

Implementation of CORDIC based DCT on FPGA for SAR Image Compression

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Abstract— At present, the Synthetic Aperture Radar (SAR) technologies are rapidly developing in various countries and enhancing the unique and variable functionality of observing the global earth and its environmental characteristics. The SAR is a form of a Radio Detection and Ranging (RADAR) system which can acquire data with very high resolution. At present, a huge amount of SAR raw data compression is required to store and the purpose of statistical data analysis. In this regard, two encoding techniques are generally used for SAR. First, direct encoding in spatial domain and second, transform encoding in transform domain. In transform domain, we use discrete cosine transform (DCT) for efficient coding cum compressing. The DCT based compression techniques play an important role in today's digital SAR image compression applications. The DCT is a fast transform, perform real operations using low complexity arithmetic and has very good energy compaction. Implementing DCT using CORDIC algorithm reduces the number of computations while using shift-and-add operations during processing. The proposed architecture is simulated and synthesized by using Xilinx's software and the same was implemented on targeted FPGA.

Index Terms— CORDIC, DCT, FPGA, RADAR, SAR, etc...

I. INTRODUCTION

The RADAR means Radio Detection and Ranging which is a communication medium to detect objects which are at a distance that cannot be observed visually. The RADAR provides a capability of detecting objects over great ranges through air at night and in poor weather. The SAR is a form of RADAR whose defining characteristics is its use of relative motion, between an antenna and its target region. Synthetic Aperture Radar is remote sensing system which is used for earth observation. The SAR system has the ability to take images in all weather conditions and darkness. With the improvement of SAR technology larger areas are being imaged and the resolution of the images has increased. This causes larger images to be transmitted and stored [1]. The SAR images carry information in low frequency bands as well as high frequency bands. The SAR images have larger dynamic range and higher entropy. The higher entropy and large dynamic range of SAR images results in very low compression ratio when lossless compression techniques are used. Thus, if a small amount of information loss is

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acceptable lossy techniques can be used. The DCT is one of the best SAR image compression technique.

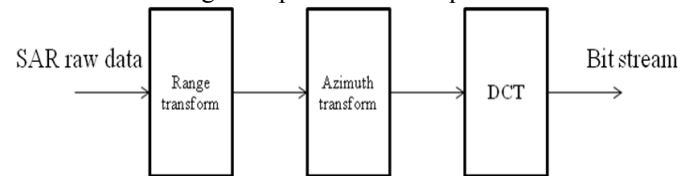


Fig 1. Block diagram of SAR

The block diagram of a SAR data compression by using DCT is shown in figure 1. The transform has been split into two separate processing steps, one is Range transform and another one is Azimuth transform. The main aim of the range transform is to remove the chirp pulse from the collected raw data. This can be done by a matched filter. The range transform can be performed in frequency domain. The Azimuth compression or azimuth focusing involves generation of a frequency-modulated chirp in azimuth based on the knowledge of the spacecraft orbit[2]. In this Section I describes the SAR in RADAR, Section II is mathematical representation of Discrete Cosine transformation and section III explains the design and analysis of CORDIC based DCT algorithm/architecture. Implementation results are shown in Section IV. Finally, Concluded in Section V.

II. MATHEMATICAL REPRESENTATION OF DCT

The DCT was proposed by Ahmed[3], it is fast algorithm, which are classified into non-radix and radix algorithms. The non radix category attempt to reduce computational complexity and more efficient. In addition to above we introduce CORDIC based flexible algorithms. Due to extensive design optimization for cost reduction and performance enhancement, these DCT algorithms are often complicated and hardly scalable to more than 8-point DCTs. In this, we propose a CORDIC based radix-2 fast DCT algorithm. Based on the proposed algorithm, signal flows of DCTs and inverse DCTs (IDCTs) are developed and deduced using their orthogonal properties, respectively. Similar to the Cooley-Tukey fast Fourier transformation (FFT) algorithm, the proposed algorithm can generate the next higher order DCT from two identical lower order DCTs. By using the unfolding CORDIC technique, this algorithm can overcome the problem of difficult to realize pipeline that in conventional CORDIC algorithms. Compared to existing DCTs, the proposed algorithm has computational complexity, and is highly scalable, modular, regular, and able to admit efficient pipelined implementation. For an N-point signal, $x(n)$, the DCT is defined as:

$$\tilde{C}[k] = \alpha(k) \sum_{n=0}^{N-1} x[n] \cos\left[\frac{(2n+1)k\pi}{2N}\right], (k = 0, \dots, N-1) \quad (1)$$

Where $\alpha(0) = 1/\sqrt{N}$ if $k = 0$, and $\alpha(k) = \sqrt{2/N}$ otherwise.

