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PERFORMANCE ANALYSIS OF DIRECTION OF ARRIVAL ESTIMATION ALGORITHMS IN SMART ANTENNAS

T.ESWAR PRASAD K.YASWANTH KRISHNA

DEPARTMENT OF ECE, SIR C R REDDY COLLEGE OF ENGINEERING,ELURU

ABSTRACT:

The goal of this paper is to give a brief overview on smart antennas, before analyzing some popular algorithms from this domain in MATLAB and providing simulation results(especially a parametric study of the algorithms), in order to enhance understanding of the possibilities, but also the limitations of these algorithms. The main focus is on Direction of Arrival (DOA) algorithm called MUSIC and ESPRIT algorithms.

A smart antenna combines antenna arrays with digital signal processing units in order to improve reception and emission radiation patterns dynamically in response to the signal environment. It can increase channel capacity, extend range coverage, steer multiple beams to track many mobiles, compensate aperture distortion or reduce multipath fading and co-channel interference. In estimating DOA,MUSIC deals with the decomposition of covariance matrix into two orthogonal matrices, i.e., signal-subspace and noise-subspace. The ESPRIT algorithm reduces the computational and storage requirements and its main goal is to exploit the rotational invariance in signal subspace.

KEY WORDS:

Smart antenna,SDMA, DOA, MUSIC , ESPRIT .

1.0 INTRODUCTION:

Smart antennas (also known as adaptive array antennas, multiple antennas and, recently) are antenna arrays with smart signal processing algorithms used to identify spatial signal signature such as the direction of arrival (DOA) of the signal, and use it to calculate beamforming vectors, to track and locate the antenna beam on the mobile/target. In actual, antennas are not Smart Antenna, systems are smart. Generally co-located with a base station, a smart antenna system combines an antenna array with a digital signal-processing capability to transmit and receive in an adaptive, spatially sensitive manner.

- ❖ MAIN FUCTIONS OF SMART ANTENNAS:
  - Direction of Arrival(DOA) Estimation
  - Beam Forming

2.0 DIRECTION OF ARRIVAL:

The need for Direction-of-Arrival estimation arises in many engineering applications including wireless communications, radar, radio astronomy, and other emergency assistance devices. It can be defined as the ability to select various frequency components out of a collection of signals. The array based direction of arrival (DOA) estimation techniques considered here can be broadly divided into four different types: conventional techniques, subspace based techniques, maximum likelihood techniques and the integrated techniques which combine property restoral techniques with subspace based techniques.

2.1 Subspace Methods for DOA Estimation:

Though many of classical beam forming based methods such as the Capon's minimum variance method are often successful and widely used, these methods have some fundamental limitations in resolution. Most of

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these limitations arise due to the fact they do not exploit the structure of the narrowband input data model of the measurements. Schmidt and Bienvenu and Kopp were the first to exploit the structure of a more accurate data model for the case of sensor arrays of arbitrary form. Schmidt derived a complete geometric solution to the DOA estimation problem in the absence of noise, and extended the geometric concepts to obtain a reasonable approximation to the solution in the presence of noise. The technique proposed by Schmidt is called the Multiple Signal Classification (MUSIC) algorithm. The geometric concepts upon which MUSIC is founded form the basis for a much broader class of subspace based algorithms. Apart from MUSIC, the primary contributions to the subspace based algorithms include the Estimation of Signal Parameters via Rotational Invariance technique (ESPRIT).

