Performance Comparison of Wavelet based OFDM with Fourier based OFDM under Rayleigh and Nakagami-m Fading Channels

Prashant Sharma, Dr. Neetu Sood
Deptt. of Electronics and Communication Engineering
Dr. B.R Ambedkar National Institute of Technology
Jalandhar 144011, Punjab, India.

Abstract
Next-generation wireless communication systems must fulfill the high data rate demand of intensive applications like multimedia services, data transfer, audio, streaming video etc. This has lead to the demand for future wireless terminals of being capable of connecting to various networks to support services like switched traffic, Internet Protocol (IP) data packets and broadband streaming services. Additionally, due to the growth of Internet applications and wireless users, many Wireless Local Area Network (WLAN) standards have been proposed including IEEE802.11 which permit mobile connectivity to the Internet. With a surging demand for wireless Internet connectivity, new WLAN standards have been developed. As a solution to their requirements for high data rates, all these standards use multicarrier communications, and in most cases, Orthogonal Frequency Division Multiplexing (OFDM) is used.
Increase in demand of higher data rates and need of efficient use of bandwidth exposed few shortcomings of conventional OFDM like inefficient use of bandwidth, overhead of cyclic prefix to prevent Inter Symbol Interference (ISI) and Inter Channel Interference (ICI) due to use of a rectangular window in Fast Fourier Transform (FFT).

Wavelet based OFDM has emerged as a technique that can provide efficient use of bandwidth, prevent ISI without using a cyclic prefix and no ICI as wavelets offer very good spectral containment. In this paper, an orthonormal wavelet basis functions is proposed to be used as the orthogonal signals on which QAM/QPSK data symbols are to be placed. Specifically, the purpose of this paper is the development of a general, unified approach to orthogonally multiplexed communications using wavelet bases and the implementation of these systems using perfect reconstruction digital filter bank and analyze the performance of DWT-OFDM as a new method to achieve a nonuniform partitioning of the data bandwidth in orthogonally multiplexed communications over Rayleigh and Nakagami channels.

Index Terms—**About four key words or phrases in alphabetical order, separated by commas.**

I. INTRODUCTION

OFDM is a multicarrier modulation technique which divides a broadband frequency selective channel into a number of narrowband frequency flat subchannels so that standard QAM/QPSK can be used in each subchannel [1]. As the demand for higher data rate is increasing, some of the shortcomings of conventional FFT based OFDM like loss of achievable data rate and presence of objectionable ICI has been exposed. The reason for former is usage of cyclic prefix to counter ICI and realize circular convolution in time domain while the reason for latter is the usage of raised cosine rectangular window which leads to the formation of more prominent sidelobes.

The power of Wavelet Transform comes from the fact that it provides excellent localization in both time and frequency [6-8, 17]. As compared to complex exponential bases used in conventional FFT based OFDM, wavelet based OFDM uses far more flexible bases which are less susceptible to techniques offer excellent digital filter bank implementation. The filters constituting these filter banks satisfy perfect reconstruction property and are termed as Quadrature Mirror Filters (QMFs). MRA is a framework for constructing orthonormal dyadic wavelet bases [4]. It is a recursive algorithm surfaced for computing the series expansion coefficients of wavelet and scaling functions, it has provided a significant boost to the wavelet analysis as an alternative to Fourier transform for OFDM systems. Section 2 provides the basic theory related to wavelet OFDM like MRA and digital filter bank implementations is explained in this section. A brief review of multipath fading channel (Nakagami) is also presented towards the end. Section 3 provides the simulation setup used in the implementation of the system. Section 4 lists the simulation results in graphical and tabular form and discussions on the results are also made.

All Rights Reserved © 2014 IJARECE

II. Wavelet based OFDM

As opposed to complex exponential basis functions in conventional OFDM, wavelet based OFDM use wavelet based basis functions which provide them the flexibility and immunity.