According to (1), neglecting the post-scaling factor without loss of generality, the main operation of an N-point DCT denoted as DCT can be written as:

$$\tilde{C}[k] = \sum_{n=0}^{N-1} x[n] \cos\left[\frac{(2n+1)k\pi}{2N}\right], (k = 0, \dots, N-1) \quad (2)$$

A length- N input sequence $x(n)$, with N is power-of-two, can be decomposed into $x_L(n)$ and $x_H(n)$, which are defined as:

$$x_L[n] = \frac{1}{2} \{x[2n] + x[2n+1]\} \quad (3a)$$

$$x_H[n] = \frac{1}{2} \{x[2n] - x[2n+1]\}; \quad (3b)$$

Where $n = 0, 1, 2, \dots, (N/2) - 1$

So the original signal, $x(n)$, can be obtained from $x_L(n)$ and $x_H(n)$ as follows:

$$x[2n] = x_L[n] + x_H[n] \quad (4a)$$

$$x[2n+1] = x_L[n] - x_H[n] \quad (4b)$$

Substituting (4a) and (4b) into (2), (2) can be rewritten as:

$$\tilde{C}(k) = 2 \cos\left(\frac{\pi k}{2N}\right) \sum_{n=0}^{N/2-1} x_L[n] \cos\left[\frac{(2n+1)k\pi}{N}\right] + \sin\left(\frac{\pi k}{2N}\right) \sum_{n=0}^{N/2-1} x_H[n] \sin\left[\frac{(2n+1)k\pi}{N}\right] \quad (5)$$

Where $k = 0, \dots, N-1$.

Since

$$\cos\left[\frac{(2n+1)(\frac{N}{2}-k)\pi}{N}\right] = \sin\left[\frac{(2n+1)\pi}{N}\right] \sin\left[\frac{(2n+1)k\pi}{N}\right] \quad (6)$$

We get (7) and (8)

$$\tilde{C}(k) = 2 \cos\left(\frac{\pi k}{2N}\right) \sum_{n=0}^{N/2-1} x_L[n] \cos\left[\frac{(2n+1)k\pi}{N}\right] + \sin\left(\frac{\pi k}{2N}\right) \sum_{n=0}^{N/2-1} (-1)^n x_H[n] \cos\left[\frac{(2n+1)(\frac{N}{2}-k)\pi}{N}\right] \quad (7)$$

$$\tilde{C}(N-k) = -2 \sin\left(\frac{\pi k}{2N}\right) \sum_{n=0}^{N/2-1} x_L[n] \cos\left[\frac{(2n+1)k\pi}{N}\right] + \cos\left(\frac{\pi k}{2N}\right) \sum_{n=0}^{N/2-1} (-1)^n x_H[n] \cos\left[\frac{(2n+1)(\frac{N}{2}-k)\pi}{N}\right] \quad (8)$$

Where

$$k = 0, \dots, \frac{N}{2} - 1$$

III. CORDIC BASED DCT

The Coordinate Rotation Digital Computer (CORDIC), provides an iterative solution to perform vector rotations by arbitrary angles using only shifts and adds. The generalized CORDIC is formulated as follows

$$x_{i+1} = x_i - m\sigma_i \cdot 2^{-i} \cdot y_i \quad (9)$$

$$y_{i+1} = y_i + \sigma_i \cdot 2^{-i} \cdot x_i \quad (10)$$

$$z_{i+1} = z_i - \sigma_i \cdot \alpha_i \quad (11)$$

Where

$\sigma_i = \text{sign}(z_i)$ for rotation mode

$\sigma_i = -\text{sign}(y_i)$ for vectoring mode

For $m = 1, 0$ or -1 which is suitable to perform rotations in circular, hyperbolic & linear coordinate system correspondingly and α represents the angle. According to the modes of operation and coordination systems the CORDIC, can be available in six combinations of operations to calculate various computations. These are generally, Circular rotation mode, Circular vectoring mode, Linear rotation mode, Linear vectoring mode, Hyperbolic rotation mode and Hyperbolic vectoring mode. Table 1, shows relevant CORDIC based Computations. In this, 8-point CORDIC based DCT is proposed to design and implemented. According to above equations (7) and (8), we find that each equation has two $N/2$ -point DCT with two different coefficients, and the four coefficient just make one CORDIC. Hence, we combine the above two equations to realize a CORDIC based fast DCT algorithm.

