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Analysis of rectangular infinite flanged two element waveguide array radiator

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Abstract— **In this analysis, a standard 2.286×1.016 cm X-band WR-90 open ended rectangular waveguides with infinite ground plane operating at 8 to 12 GHz is used as radiating array. The field inside the rectangular waveguide in an infinite ground plane is determined from the Maxwell's equation using the vector potential approach. The field inside a waveguide due to an aperture in the transverse plane of the rectangular waveguide is expressed in terms of modal vectors and modal functions. The field radiated into free space by an aperture is obtained using the spectral domain approach. The radiator aperture field is described by a sum of weighted basis function in the form of sinusoidal functions, defined over the entire region of the aperture. Galerkin's Moment method is used to solve for the coefficients of the basis functions. The reflection coefficient by the waveguide array is evaluated from amplitude coefficients determined from matrix inversion. These values are compared with the reflection coefficients of the single waveguide radiator. The computations have also been carried out to find the admittance (conductance and susceptance) of the array radiator and compared with single radiator. Because of the mutual coupling and multiple reflections between radiators, radiators reflection coefficient and near field values are changed. So that the radiation properties of the radiator array improves.**

Index Terms— **Array radiator, Ground plane, Galerkin's method, Waveguide.**

I. INTRODUCTION

Rectangular waveguides were one of the earliest types of transmission lines used to transport microwave signals. Waveguides were used in many applications such as high power systems, millimeter wave systems, and in some precision test applications [1]. Usually the radiation pattern of a single element is relatively wide, and each element provides low values of directivity (gain). In many applications it is necessary to design antennas with very directive characteristics (very high gains) to meet the demands of long distance communication. This can only be accomplished by increasing the electrical size of the antenna. The way to enlarge dimension of the antenna, without necessarily increasing the size of the individual elements, is to form an assembly of radiating elements in an electrical and geometrical configuration. This new antenna, formed by multi elements, is referred as array [2].

The waveguide elements in two element array in an infinite ground plane being usedas radiating elements are assumed to be excited in the dominant TE_{10} mode. In the present analysis, the following assumptions are made [3].

1) The ground plane in Fig.1and Fig.2 are infinite in extent.
2) The x-component of the electric field at the plane of the apertures are ignored.
3) Only the x-component of the magnetic field at the apertures plane is considered.
4) The excitation in the feed waveguide array is dominant wave with the incident electric field being y-directed.
5) The electric field in the aperture varies only in the x-direction and is constant in y-direction.

The boundary conditions imposed at the radiating apertures by automatically taking mutual coupling between radiating elements into account. The field is described as a sum of M number of weighted sinusoidal basis functions, defined over the extent of the aperture at the plane of the waveguide. This field can be considered to be a magnetic current source which scattered some field into the free space and some field is scattered within the waveguide. The tangential component of scattered magnetic field within the waveguide and that scattered into the free space must be continuous at the plane of the aperture. Enforcement of this boundary condition leads to an integral equation involving the M unknowns used to

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describe the aperture electric field. This is transformed into a matrix equation by taking moments with entire domain sinusoidal weighting function [5]. A solution of this matrix equation provides the values of the unknown coefficients. The field scattered inside and outside the waveguide is obtained in terms of these coefficients. Reflection coefficients and admittance of array radiator are evaluated from these coefficients.

II. LITERATURE SURVEY

The basic radiating structure is the open end of a waveguide, generally terminated by an infinite metallic flange. Many authors using different approaches such as the variational, correlation matrix and integral equation methods have studied this subject [9]-[11]. The numbers of methods have been developed for the determination of mutual and self admittance between radiating elements [4],[8],[12] and [13]. Chakrabarty [9] developed a method for the determination of self and mutual admittance between a pair of radiators a pair of radiators. He reported impedance of an open ended waveguide using variational technique where the electric field at the radiating aperture was assumed.
The studies pertaining to waveguide aperture relied on an assumed distribution of tangential electric field at the aperture severely limit the validity of the formulae derived, particularly aperture behavior away from resonance. With the advent of fast digital computers and superior numerical techniques, attempts were made to solve the integral equation for the aperture electric field with the help of computers. Many authors used method of moment technique to solve waveguide problems [5] and [15]. It is felt, that a solution technique should be made available to asses the field at the aperture and hence impedance seen by the TE_{10} dominant wave in the feed waveguide.