A basis is a set of linearly independent functions that can be used to produce all admissible functions f(t) [10].

f(t) = combination of basis functions

$$= \sum_{j,k} b_{j,k}\psi_{j,k}(t) \qquad (1)$$

The special feature of wavelet basis is that all functions $\psi_{jk}(t)$ are constructed from a single mother wavelet. Normally it starts at time t=0 and ends at time t=N.

A typical wavelet ψ_{jk} is compressed j times and shifted k times. The formula is [7]

$$\psi_{jk}(t) = \psi(2^j t - k) \qquad (2)$$

The remarkable property that is achieved by many wavelets is orthogonality. The wavelets are orthogonal when their "inner products" are zero [7].

$$\int_{-\infty}^{+\infty} \psi_{jk}(t)\,\psi_{JK}(t)dt=0 \qquad (3)$$

MRA forms a framework for constructing orthonormal wavelet bases. Dyadic scaling and integer translations of a "mother wavelet" provide a sequence of embedded subspaces of $L^2(R)$ denoted V_{2^j} whose element approximations at a given resolution to signals are in $L^2(R)$. Details lost when changing from an approximation of fine resolution to a coarse resolution provide another sequence of embedded subspaces denoted W_{2^j} which forms orthogonal complement of the coarse resolution subspace.

Following points are worth mentioning with regard to MRA- An MRA is a sequence of closed subspaces of $L^2(R)$, denoted V_{2^j} satisfying [4]

1. $V_{2^j} \subset V_{2^{j+1}}$
2. $f(x) \in V_{2^j}$ then $f(2x) \in V_{2^{j+1}}$

W_{2^j} , the orthogonal complement of V_{2^j} in $V_{2^{j+1}}$ satisfies following rules

1. $V_{2^{j+1}} = V_{2^j} \oplus W_{2^j}$
2. $V_{2^j} \perp W_{2^j}$

Let h_n and g_n be the series coefficients of the scaling and wavelet functions in an MRA, then

$$\phi(x) = \sqrt{2}\sum_{n\in I} h_n \phi(2x-n) \qquad (4)$$

$$\psi(x) = \sqrt{2}\sum_{n\in I} g_n \phi(2x-n) \qquad (5)$$

The sequences h_n and g_n form the coefficients of the High Pass and Low Pass filter respectively.

The basic building blocks of analysis and synthesis filter banks can be realized using simple mathematical operations.

$$a_{2^{j+1}}(n) = \sum_{k\in I} h(n-2k)a_{2^j}(k) + \sum_{k\in I} g(n-2k)d_{2^j}(k) \qquad (6)$$

Moreover

$$a_{2^j} = \sum_{k\in I} h(k-2n)\, a_{2^{j+1}}(k) \qquad (7)$$

$$d_{2^j} = \sum_{k\in I} g(k-2n)\, a_{2^{j+1}}(k) \qquad (8)$$

Equation (7) defines one stage of the inverse DWT while equations (8) and (9) define one stage of the DWT. The signal processing operations associated with these equations are illustrated in Figures (1) and (2), respectively.

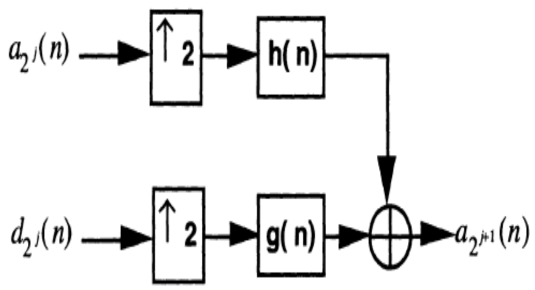

Fig1. One Stage of Reconstruction [4]