Let,

$$\hat{x}_H[n] = (-1)^n x_H(n) \quad (12)$$

Table.1 Computations using CORDIC algorithm

Operation	Configuration	Initialization	Output
$\cos\theta, \sin\theta, \tan\theta$	Circular rotation mode	$x_0 = 1$ $y_0 = 0$ and $z_0 = \theta$	$x_n = \cos\theta$ $y_n = \sin\theta$
$\cosh\theta, \sinh\theta$ $\tanh\theta, \exp(\theta)$	Hyperbolic rotation mode	$x_0 = 1$ $y_0 = 0$ and $z_0 = \theta$	$x_n = \cosh\theta$ $y_n = \sinh\theta$
$\ln(a), \sqrt{a}$	Hyperbolic vectoring mode	$x_0 = a + 1$ $y_0 = a - 1$ and $z_0 = 0$	$x_n = \sqrt{a}$ $z_n = \frac{1}{2} \ln(a)$
$\arctan(a)$	Circular vectoring mode	$x_0 = a$ $y_0 = 1$ and $z_0 = 0$	$z_n = \arctan(a)$
$\text{division}(b/a)$	Linear vectoring mode	$x_0 = a$ $y_0 = b$ and $z_0 = 0$	$z_n = b/a$
$\text{polar to rectangular}$	Circular rotation mode	$x_0 = R$ $y_0 = 0$ and $z_0 = \theta$	$x_n = R \cos\theta$ $y_n = R \sin\theta$
$\text{rectangular to polar}$ $\tan^{-1}(b/a)$ and $\sqrt{a^2 + b^2}$	Circular vectoring mode	$x_0 = a$ $y_0 = b$ and $z_0 = 0$	$x_n = \sqrt{a^2 + b^2}$ $w_n = \arctan\left(\frac{b}{a}\right)$

Combining the constant values 2 and $\sqrt{2}$ in recursively decomposing stages with the post-scaling factor, the DCT can be written as:

$$C(k) = \frac{1}{\sqrt{N}} \begin{cases} \tilde{C}_L[0], k = 0 \\ \tilde{C}_H[0], k = N/2 \\ \begin{bmatrix} \tilde{C}(k) \\ \tilde{C}(N-k) \end{bmatrix} = \begin{bmatrix} \cos\left(\frac{k\pi}{2N}\right) & \sin\left(\frac{k\pi}{2N}\right) \\ -\sin\left(\frac{k\pi}{2N}\right) & \cos\left(\frac{k\pi}{2N}\right) \end{bmatrix} \\ \begin{bmatrix} \tilde{C}_L[k] \\ \hat{C}_H[N-k] \end{bmatrix}, k = 1, \dots, N/2 \end{cases} \quad (13)$$

According to equation (13), we can decompose the N-point DCT into two $N/2$ point DCTs based on the CORDIC algorithm. For power of two point DCT, the proposed algorithm computes the DCT by recursively decomposing it into 2- point DCT. In addition, the rotation angles of the

CORDICs are arithmetic sequences with a common difference of $-\pi/2N$ another important aspect is that all outputs, $C(k), k = 1, \dots, N/2 - 1$, have the uniform post scaling factor can be merged into negative powers of two in the 2-D DCT, which can be implemented with shifting operations.

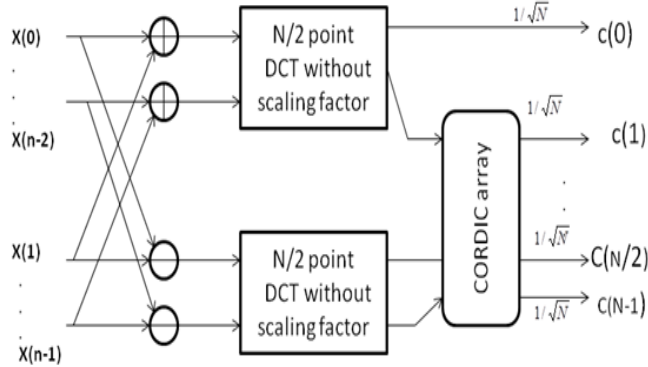


Fig. 2 N-point discrete cosine transformation(DCT)