III. PROBLEM FORMULATION

The geometry of the problem is two-element waveguide array radiator shown in Fig 2. The aperture dimension of each waveguide is 2a×2b. Before analyzing the two-element waveguide array radiator, the performance of single waveguide radiator shown in Fig.1 is given [3] by:

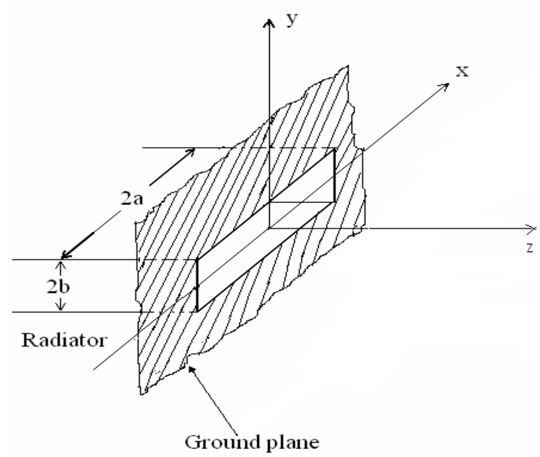

Fig. 1 Schematic diagram of open ended waveguide radiator.

The incident magnetic field at the radiating aperture for the dominant TE_{10} mode is given by:

$$H_x^{inc1} = -Y_0 \cos\left(\frac{\pi x}{2a}\right) e^{-j\beta z} \tag{1}$$

And the electric field at the radiating aperture 1 is described by a sum of weighted basis function s in the form of sinusoidal function defined over entire region of the aperture and is give by:

$$\overrightarrow{E^1}(x', y', 0) = \hat{u}_y \sum_{p=1}^{M} E_p \, e_p \tag{2}$$

Where the entire domain basis functions e_p (p=1,2,….M) are defined by:

$$e_p = \begin{cases} \sin\left\{\frac{p\pi}{2a}(x+a)\right\} & \begin{cases} -a \le x \le a \\ -b \le y \le b \end{cases} \\ 0 & \text{elsewhere} \end{cases} \tag{3}$$

Where e_p is the entire domain basis function and E_p is the coefficient of the basis function to be determined.
The boundary condition at the region of the waveguide aperture is enforced such that the tangential component of the magnetic field both inside and outside the waveguide is identical. At z=0, plane the boundary condition at the waveguide aperture is given by:

$$2H_x^{inc} + H_x^{int} = H_x^{ext} \tag{4}$$

Where H_x^{int} is the internally scattered magnetic field, evaluated using modal expansion method [4], H_x^{ext} is the magnetic field at the plane of the aperture, evaluated using the plane wave spectrum approach[4]. Taking the moment for equation (4) such that testing function at the waveguide aperture is entire domain function, the integral equation (4) is converted into matrix form:

$$2\left[L_1^{inc}\right] + \left[L_1^{int}\right]\left[E_p\right] = \left[L_1^{EXT}\right]\left[E_p\right] \tag{5}$$

Where

$L_1 = \int H . w_q \, ds$

From equation (5), coefficient of the entire domain basis function is determined. From E_p, reflection coefficient of the waveguide aperture is determined.