3.0 The MUSIC Algorithm:

MUSIC stands for Multiple Signal Classification, one of the high resolution subspace DOA algorithms, which gives the estimation of number of signals arrived, hence their direction of arrival. MUSIC deals with the decomposition of covariance matrix into two orthogonal matrices, i.e., signal-subspace and noise-subspace. Estimation of DOA is performed from one of these subspaces, assuming that noise in each channel is highly uncorrelated. This makes the covariance matrix diagonal. The popularity of the MUSIC algorithm is in large part due to its generality

3.1 Data Model:

In order to demonstrate the algorithm, a few feasible assumptions must be proposed. First of all the transmission space in MUSIC algorithm is assumed to be isotropic and non-dispersed that means the radiation propagating straight and the signals are considered to be in the far-field of the smart antenna, so that the radiation received by the array elements is in the form of a linear sum of plane waves.Mathematically,assume that the linear combinations of the D incident signals as well as noise are received by the smart antenna with M array elements and $D \leq M$. The received complex matrix X of the smart antenna in the multiple signal classification algorithms can be formulated as:

$$\begin{bmatrix} X1 \\ X2 \\ X3 \\ . \\ . \\ . \\ Xm \end{bmatrix} = [a(\theta_1)a(\theta_2)a(\theta_3)\ldots\ldots a(\theta_m)] \begin{bmatrix} F1 \\ F2 \\ F3 \\ . \\ . \\ . \\ Fm \end{bmatrix} + \begin{bmatrix} W1 \\ W2 \\ W3 \\ . \\ . \\ . \\ Wm \end{bmatrix}$$

Or

$$\mathbf{X = A\ F_i + W}$$

The incident signals are denoted in phase and amplitude at some arbitrary reference point by the complex parameters $F_1, F_2 \ldots F_D$, and appear as the complex vector F_i.

MUSIC is an Eigen structure algorithm and it means that Eigen value decomposition of the estimated covariance matrix **R** is the first step of this algorithm.

$$\mathbf{R=E[XX^T]=E[AF_i\,F_i^{\,T}A^T\ + WW^T]}$$

Here E denotes the expectation value. We can regard the noise as white noise. The factor that the incident signals and the noise are uncorrelated is the basic assumption, so that we can analyze the Eigen values of the covariance matrix and get

$$\mathbf{R=AE[F_iF_i^T]A^T+\lambda_eR_0}$$

Where λ_e is the Eigen value.The incident signals represented by the complex vector F_i may be uncorrelated or may contain completely correlated pairs. Then, $E[F_iF_i^T]$ reflects the degrees of arbitrary in pair-wise correlations occurring among the incident signals and it will be positive definite**.**

As assumed in data modulation section, the number of incident wave fronts D is less than the number of array elements M, so the matrix $AE[F_iF_i^T]A^T$ is singular, and it has a rank less

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than M. Therefore, the determinant of $AE[F_iF_i^T]A^T$ is zero

$$|AE[F_iF_i^T]A^T| = |R-\lambda_e R_0| = 0$$

Only when λ_e equal to one of the Eigen values of R, this above eqution is satisfied. thus λ_e is the Eigen values of R. Definitely, $AE[F_iF_i^T]A^T$ must be nonnegative. And since A is full rank and $E[F_iF_i^T]$ Is positive, λ_e must be the minimum Eigen value denoted as λ_{min}. Then, any measured covariance matrix $R=E[XX^T]$ matrix can be written as:

$$R= AE[F_iF_i^T]A^T + \lambda_{min} R_0 \ , \lambda_{min} \geq 0$$

Where λ_{min} is the smallest solution to $|R-\lambda_e R_0|=0$.

$$\lambda_{min} R_0 = \sigma^2 I$$

Eigen value and Eigen structure are the key points of MUSIC. After decomposition we can get Eigen values of R which directly determine the rank of $AE[F_iF_i^T]A^T$ (it is D). Because of the complete set of Eigen values of R, λ_{min} is not always simple.

Actually, in all cases, the Eigen values of R and those of $|AE[F_iF_i^T]A^T| = |R-\lambda_e R_0|$ differ by λe, so λ_{min} occurs repeated N= M-D times. λ_{min} must occur repeated N times, since the minimum Eigen value of $AE[F_iF_i^T]A^T$ is zero because of being singular. Thus, the number of incident signals sources satisfies

$$D=M-N$$

Where N is the multiplicity of λ_{min} (R,R_0), which means " λ_{min} of R in the metric of R_0".