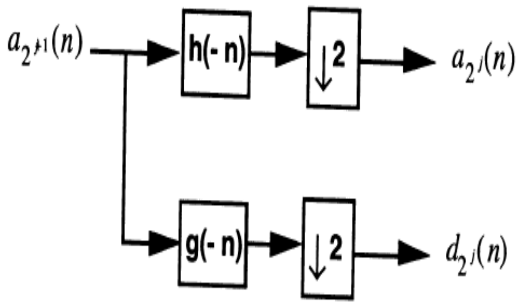

Fig 2. One Stage of Decomposition [4]

II SYSTEM ARCHITECTURE FOR DWT-OFDM

DWT-OFDM is developed using orthonormal dyadic wavelet basis functions as the orthogonal signals on which the QPSK/QAM sequences are placed. In this paper QAM is used as the baseband modulation scheme for modulating narrowband subchannels.

A quadrature amplitude modulated (QAM) digital signal is defined as [1]

$$m(t) = \sqrt{\frac{E}{T}}\sum_{k\in I} d_k\, \phi(\tfrac{t}{T} - k) \qquad (10)$$

Where ϕ is the pulse shape

E is the average symbol energy

d_k are complex-valued QAM symbols

V_1 Vector space can be decomposed into the direct sum-

$$V_1 = V_{2^{-(J-1)}} \oplus W_{2^{-(J-1)}} \oplus \ldots\ldots \oplus W_{2^{-1}} \qquad (11)$$

Where: J is a positive integer.

All Rights Reserved © 2014 IJARECE

It follows that ϕ can be expanded in the basis functions for the subspaces on the right hand side of (11). We now rewrite (10) as the multi dimensional signal [5]

$$m(t) = \sqrt{2^{-(J-1)}\frac{E}{T}}\sum_{j\in I} a_j^0 \phi(2^{-(J-1)}\frac{t}{T} - j) + \sum_{n=1}^{J-1}\sqrt{2^{-(J-n)}\frac{E}{T}}\sum_{j\in I} a_j^n \psi(2^{-(J-n)}\frac{t}{T} - j)$$

(12)

Where: a_j^n are complex-valued QAM symbols for n=0... J-1

Since data has been placed at different scales, (12) can also be referred to as multiscale modulation (MSM). Orthonormality of the individual basis functions prevents ISI. Their mutual orthogonality prevents interference across scales.

To obtain the relationship between x_k and the data symbols we apply equation 6 in a recursive manner. The signal processing is illustrated in Figure 3 and corresponds to a non-uniform synthesis filter bank.

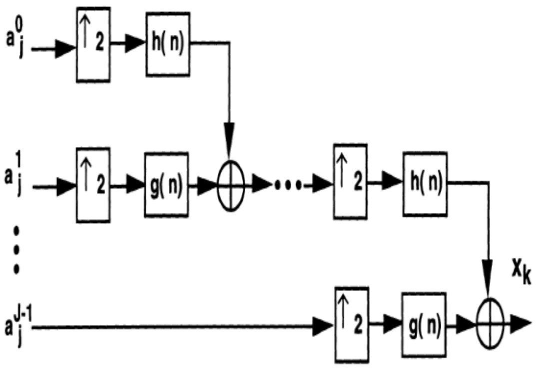

Fig 3. Synthesis Filter Bank of DWT OFDM Transmitter [5]

If it is assumed that r_k is the received sequence and n_k is the sampled Additive White Gaussian Noise (AWGN),

$$r_k = \sqrt{E}x_k + n_k \qquad (13)$$

In DWT-OFDM wavelet basis functions are used as pulse shapes to carry the data as opposed to rectangular pulses used in Fourier-OFDM. This difference leads to excellent spectral containment in DWT-OFDM and removes ICI. We can now recursively apply equations 7 and 8 to the sequence r_k producing estimates of the data symbol [9]. The signal processing is illustrated in Figure 4 and corresponds to a non-uniform analysis filter bank.