Next consider the open ended waveguide of two element array radiator as in fig.2. For such a radiator array, the incident magnetic field at the radiating apertures 1 and 2 for the dominant TE_{10} mode are given by [6]:

$$H_x^{inc1} = -Y_0 \cos\left(\frac{\pi x}{2a}\right) e^{-j\beta z} \tag{6}$$

$$H_x^{inc2} = -Y_0 \cos\left(\frac{\pi x}{2a}\right) e^{-j\beta z} = H_x^{inc1} \tag{7}$$

And the electric fields at the radiating aperture 1and 2 are described by:

$$\overrightarrow{E^1}(x', y', 0) = \hat{u}_y \sum_{p=1}^{M} E_p^1 \, e_p^1 \tag{8}$$

$$\overrightarrow{E^2}(x', y', 0) = \hat{u}_y \sum_{p=1}^{M} E_p^2 \, e_p^2 \tag{9}$$

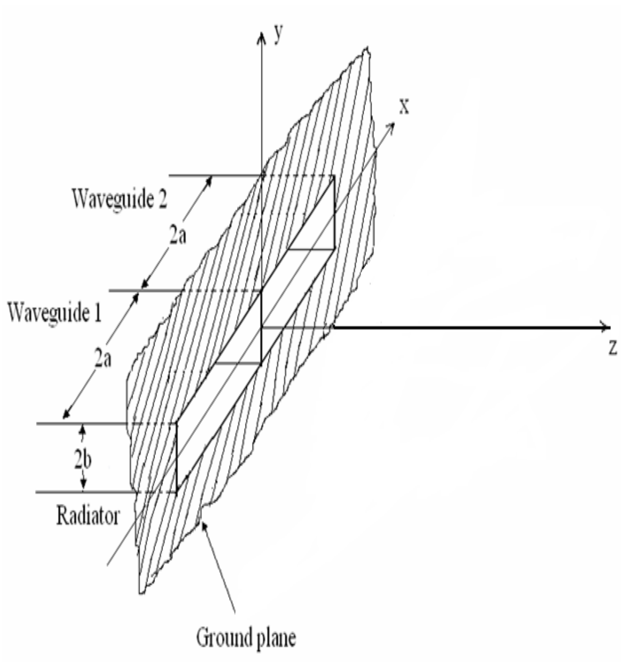

Fig. 2 Schematic diagram of open ended waveguide of two element array radiator.

Where the entire domain basis functions e_p (p=1, 2….M) are defined by

$$e_p^1 = \begin{cases} \sin\left\{\frac{p\pi}{2a}(x+a)\right\} & \begin{cases} -a \le x \le a \\ -b \le y \le b \end{cases} \\ 0 & \text{elsewhere} \end{cases}$$
$$= e_p^2 \tag{10}$$

The equivalent magnetic current at aperture 1 for computing the externally radiated magnetic field using the plane wave spectrum approach is given by [4]:

$$\overrightarrow{M_s^1} = 2\overrightarrow{E^1}(x', y', 0) \times \hat{u}_z$$

$$= \hat{u}_x \sum_{p=1}^{M} 2E_p^1 \sin\left\{\frac{p\pi}{2a}(x+a)\right\} \begin{cases} -a \le x \le a \\ -b \le y \le b \end{cases} \tag{11}$$

The electric vector potential $\overrightarrow{F^1}$ at any point in the space due to magnetic current at aperture 1is given by:

$$\overrightarrow{F^1} = \iint_{aper} \frac{\overrightarrow{M_s^1} \, e^{-jk|r-r'|}}{4\pi|r-r'|} \, ds' \tag{12}$$

The electric and magnetic field at any point in the space are given by $\overrightarrow{EM_s^1}$

$$\overrightarrow{E^1} = -\nabla \times \overrightarrow{F^1} \tag{13}$$

$$H^1 = -\frac{\nabla \times E^1}{jw\mu} = \frac{1}{jk\eta} \nabla \times \nabla \times \overrightarrow{F^1} \tag{14}$$

From equation (2) through (9), the externally scattered magnetic field at the plane of the aperture 1 (z=0) is obtained as:

$$\overrightarrow{H^{ext1}} = \frac{1}{2\pi k\eta} \iint_{-\infty}^{\infty} k \times k \times \overrightarrow{\varepsilon_s} \times \frac{\hat{u}_x}{k_z} e^{j(k_x x + k_y y)} \, dk_x \, dk_y \tag{15}$$

