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Pilot-Based Channel Estimation Techniques in OFDM Systems

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Abstract—**Orthogonal frequency division multiplexing (OFDM) is an efficient multi-carrier modulation. For OFDM systems, channel estimation and tracking must be performed since the receiver requires channel state information for decoding. In this paper, block-type and comb-type channel estimation algorithms for OFDM systems over multipath fading channels are studied and simulated. Performance results using simulated frequency-selective channels are presented.**

Index Terms— **Channel estimation,LS, MMSE, OFDM.**

I. INTRODUCTION

The Orthogonal Frequency Division Multiplexing (OFDM) technique is considered to be a highly suitable technique for high-speed wireless transmissions with high bandwidth efficiency that is also robust to multipath delays. Compared to traditional single carrier techniques, OFDM has also a low complexity implementation for high-speed systems. OFDM signals can be demodulated either coherently or differentially coherent manners. The most important advantage of differential demodulation is not requiring channel information, and the receiver is relatively simple. However, compared with coherent demodulation, system performance will be degraded from 3 to 4 dB [1]. OFDM system channel estimation methods can be divided into two ways, pilot-based channel estimation and blind channel estimation. The pilot channel estimation methods are based on the pilot channel and pilot symbol. However, due to two-dimensional time-frequency structure of OFDM system, pilot symbol assisted modulation (PSAM) is more flexible [4]. PSAM is the method doing channel estimation by using pilot sequence and symbol, which are inserted into some fixed positions of signals sent by transmitter. The pilot symbol sent by transmitter makes spectral efficiency and power utilization lower with the trade-off of quick response to the channel variation. Blind channel estimation is focusing on the correlation between the data sent and received, without knowing the information of the transmitted data. Although it yields higher spectral and power efficiencies by using blind channel estimation, it needs more data to analyze. Hence it is suitable for slow varying channel [8]. This paper is concentrated on PSAM.

II. OFDM SYSTEM MODEL

The main idea of Orthogonal Frequency Division Multiplexing (OFDM) transmission is to turn the channel convolutional effect into multiplicative one. The complete base band OFDM system model is shown in Figure 1 and described.

The binary information is first grouped and mapped according to the modulation in “signal mapper”. After inserting pilots either to all sub-carriers with a specific period or uniformly between the information data sequence, IDFT block is used to transform the data sequence of length N {X(k)} into time domain signal {x(n)} with the following equation

$$x(n) = IDFT\{X(k)\} = \sum_{\mathrm{k}=0}^{N-1} X(k) e^{-j2\pi\frac{kn}{\mathrm{N}}} \qquad (1)$$

where N is the DFT length. Following IDFT block, guard time, which is chosen to be larger than the expected delay spread, is inserted to prevent inter-symbol interference. This guard time includes the cyclically extended part of OFDM symbol in order to eliminate inter-carrier interference (ICI). The resultant OFDM symbol is given as follow

$$x_f(n) = \begin{cases} x(N+n), & n = -N_g, \dots, -N_g + 1 \\ x(n), & n = 0,1,\dots,N-1 \end{cases} \qquad (2)$$

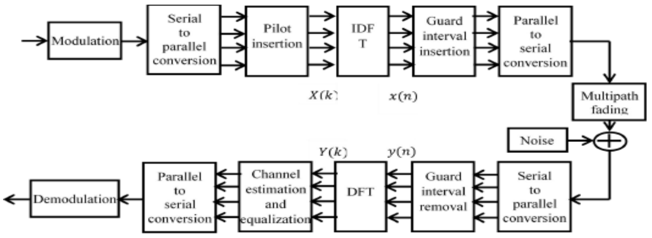

Fig. 1 OFDM baseband model

where N_g is the length of the guard interval. After following D/A converter, this signal will be sent from the transmitter with the assumption of the baseband system model. The transmitted signal$x_f$will pass through the frequency selective time varying fading channel with additive noise. The received signal $y_f$is given by:

$$y_f(n) = x_f(n) * h(n) + W(n) \qquad (3)$$

where *w(n)*is additive white Gaussian noise and *H(n)*is the channel impulse response, which is equal to

$$H(n) = \sum_{i=0}^{r-1} h_i e^{\frac{j2\pi f_{D_i} Tn}{N}} \delta(\lambda - \tau_{\_}) \quad 0 < n < N-1 \qquad (4)$$

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where r is the total number of propagation paths, $h_i$is the complex impulse response of the i^{th}path, f_{D_i} is the ith path Doppler frequency shift, λ is delay spread index, T is the sample period and τ_i is the ith path delay normalized by the sampling time. At the receiver, after passing to discrete domain through A/D and low pass filter, guard time is removed:

$$y(n) = y_f\left(n + N_g\right) \qquad n = 1,2,\dots,N-1 \qquad (5)$$

Then y(n) is sent to DFT block for the following operation Assuming there is no ISI, then

$$Y(k) = DFT\{y(n)\} \qquad (6)$$

III. PILOT-BASED CHANNEL ESTIMATION ALGORITHMS

A. Types of Pilot

In order to obtain the channel information, pilot symbols are inserted in the information from transmitter, and the receiver get the channel information by using pilot symbols received. In essence, the problem of pilot pattern design is to determine where to insert the pilot and how closely between pilots. A suitable way of inserting could be calculated according to the known communication environment and estimated speed from the terminal.