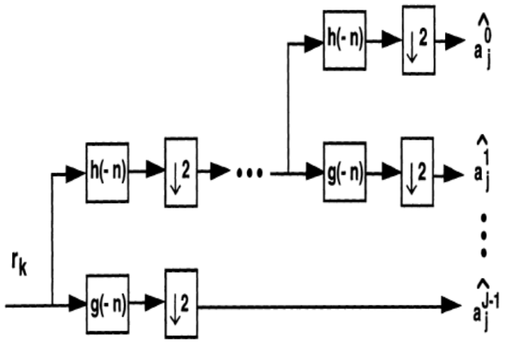

Fig4. Analysis Filter Bank of DWT-OFDM Receiver [5]

2.1.1. Nakagami-m Fading Channel: It is possible to describe both Rayleigh and Rician fading with the help of a single model using the Nakagami distribution. The fading model for the received signal envelope, proposed by Nakagami, has the PDF given by [2]

$$f(r) = \frac{2m^m r^{2m-1}}{\Gamma(m)\Omega^m} exp\left\{-\frac{mr^2}{\Omega}\right\}, \quad r \geq 0$$

(14)

Where (m) is the Gamma function and m is the shape factor (with the constraint that m ≥ ½) given by [2]

$$m = \frac{E^2\{r^2\}}{E\{[r^2 - E(r^2)]^2\}} \qquad (15)$$

In the special case m = 1, Nakagami reduces to Rayleigh distribution. For m > 1, the fluctuations of the signal strength reduce compared to Rayleigh fading, and Nakagami tends to Rician.

IV. SIMULATION SETUP

Performance analysis of DWT-OFDM over Rayleigh and Nakagami fading channels is performed with comparison made on the basis of the BER performance of the models that have been simulated using MATLAB. Monte Carlo technique of BER calculation for DWT-OFDM is followed for simulation.

Table 1 lists the parameters that have been taken for the purpose of simulation.

Table 1. Simulation Parameters

PARAMETERS	**FFT-OFDM**	**DWT-OFDM**
N(Number of Channels)	30	30
Size of FFT	64	NA
Decomposition levels	NA	6
Ns(Bits per sample)	100	100
Modulation	QAM	QAM
Wavelet	NA	Sym, Haar, db
Channel	AWGN+Rayleigh,AWGN+Nakagami m	

All Rights Reserved © 2014 IJARECE

From Figure 5, it is obvious that the transmitter first uses a QAM digital modulator which maps the serial bits *d* into the OFDM symbols X_m, within *N* parallel data stream $X_m(i)$ where $0 \le i \le N-1$.

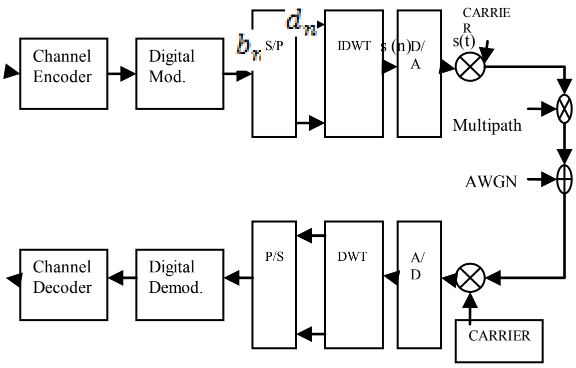

Fig5. Simulation Model for DWT-OFDM

The main task of the transmitter is to perform the discrete wavelet modulation by constructing orthonormal wavelets. As shown in Figure 6, each $X_m(i)$ is first converted to serial representation having a vector *xx* which will next be transposed into *CA*. This means that *CA* not only its imaginary part has inverting signs but also its form is changed to a parallel matrix. Then, the signal is up-sampled and filtered by the LPF coefficients or namely as approximated coefficients. These coefficients are also called scaling coefficients. Since our aim is to have low frequency signals, the modulated signals *xx* perform circular convolution with LPF filter whereas the HPF filter also perform the convolution with zeroes padding signals *CD* respectively. Note that the HPF filter contains detailed coefficients or wavelet coefficients and LPF filter contains approximated coefficients or scaling coefficients.