Where $\overrightarrow{\varepsilon_s}$ is the Fourier Transform of the aperture electric field $\overrightarrow{E_s^1}$; it is given by

$$H_x^{ext1} = \frac{1}{2\pi k\eta} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{k_x \mathcal{E}_x k_y + (k^2 - k_x^2)}{k_z} \times e^{j(k_x x + k_y y)} \, dk_x \, dk_y \tag{16}$$

$$H_x^{ext11} = -\frac{ab}{\pi^2 k\eta} \sum_{p=1}^{M} E_p^1 \times \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{k^2 - k_x^2}{(k^2 - k_x^2 - k_y^2)^{\frac{1}{2}}} \, sinc\left(k_y b\right) \times \frac{\left\{\begin{matrix} j\sin(k_x a) & p \text{ even} \\ \cos(k_x a) & p \text{ odd} \end{matrix}\right\}}{\frac{p\pi}{2}\left\{1 - \left(\frac{2ak_x}{p\pi}\right)^2\right\}} \times$$

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$$e^{j(k_x x + k_y y)} \; dk_x \, dk_y \qquad (17)$$

Equation (17) gives the x-component of the externally scattered magnetic field at the aperture 1 due to the magnetic current source at the aperture 1. Similarly, the externally scattered magnetic field at the plane of the aperture 2 due to the magnetic current source at the aperture 2 is obtained as:

$$H_x^{ext22} = -\frac{ab}{\pi^2 k\eta} \sum_{P=1}^{M} E_P^2 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{k^2-k_x^2}{(k^2-k_x^2-k_y^2)^{\frac{1}{2}}} \times$$

$$sinc\,(k_y b)\, \frac{\begin{Bmatrix} j\sin(k_x a) & p\ even \\ \cos(k_x a) & p\ odd \end{Bmatrix}}{\frac{p\pi}{2}\left\{1-\left(\frac{2ak_x}{p\pi}\right)^2\right\}} \times$$

$$e^{j(k_x x + k_y y)} \; dk_x \, dk_y \qquad (18)$$

The internally scattered field at the waveguide aperture is obtained by using the modal expansion approach [4]. The x-component of the internally scattered magnetic fields at apertures 1 and 2 are obtained as:

$$H_x^{int1} = \sum_{P=1}^{M} E_P^1 Y_{po}^e \sin\left\{\frac{m\pi}{2a}(x+a)\right\} \qquad (19)$$

$$H_x^{int2} = \sum_{P=1}^{M} E_P^2 Y_{po}^e \sin\left\{\frac{m\pi}{2a}(x+a)\right\} \qquad (20)$$

The boundary conditions are simultaneously imposed at the plane of the radiating waveguide apertures 1 and 2. The boundary condition at the region of the waveguide aperture is the tangential component of the magnetic field both inside the waveguide and outside it should be identical. At z=0 plane, the x-component of the magnetic field at the plane of the radiating aperture 1 is given by:

$$2H_x^{inc1} + H_x^{int1} = H_x^{ext11} + H_x^{ext12} \qquad (21)$$

Similarly the x-component of the magnetic field at the plane of the radiating aperture 2 is given by:

$$2H_x^{inc2} + H_x^{int2} = H_x^{ext22} + H_x^{ext21} \qquad (22)$$

Since the field is described by M basis functions, M unknowns are to be determined from the boundary condition. The Galerkin's method of moments is used to obtain M different equations from the boundary condition to enable the determination of the Ep. The weighting functions w_q is selected to be of the same as the basis function. The integral equations are then converted into matrix form as:

$$2[L^{inc1}] + [L^{int1}]\,[E_p^1] = [L^{ext11}]\,[E_p^1] + [L^{ext12}]\,[E_p^1] \qquad (23)$$

$$2[L^{inc2}] + [L^{int2}]\,[E_P^2] = [L^{ext22}]\,[E_p^2] + [L^{ext21}]\,[E_P^2] \qquad (24)$$