It is important to know that the Eigen values of R can be subdivided into two parts when the data consist of uncorrelated desired signals corrupted by uncorrelated white noise. The eigenvectors associated with λ_{min} (R,R_0) are perpendicular to the space which is spanned by the columns of the incident signal mode vectors *A*, so it is acceptable that for each of which is λ_i equal to λ_{min} (there are N), we have

$$AE[FiFi^T]A^T * e_i = 0 \text{ or } A * e_i = 0$$

Therefore, we can define the N×M dimensional noise subspace which is spanned by the N noise eigenvectors and the D dimensional signal subspace which is spanned by the D incident signal Eigen vectors. These two subspaces are orthogonal.

Then we can turn to solve for the incident signal vectors, the search for directions is made by scanning steering vectors that are as perpendicular to the noise subspace as possible, once the noise subspace has been estimated.

If E_N is defined to be the M × N dimensional noise subspace whose columns are the N noise eigenvectors, and we use the ordinary Euclidean distance from a vector $a(\theta)$ which is a continuum function of azimuth θ ,to the signal subspace for the judgment standard:

$$d^2 = \vec{a}(\theta)^T E_N E_N^T \vec{a}(\theta)$$

For the convenient of distinction, we use the graph of $1/d^2$ rather than d^2, and define $P_{MU}(\theta)$ that is:

$$P_{MU}(\theta) = 1 / (\vec{a}(\theta)^T E_N E_N^T \vec{a}(\theta))$$

Where $a(\theta)$ does not depend on the data. In this case, we get the DOA by searching for peaks in the $P_{MU}(\theta)$ spectrum. Clearly, R is asymptotically perfectly measured so E_N is asymptotically perfectly measured. Then, it is acceptable that even for multiple incident signals MUSIC is asymptotically unbiased.

The A matrix becomes available to compute other parameters of the incident signals, after finding the directions of arrival (DOA) of the D incident signals.

MUSIC algorithm greatly improves the resolution direction finding, while adapting to the antenna array of arbitrary shape. But the

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prototype of the MUSIC algorithm requirements wave signal is irrelevant.

The steps of the MUSIC algorithm in practice can be shown in summary as:

Step 0:Collect data, form correlation matrix R.
Step 1: Calculate Eigen structure of R in metric of R_0.
Step 2: Decide number of signals D.
Step 3: Choose N columns to form the noise subspace E_N.
Step 4: Evaluate $P_{MU}(\theta)$ versus θ.
Step 5: Pick D peaks of $P_{MU}(\theta)$.

.4.0 ESPRIT ALGORITHM:

Estimation of signal parameters via rotational invariance techniques (ESPRIT) is a computationally efficient and robust method of DOA estimation. It uses two identical arrays in the sense that array elements need to form matched pairs with an identical displacement vector, that is, the second element of each pair ought to be displaced by the same distance and in the same direction relative to the first element.

The Pros/Cons of The MUSIC Algorithm are works for other array shapes, need to know sensor positions and it is very sensitive to sensor position, gain, and phase errors, need careful calibration to make it work well and Searching through all could be computationally expensive.

So, the ESPRIT algorithm overcomes such shortcomings to some degree and it relaxes the calibration task somewhat and it takes much less computation .But ESPRIT takes twice as many sensors.

However, this does not mean that one has to have two separate arrays. The array geometry should be such that the elements could be selected to have this property. Let us consider the two arrays are displaced by the distance d. The way ESPRIT exploits this sub array structure for DOA estimation is now briefly described Based on "doublets" of sensors, i.e., in each pair of sensors the two should be identical, and all doubles should line up completely in the same direction with a displacement vector having magnitude. The positions of the doublets are also arbitrary. This makes calibration a little easier. Assume N sets of doublets, i.e., 2*N* sensors. Assume I sources, N > I and it is represented in the figure(4.1).