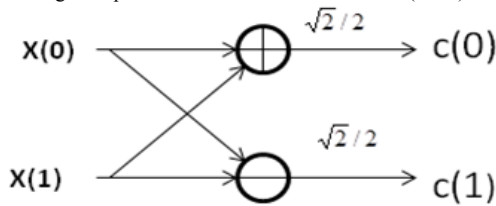


Fig. 3 2-point discrete cosine transformation (DCT)

The general signal-flow graph for the proposed CORDIC based fast DCT algorithm given in (13) is shown in figure 2, while the signal-flow graphs of 2-point DCT, 4-point DCT, and 8-point DCT are respectively represented in figures.2-5, where the angles in the circles are used to represent CORDICs with this rotational angles.

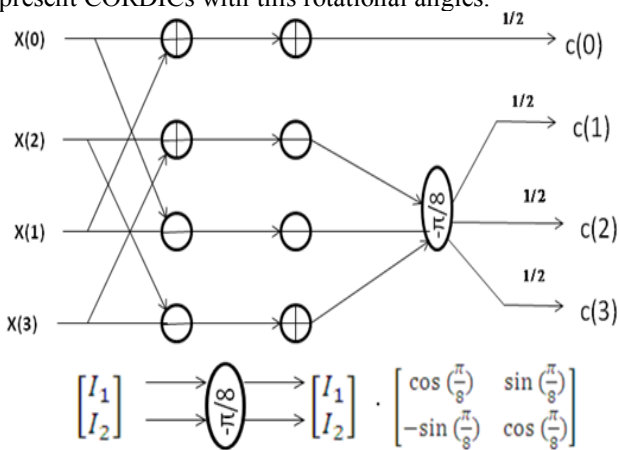


Fig. 4 4-point discrete cosine transformation(DCT).

In figure. 2, there are two separate $N/2$ point DCTs and one CORDIC array. As mentioned above, the CORDIC array has $N-1$ CORDICs with arithmetic-sequence rotational angles. The inputs are addressed in bit-reverse order and the outputs are addressed in natural order. In table 2, the CORDICs in the DCT and IDCT have the same rotation angle but opposite rotation directions. When changing the CORDIC from a clockwise to an anticlockwise rotation with the same angle, the only thing that is required is to change all adders to subtractors, and subtractors to adders in the rotation iteration stage. It is found that, the IDCT algorithm has the same arithmetic complexity as does the DCT one.

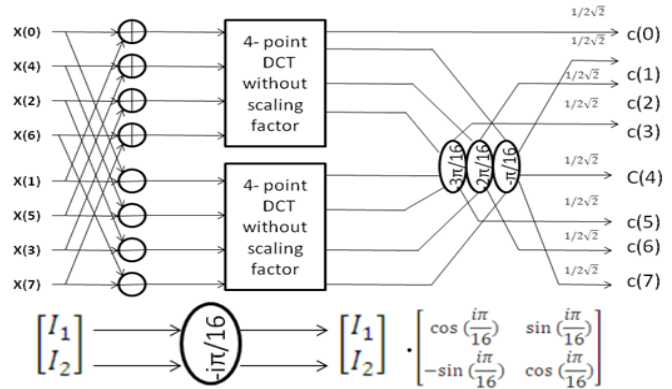


Fig. 5 8-point fast discrete cosine transformation(DCT).

Table. 2 Transfer functions of the DCT and IDCT

Symbol	DCT	IDCT
Butterfly	$x_{out} = x_{in} + y_{in}$ $y_{out} = x_{in} - y_{in}$	$x_{out} = (x_{in} + y_{in})/2$ $y_{out} = (x_{in} - y_{in})/2$
Multiply constant	K	1/K
CORDIC	Clock wise(- θ)	Anti clock wise(- θ)

IV. ANALYSIS AND SIMULATION RESULTS

According to equation (2), it is found that, the computation of DCT involves multiplication of $x(n)$ with resultant value of cosine term. The process will be continued until the k reaches the N . Finally, the computed results are added with n -bit adder. Initially, the cosine value is computed by using CORDIC in circular rotation mode operation. Further, multiplication operation was done by using CORDIC in linear rotation mode. The square root operation is executed by using CORDIC in hyperbolic vectoring mode and inversion operation is done by using CORDIC linear vectoring mode. We have alternative solution for calculation of square root term is simply applying the normalization technique before assigning to the CORDIC. The computation of the $\cos\theta$, $x(n)$ multiplied with $\cos\theta$, and its summation according to above equation are shown figure 6,7, and 8.