Where E_p^1 and E_P^2 are the coefficients of the basis functions at the aperture 1 and aperture 2 respectively. The elements of the matrix are defined as

$$\begin{bmatrix} [E_p^1] \\ [E_p^2] \end{bmatrix} = \begin{bmatrix} [L^{ext1}] - [L^{int1}] & [L^{ext12}] \\ [L^{ext21}] & [L^{ext22}] - [L^{int2}] \end{bmatrix}^{-1} \times \begin{bmatrix} 2[L^{inc}1] \\ 2[L^{inc2}] \end{bmatrix} \qquad (25)$$

The moment elements are obtained as:

$$[L^{inc1}] = L_q^{inc1} = \langle H_x^{inc1}, w_q^1 \rangle = \begin{Bmatrix} -2abY_0 & q=1 \\ 0 & otherwise \end{Bmatrix} \qquad (26)$$

$$[L^{int1}] = \langle H_x^{inc1}, (e_p^1), w_q^1 \rangle = \begin{Bmatrix} -2ab\,Y_P^e & p=q=m=1\ and\ n=0 \\ 0 & otherwise \end{Bmatrix} \qquad (27)$$

$$[L^{ext11}] = L_{q,p}^{ext11} = \langle H_x^{ext11}, (e_p^1), w_q^1 \rangle$$

$$[L^{ext11}] = \frac{-4a^2b^2}{\pi^2 k\eta} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{k^2-k_x^2}{(k^2-k_x^2-k_y^2)^{\frac{1}{2}}} \times$$

$$sinc^2\,(k_y b)\, \frac{\begin{Bmatrix} j\sin^2(k_x a) & p,q\ both\ even \\ \cos^2(k_x a) & p,q\ both\ odd \\ 0 & otherwise \end{Bmatrix}}{\frac{p\pi}{2}\left\{1-\left(\frac{2ak_x}{p\pi}\right)^2\right\}\frac{q\pi}{2}\left\{1-\left(\frac{2ak_x}{q\pi}\right)^2\right\}} \times$$

$$e^{j(k_x x + k_y y)} \; dk_x \, dk_y \qquad (28)$$

$$[L^{ext12}] = L_{q,p}^{ext12} = \langle H_x^{ext12}, (e_p^2), w_q^1 \rangle$$

$$[L^{ext12}] = \frac{-4a^2b^2}{\pi^2 k\eta} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{k^2-k_x^2}{(k^2-k_x^2-k_y^2)^{\frac{1}{2}}} \times$$

$$sinc^2\,(k_y b)\, \frac{\begin{Bmatrix} j\sin^2(k_x a) & p,q\ both\ even \\ \cos^2(k_x a) & p,q\ both\ odd \\ 0 & otherwise \end{Bmatrix}}{\frac{p\pi}{2}\left\{1-\left(\frac{2ak_x}{p\pi}\right)^2\right\}\frac{q\pi}{2}\left\{1-\left(\frac{2ak_x}{q\pi}\right)^2\right\}} \times$$

$$e^{j(k_x x + k_y y)} \; dk_x \, dk_y \qquad (29)$$

Solving equations (28) and (29) simultaneously, we obtain the coefficients of the basis functions E_p^1 and E_p^2. from these coefficients, radiating elements reflection coefficient and admittance are determined.

IV. NUMERICAL RESULTS AND DISCUSSION

The two standard X-band WR-90 rectangular waveguide in elements collinear in the x-axis at z=0 plane, in an infinite ground plane is used as a radiating array. For the determination of the coefficients of the basis functions, the program written in MATLAB12 was run on Pentium4, CPU 3 GHz computers. Convergence is obtained with M=11.

Computations have been carried out to determine the radiator array reflection coefficient and admittance. Fig. 3 and Fig.4 shows the radiator array phase of reflection coefficients, reflection coefficient (absolute) as a function of frequency and Fig.5 and Fig.6 shows the radiation conductance and susceptance as a function of frequency. These values are compared with single element radiator. Fig.7 shows the absolute reflection coefficient, radiation conductance and susceptance as a function of frequency, and Fig.8 shows the phase of the reflection coefficient for the single radiator.