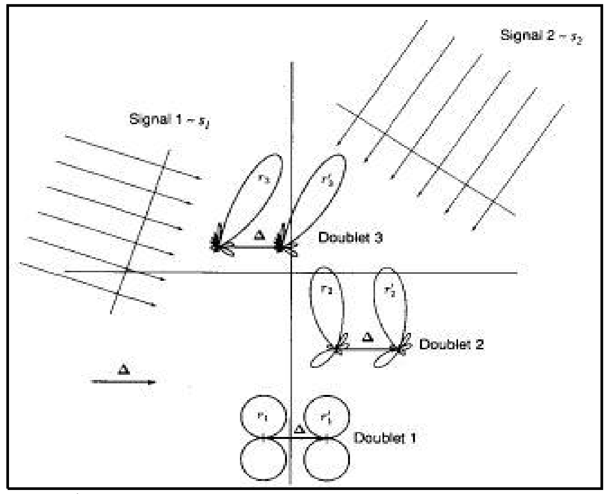

Figure 4.1 **A Sensor Array Of Doublets.**

4.1 Data Model:

Let the signals induced on the l^{th} pair due to a narrowband source in direction θ be denoted by $x_l(t)$ and $y_l(t)$. The phase difference between these two signals depends on the time taken by the plane wave arriving from the source under consideration to travel from one element to the other. Assume that the two elements are separated by the displacement Δ_0.Thus, it follows that

$$\mathbf{y_1(t) = x_1(t)e^{j2\pi\Delta_0 \cos\theta}}$$

Where Δ_0 is measured in wavelengths.

Note that Δ_0 is the magnitude of the displacement vector.. Let the array signals received by the two K-element arrays be denoted by x (t) and y (t). These are given by

$$\mathbf{x(t) = As(t) + n_x(t)} \quad \text{and}$$

$$\mathbf{y(t) = A\varphi s(t) + n_y(t)}$$

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where A is a K × M matrix with its columns denoting the M steering vectors corresponding to M directional sources associated with the first sub array, Φ is an M × M diagonal matrix with its m[th] diagonal element given by ,

$$\phi_{m,m} = e^{j2\pi\Delta_0 \cos\theta}$$

s(t) denotes M source signals induced on a reference element, and $n_x(t)$ and $n_y(t)$, respectively, denote the noise induced on the elements of the two subarrays. Let Ux and Uy denote two K × M matrices with their columns denoting the M eigenvectors corresponding to the largest Eigen values of the two array correlation matrices Rxx and Ryy, respectively.The two matrices Ux and Uy are related by a unique nonsingular transformation matrix ψ, that is,

$$U_x = \psi U_y$$

Similarly, these matrices are related to steering vector matrices A and AΦ by another unique nonsingular transformation matrix T as the same signal subspace is spanned by these steering vectors. Thus,

$U_x = AT$. and,

$U_y = A\phi T$

Substituting for Ux and Uy and the fact that A is of full rank,

$$T\psi T^{-1} = \phi$$

According to this statement, the eigen values of ψ are equal to the diagonal elements of Φ, and columns of T are eigenvectors of ψ.This is the main relationship in the development of ESPRIT.

$$\theta_m = \cos^{-1}\{Arg(\lambda_m)|2\pi\Delta_0\}, m = 1, \dots, M.$$

Other ESPRIT variations include beam space ESPRIT,beam space ESPRIT, resolution enhanced ESPRIT etc., Use of ESPRIT for DOA estimation employing an array at a base station in the reverse link of a mobile communication system has been studied.

5.0 SIMULATION RESULTS:

MATLAB tool has been used for the simulation of MUSIC DOA estimation techniques simulation has been run for the three signals coming from different angles 40, 80,85.