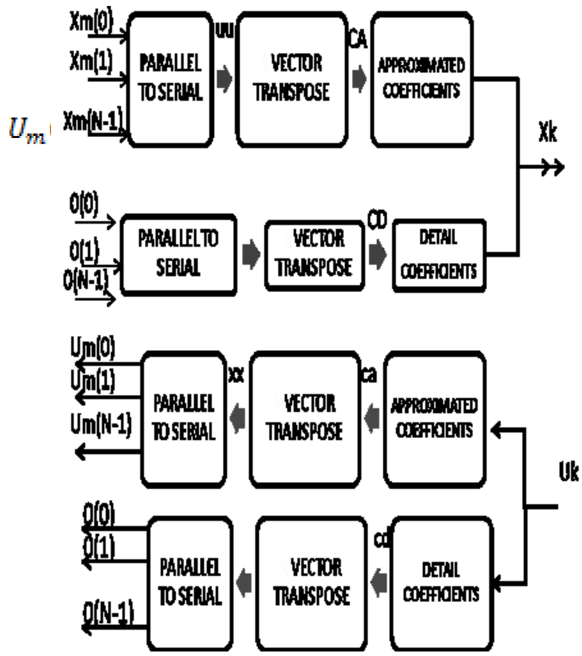

Fig6. Model for IDWT/DWT blocks in DWT-OFDM

The number of *CA* and *CD* depends on the OFDM subcarriers *N*. These models have been developed for ideal conditions and there is no need for synchronization.

4. SIMULATION RESULTS

In this section performances of DWT-OFDM systems have been compared with different wavelets families like symlet, Haar and daubechies. Wireless channels used in simulation are Rayleigh and Nakagami-m.

4.1. Under Rayleigh Fading channel

In this subsection BER performance of DWT-OFDM and FFT-OFDM are compared, the modulation format is QAM and the wavelets used are symlet and haar. MC simulation technique is used

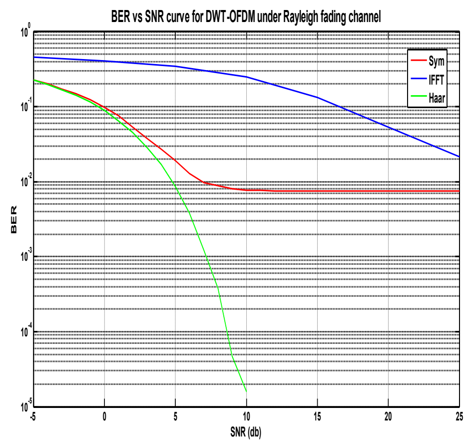

Fig. 7 BER vs SNR curve for DWT-OFDM and FFT-OFDM under Rayleigh Fading channel

All Rights Reserved © 2014 IJARECE

Table. 2 BER vs SNR values for DWT and FFT-OFDM under Rayleigh channel

SNR (db)	0	5	10	15	20	25
FFT OFDM	4.078 e-01	3.450e -01	2.521e -01	1.334e -01	5.360e -02	2.162e -02
DWT OFDM (sym)	9.690 e-02	1.901e -02	7.668e -03	7.452e -03	7.452e -03	7.452e -03
DWT-OFDM (haar)	8.906 e-02	8.672e -03	1.563e -05	6.724e -07	2.342e -08	1.894e -08

Fig. 7 show that DWT-OFDM with haar wavelet filter is superior then DWT-OFDM with symlet wavelet filter, Table 2 shows an SNR improvement of about 2 dB at 0.01 BER for symlet based DWT-OFDM over FFT-OFDM. Haar based DWT-OFDM outperforms FFT-OFDM by about 15 dB at 0.02 BER. The reason for very low BER in case of Haar wavelet is that Haar wavelet offers excellent time and spectral containment but unfortunately it is discontinuous hence it cannot be used for practical purposes.