When the radiating elements are open ended waveguides, because of multiple reflections and mutual coupling effect, and equivalent circuit properties of the dominant mode, the radiating elements reflection coefficient improves and hence the admittance. Therefore it is concluded that radiation properties of waveguide radiator array improves.

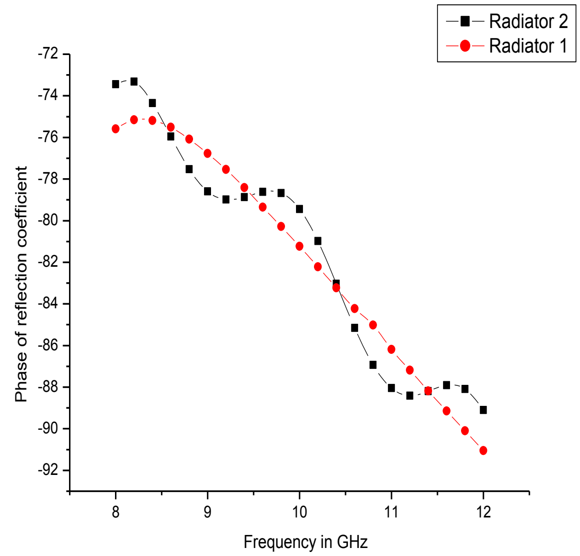

Fig.3 Phase of the Reflection coefficient for the radiating array element

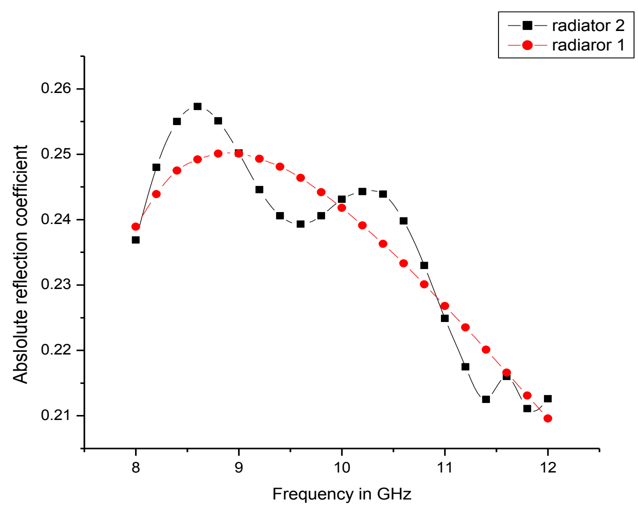

Fig.4 Reflection coefficient (absolute) for the radiating array element

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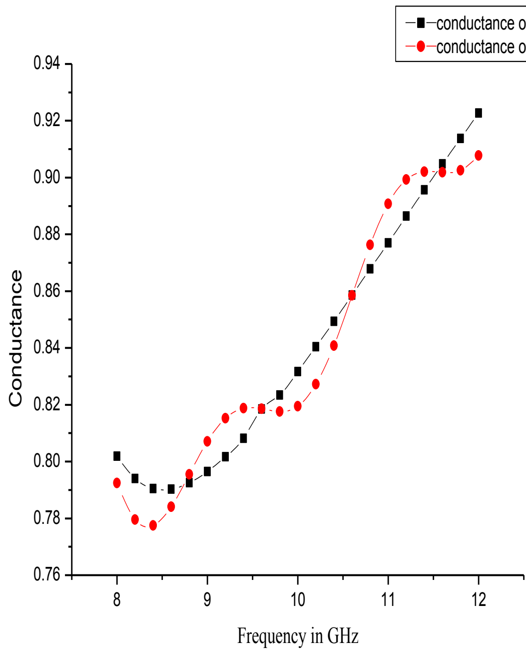