5.1.1 Music spectrum for varying no of elements:

Withconstant100snapshots,stepsize=0.01,SNR =10dB for static case

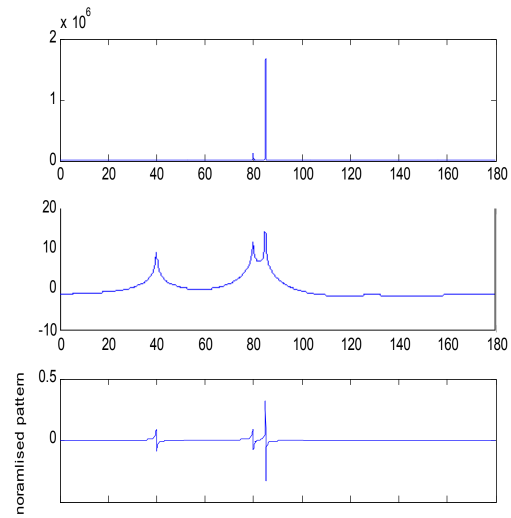

Fig 1 - **MUSIC spectrum for varying no of array elements(M=5)**

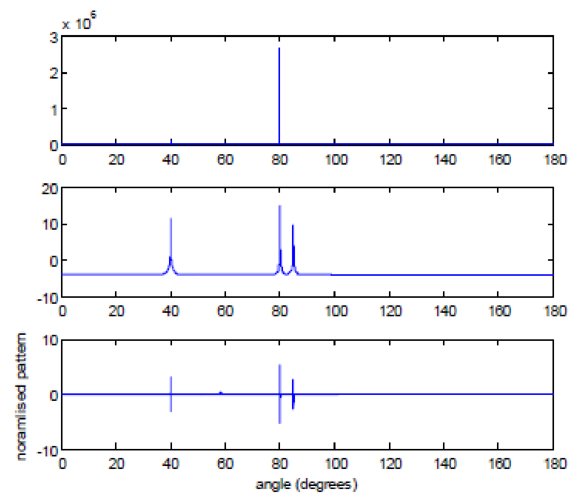

Fig 2- Music spectrum for M=50

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Table 1: Varying no. of array elements

DOA	M=5	M=20	M=50
40	40.0750	40.0350	40.0050
80	79.9250	80.0350	80.0150
85	85.0350	85.0370	85.0150

Table 2:Varying step size

DOA	dθ=0.01	dθ=0.1	dθ=1
40	40.2500	40.3000	45.5000
80	80.1500	80.2500	80.5000
85	85.0500	85.3000	85.5000

Table 3: Varying no. of snap shots

DOA	N=200	N=500	N=1000
40	40.0150	40.0050	40.0050
80	80.0170	80.0150	80.0150
85	85.1250	84.9250	85.0150

Table 4: Varying Signal to Noise ratio (SNR)

DOA	SNR=0dB	SNR=20dB	SNR=50dB
40	40.0150	40.0050	40.0050
80	79.9950	80.0150	80.0150
85	84.8550	85.0350	85.0150

5.2 Simulation Results Of MUSIC Algorithm (Dynamic Case):

Let's now have a look at the simulations for the dynamic case. This time we have M=9 and L=5 and the main user is moving from position 50° to position 90° in 1000 samples which means that we have 100 windows with 10 samples. This corresponds to a speed of 0.04 deg/sample. The result is as shown in fig 3. Here, SNR is 20dB. The spectrum step size is 0.1 and not 0.01 in order to make the simulation faster. Of course; at the moment when the moving user is crossing the user at 80^0 the algorithm will only detect 4 users, because the two crossing users mask each other.