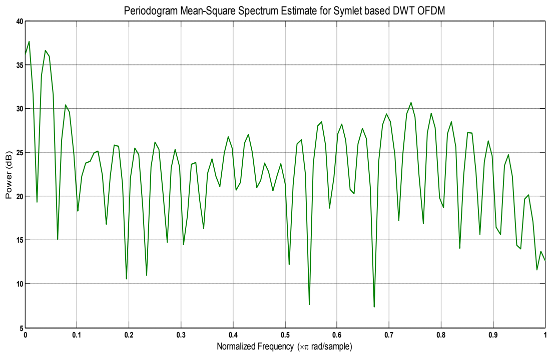

Fig. 8 Power Spectrum of Symlet based DWT OFDM in Rayleigh Channel

When the performance under narrowband interference is investigated, the results show that the advantage of higher sidelobe suppression is present but the frequency behavior of the wavelets is however very complicated, and makes optimization using the time-frequency tiling a rather unrealistic task.

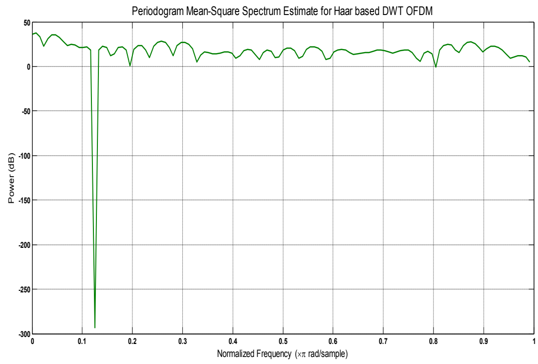

Fig. 9 Power Spectrum of Haar based DWT OFDM

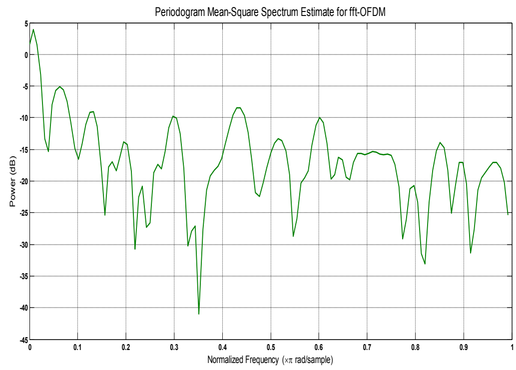

Fig. 10 Power Spectrum of Haar based DWT OFDM

4.3 Under Nakagami-m Fading channel

BER curves for FFT-OFDM and DWT-OFDM in Nakagami-m fading channel have been compared. Parameter m has been taken 1.5. Wavelets considered for DWT-OFDM are Haar and Symlet.

In this subsection BER performance of DWT-OFDM and FFT-OFDM are compared, the modulation format is QAM and the wavelets used are symlet and haar.

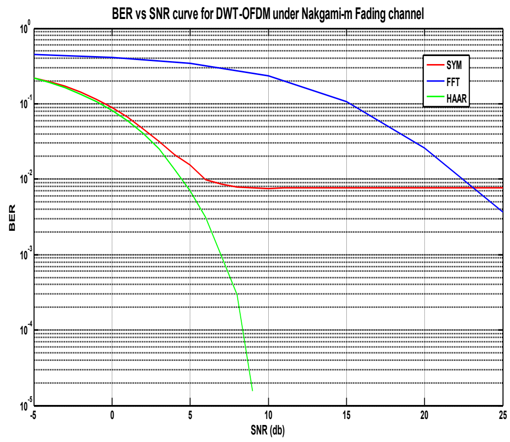

Fig 11. BER vs. SNR curve for DWT-OFDM and FFT-OFDM under Nakagami-m Fading channel