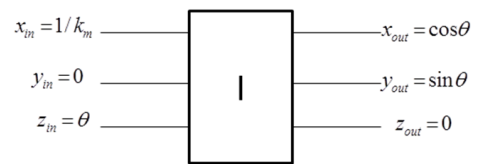


Fig. 6 Circular rotation mode

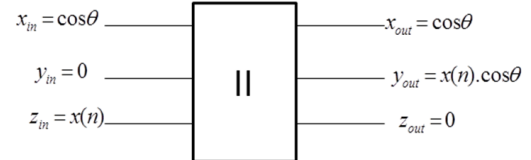


Fig. 7 Linear rotation mode

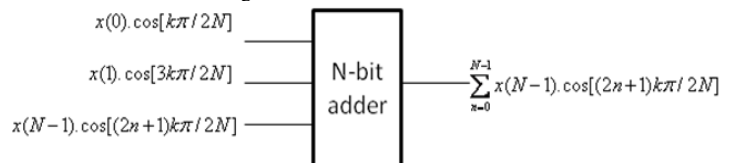


Fig. 8 Block diagram of a N-bit adder

The scaling factor $1/\sqrt{N}$ can be merged in to negative powers of two and it can be implemented by using shifting

operation. The other way of implementing $1/\sqrt{N}$ term using hyperbolic vectoring mode.

Initially the DCT was decomposed in terms of COS and SINE terms by using Euler's formula, then for the computation of these trigonometric components we used pipelined CORDIC processor. For hardware implementation, we developed Verilog code and compiled using ModelSim software. Further simulated and synthesized by using Xilinx ISE design suite version 12.0 and implemented on Spartan 6.0 FPGA. Finally synthesis report and delay report are noted down. From the results it is observed that, the total real time taken for execution is 1.00secs, the total CPU time taken for execution is 0.94 sec.

The macro statistics of CORDIC requires only single ROM, one 4x8-bit ROM, 20-adders and subtractions, three 8-bit adders, one 8-bit subtraction, 32 registers, eight 2-bit registers, 24 8-bit registers and 2 – multiplexers.

Table 3. Comparisons of different algorithms

Methods	Additions	Multiplications	CORDICs
Chen's algorithm	$3N/2(\log_2^N - 1) + 2$	$N \log_2^N - 3N/2 + 4$	-
DCT through DHT	$N/2$	-	$N/2 + 4$
DCT based SDFT	$2 \log_2^N + 2$	-	\log_2^N
CORDIC based DCT	$N \log_2^N$	-	$N/2 \log_2^N - N + 1$

Table 4 4-Point DCT computations

Methods	Additions	Multiplications	CORDICs
Chen's algorithm	20	6	-
DCT through DHT	2	-	6
DCT based SDFT	6	-	2
CORDIC based DCT	4	-	1

Table 5 8-Point DCT computations

Methods	Additions	Multiplications	CORDICs
Chen's algorithm	26	16	-
DCT through DHT	4	-	8
DCT based SDFT	8	-	3
CORDIC based DCT	24	-	5

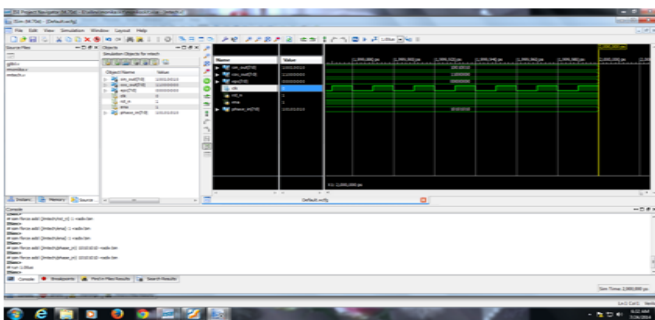


Fig 9 Simulation results

V. CONCLUSION

In this work, we proposed a novel CORDIC-based DCT algorithm. This algorithm can generate the next higher-order DCT from two identical lower-order DCTs. Compared to existing DCT algorithms, our proposed algorithm has several distinct advantages, such as low computational complexity, and being highly scalable, modular, regular, and able to admit efficient pipelined implementation. Furthermore, the proposed algorithm also provides an easy

way to implement a reconfigurable or unified architecture for DCTs and IDCTs, which will be researched in our future work.

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