension.

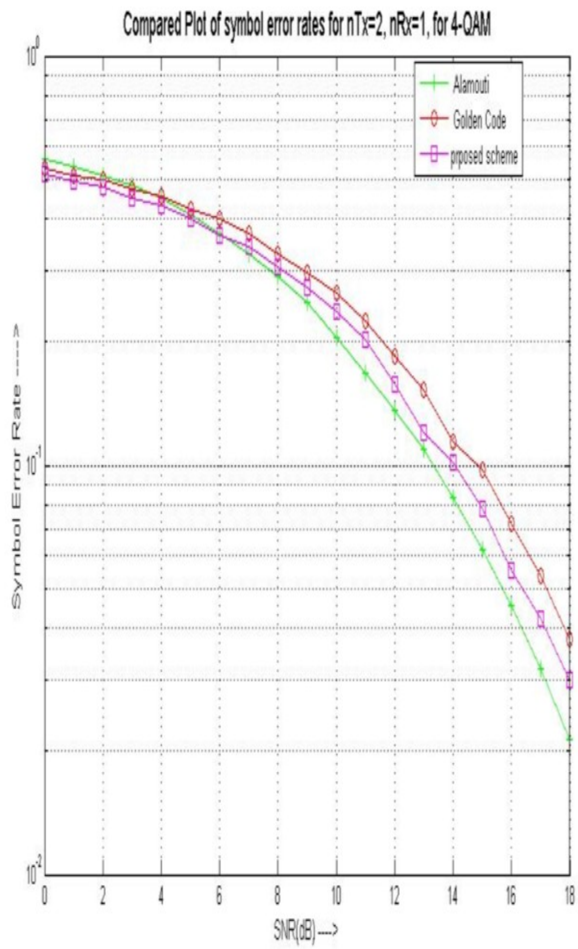

Fig.(1)Plot between SNR and Symbol error rate for Alamouti(+)Golden code (○and Proposed scheme( ) 4□QAM,nTx=2,nRx=1

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The noise is complex Gaussian with variance $2\sigma^2$ and zero mean. The signal to noise ratio at a receiver antenna is defined as

$$SNR(dB) = 10\log_{10}(P_s/2\sigma^2)$$

Where P_s is the average signal power per symbol at a receive antenna which is defined as

$$P_s = E_s(\|H\|^2 + \|G\|^2)$$

And E_s is the average energy per symbol.

The golden code is best for 2×2 system, this full rate and full rank code for this system. The figure (1) is plot between symbol error rate and SNR for 2×1 system for alamouti, golden code and proposed scheme, in this alamouti is showing best performance and proposed scheme is better than the golden code. The figure (2) is plot between symbol error rate and SNR for 2×2 system for alamouti, golden code and proposed scheme, in this alamouti is showing worst performance and proposed scheme is showing best performance. The golden code is only better than the alamouti code.

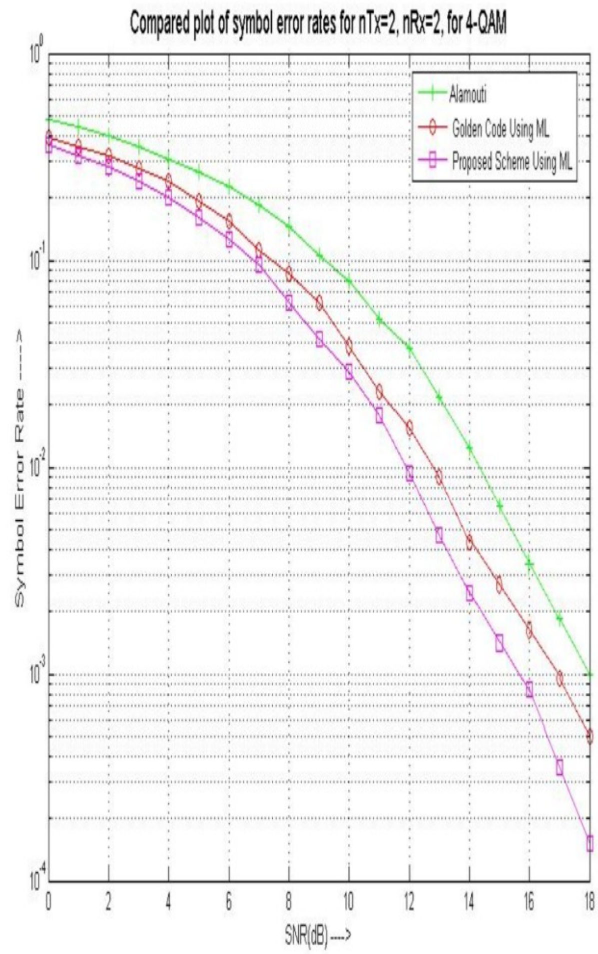

Fig.(2)Plot between SNR and Symbol error rate for Alamouti(+)Golden code(○ and Proposed scheme( ) 4□QAM,nTx=2,nRx=2

VI . CONCLUSION

This paper is on the maximum likelihood decoding of the golden code using modified golden ratio for 4-QAM constellations, we have seen some better result. There is a comparative plot between Alamouti, golden code and using our proposed modified golden ratio. In which our modified golden ratio is showing best performance. The golden code is representative of a larger class of high rate space time codes formed by multiplexing simpler blocks. Our approach applies to this larger class of codes where it provides a geometric criterion for low symbol error rate decoding with essentially ML performance.

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