All Rights Reserved © 2014 IJARECE

Table 3. BER vs. SNR values for DWT and FFT-OFDM under Nakgami-m channel

SNR(db)	0	5	10	15	20	25
FFT-OFDM	4.075e-01	3.437e-01	2.308e-01	1.082e-01	2.572e-02	4.040e-03
DWT-OFDM (sym)	8.837e-02	1.418e-02	7.452e-03	7.572e-03	7.596e-03	7.452e-03
DWT-OFDM (haar)	8.348e-02	6.281e-03	1.563e-05	7.914e-07	2.822e-08	2.097e-08

Figure 11 shows that DWT-OFDM with haar wavelet filter is superior then DWT-OFDM with symlet wavelet filter. Table 3 shows an SNR improvement of about 2 dB at 0.01 BER. Symlet based DWT-OFDM outperforms FFT by about 15 dB at 0.02 BER.

5. CONCLUSION

This paper presents a unified approach to orthogonally multiplexed communication using wavelets. In particular, we first used orthonormal dyadic wavelet functions or DWT-OFDM. This waveform achieves a non uniform partitioning of the data bandwidth. It was then shown how DWT-OFDM provided an efficient digital implementation. The modified model of DWT-OFDM was simulated using MC technique; comparison was made on the basis of BER vs. SNR curves which proved that DWT-OFDM is better than conventional OFDM with Haar based DWT giving the best results showing an improvement of 15 db at 2×10^{-2} BER. This paper has analyzed the scope of a new modulation format and has successfully applied it to a pertinent application and different fading channels- Rayleigh and Nakagami-m.

6. Acknowledgement

In the last but not the least, I also want to thank Dr. Arun Khosla (Head, Department of Electronics and Communication Engineering) for provide me the opportunity to undergo this work. I am short of words for expressing my utmost sincere and respectful gratitude to all the faculty members of ECE department for their constant support and guidance.

REFERENCES

[1] J.Proakis, "Digital Communications", *Tata McGraw Hill* **(1983).**

[2] A. Goldsmith, "Wireless Communication", *Cambridge University Press* **(2005)**.

[3] G. L. Stuber, "Principles of Mobile Communication, Second Edition", *Springer Science and business media, inc.* **(2002)**.

[4] Stephane G. Mallat, "A Theory for Multiresolution Signal Decomposition: The Wavelet Representation" *IEEE Transactions on Pattern Analysis and Machine Intelligence, Vol. 11, No. 7* **(July 1989)**

[5] A. Lindsay, "Wavelet Packet Modulation for Orthogonally Transmultiplexed Communications," *IEEE Transactions on Signal Processing, Vol. 45, pp. 1336-1339* **(May 1997).**

[6] C. Van Bouwel, J. Potemans, S. Schepers, B. Nauwelaers, A. Van de Capelle, K.U.Leuven, "Wavelet Packet Based Multicarrier Modulation," *IEEE Transactions on Signal Processing, Vol. 51, pp. 1263-1288* **(April 1998)**.

[7] I. Daubechies, "Ten Lectures on Wavelets", *Academic Press*, **(1992).**

[8] M. K. Lakshmanan, H. Nikookar, "A review of wavelets for digital wireless communication", *Wireless Personal Communications, Springer Netherlands, vol. 37, No. 3-4, pp. 387-420*, **(May 2006)**.

[9] S. Baig, F. U. Rehman, and M. J. Mughal, "Performance comparison of DFT, discrete wavelet packet and wavelet transforms, in an OFDM transceiver for multipath fading channel", *Proceedings of 9th International Multitopic Conference, INMIC'05*, **(2005)** 1-6, September, Qatar.

[10] M.K. Lakshmanan and H. Nikookar, "Construction of Optimal Bases for Wavelet Packet Modulation under Phase Offset Errors", *3rd European Wireless Technology Conference,* **(2010)** September, *Paris, France*.

All Rights Reserved © 2014 IJARECE