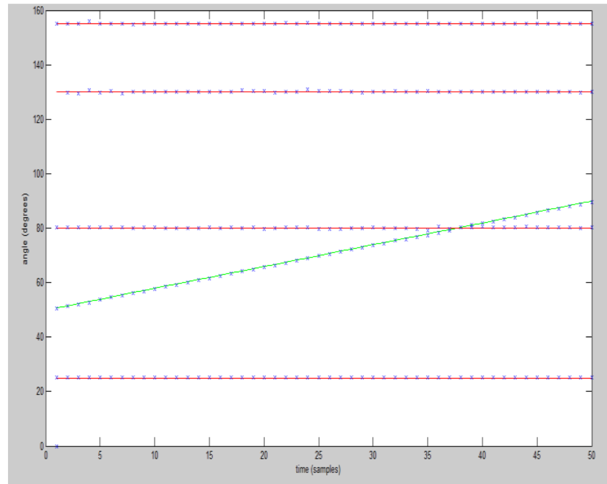

Fig 3: **Real user and calculated user positions for user speed of 0.04 deg/sample in linear scale.**

.5.3 Simulation Results of ESPRIT Algorithm:

By using ESPRIT Algorithm the angle of arrival is determined for M=9 element array where noise variance σ_n^2=.1. Approximate the correlation matrices by again time averaging over K=300 data points. The angles of arrival are θ_1=-$10^{0,}$ θ_2=$10^{0.}$

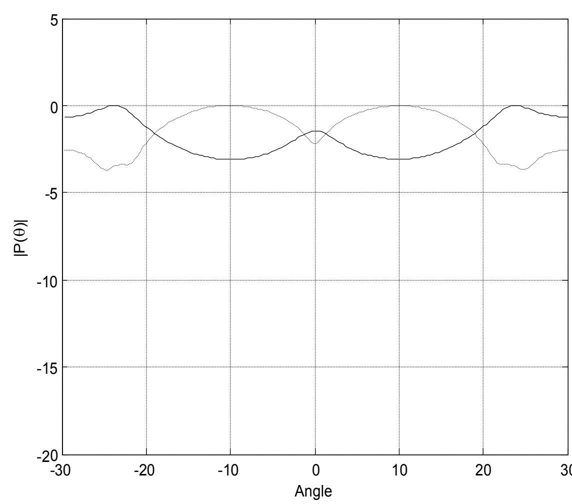

Fig 4: **ESPRIT Spectrum for DOA estimation.**

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ESPRIT is similar to MUSIC in that it correctly exploits the underlying data model. Beyond retaining most of the essential features of the arbitrary array of sensors, ESPRIT achieves a significant reduction in the aforementioned computation and storage costs. This is done by imposing a constraint on the structure of the sensor array to possess displacement invariance, i.e., sensors occur in matched pairs with identical displacement vectors

6.0 CONCLUSION:

In this paper, a novel approach for executing Subspace DOA algorithms have been studied for different source estimations.For more than one source, subspace estimation techniques can be used efficiently. Yet the choice of algorithm depends upon the number of sources to be identified and their properties.

The high resolution MUSIC algorithm is based on a single RF port smart antenna has been proposed. After presenting the configuration and the working principle of the antenna, the performance of the proposed technique for various aspects have been studied. The results have justified that the technique for a high-resolution DOA estimation of 1 degree, which is as good as a conventional MUSIC algorithm. However, ESPRIT has proven to be the most accurate method to be used as DOA algorithm. Estimated DOAs were in close agreement to each other. For radar, DOA estimation is the most important factor to localize targets. For communications, DOA estimation can give spatial diversity to the receiver to enable multi-user scenarios. Simulation studies have also pointed out that further improvements are needed to enhance system performances to make it more applicable to practical wireless communication systems.

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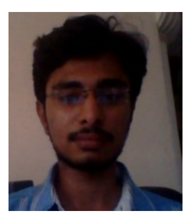

T.Eswar prasad is pursuing the bachelor's degree in electronics communication engineering from SIR C.R. Reddy College of Engineering, Eluru of Andhra University, Visakhapatnam. I have completed my project report in the area of Wireless Communication and Antennas.

K.YaswanthKrishna is pursuing the bachelor's degree in electronics communication engineering from SIR C.R. Reddy College of Engineering, Eluru of Andhra University, Visakhapatnam. I have completed my project report in the area of Wireless Communication and Antennas.

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