Fig.5 Radiation conductance for array elements

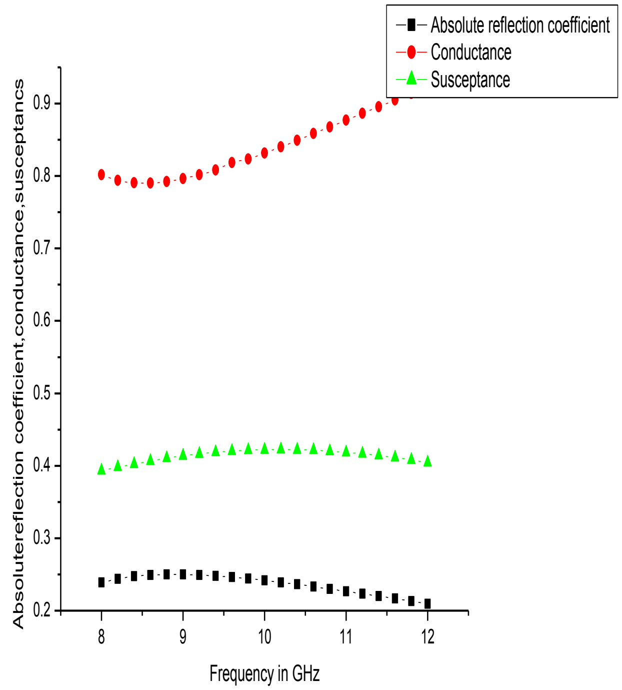

Fig.7 Absolute reflection coefficient, , radiation conductance and susceptance for single radiator

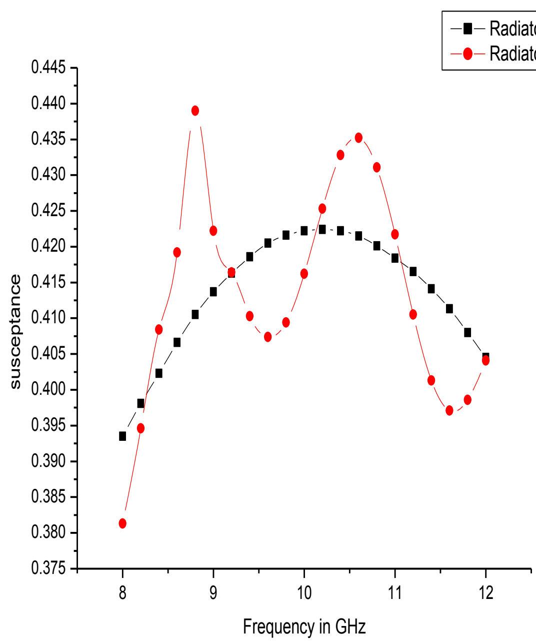

Fig.6 Radiation susceptance for array elements

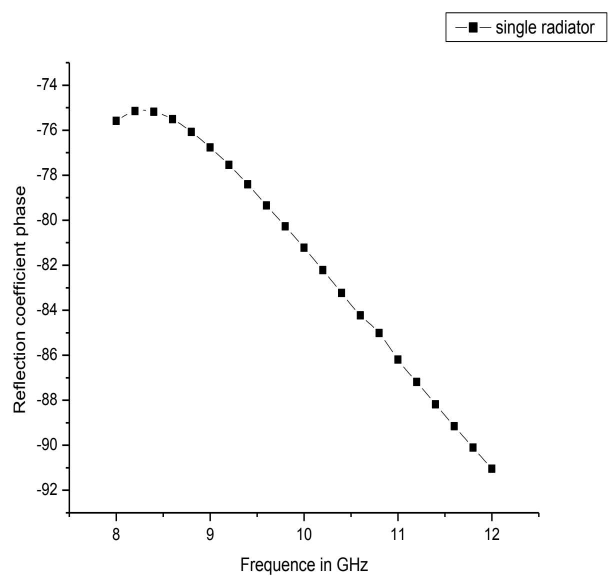

Fig. 8 Reflection Coefficient Phase, for single radiator

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