Therefore, the most two important parameters of pilot, maximum Doppler shift f_m which determines the minimum coherent time, and maximum multipath time delay τ_{max} that decides the minimum coherent bandwidth, should be discussed [3]-[5].

The fading channel of the OFDM system can be viewed as a 2D lattice in a time-frequency plane, because signal is transmitted in the fixed position. And the 2D sampling should satisfy the Nyquist sampling theorem in order to eliminate the distortion. So the minimum limit of pilot symbols inserted is decided by Nyquist theorem. From Nyquist theorem, the interval of time domain N_t and frequency domain N_f should satisfy [6]

$$f_m . T . N_t \leq \frac{1}{2}, \qquad \tau_{max} . \Delta f \, . N_f \leq \frac{1}{2} \qquad (7)$$

where Δf is bandwidth of sub-carrier and T is period of signal.
For practical application, oversampling with 2 times of the minimum is used. Hence,

$$f_m . T . N_t \approx \frac{1}{4}, \qquad \tau_{max} . \Delta f \, . N_f \approx \frac{1}{4} \qquad (8)$$

The two basic channel estimations in OFDM systems, block-type pilot and comb-type pilot, are illustrated in Fig. 2.

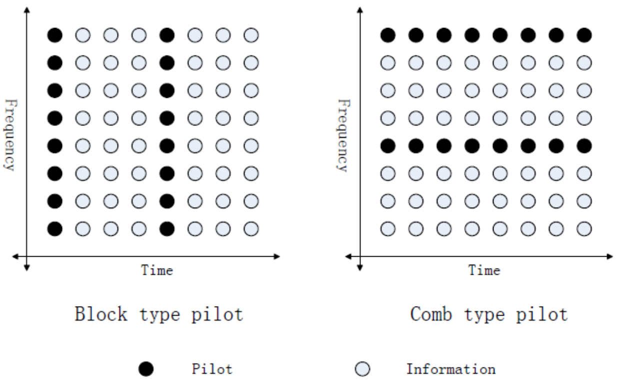

Fig. 2 Block-type pilot and comb-type pilot.

The first one, block-type pilot channel estimation, is performed by inserting pilot tones into all subcarriers of OFDM symbols with a specific period in time. The pilot symbols, because covering all frequencies, could be effective against the selective frequency fading, but more sensitive for the impact of fast fading channel. Therefore, the block-type pilot is developed under the assumption of slow fading channel. In case of same number of pilots, the performance is decided by channel change rate, known as coherent time.

The second one, comb-type pilot channel estimation, is performed by inserting pilot tones into certain subcarriers of each OFDM symbol, where the interpolation is needed to estimate the conditions of date subcarriers. Compared with the block-type pilot, because pilot symbols are inserted into subcarriers with same interval, comb-type pilot channel estimation is introduced to satisfy the need for equalizing when the channel changes even from one OFDM block to the subsequent one. In case of same number of pilots, the performance is decided by channel multipath time delay, known as coherent bandwidth.

B. Block Type Pilot-Based Channel Estimation

1) Least Square (LS) Estimator

If there is no ISI, the signal received is written as

$$\boldsymbol{Y} = \boldsymbol{XF\hat{h}} + \boldsymbol{W} \qquad (9)$$

where $\boldsymbol{Y}$ is the vector of output signal after OFDM demodulation as $\boldsymbol{Y} = [Y_0, Y_1, \dots, Y_{N-1}]^{\boldsymbol{T}}$, $^{\boldsymbol{T}}$is transpose, $\boldsymbol{X}$ is the diagonal matrix of pilots as $\boldsymbol{X} = diag\{X_0, X_1, \dots, X_{N-1}\}$, N is the number of pilots in one OFDM symbol, $\boldsymbol{\hat{h}}$ is the impulse response of the pilots of one OFDM symbol, and W is the channel noise which assumed to be Additive White Gaussian Noise (AWGN). Also F is the Fourier transfer matrix as below,

$$\boldsymbol{F} = \begin{bmatrix} W_N^{00} & \cdots & W_N^{0(N-1)} \\ \vdots & \ddots & \vdots \\ W_N^{(N-1)0} & \cdots & W_N^{(N-1)(N-1)} \end{bmatrix} \qquad (10)$$

Where $W_N^{i,k} = \frac{1}{\sqrt{N}} e^{-j2\pi\left(\frac{ik}{N}\right)}$
The cost function of LS algorithm is written as,

$$\begin{aligned} J &= \left|Y - XF\hat{h}\right|^2 \\ &= \left(Y - XF\hat{h}\right)^H \left(Y - XF\hat{h}\right) \qquad (11) \\ &= Y^H Y - Y^H XF\hat{h} - X^H Y F^H \hat{h}^H + X^H F^H \hat{h}^H XF\hat{h} \end{aligned}$$

where ^{H} denotes conjugate transpose.
the purpose of LS algorithm [2] is to minimize the cost function J without noise. For the minimization of J, let

$$\frac{\partial J}{\partial \hat{h}^H} = 0.$$

Then, from (11),

$$\begin{aligned} \frac{\partial J}{\partial \hat{h}^H} = 0 - 0 - \frac{\partial J}{\partial \hat{h}^H}\left(X^H Y F^H \hat{h}^H\right) \\ + \frac{\partial J}{\partial \hat{h}^H}\left(X^H F^H \hat{h}^H XF\hat{h}\right) \end{aligned} \qquad (12)$$

$$= -F^H X^H Y + F^H X^H X F \hat{h} = 0$$

Then we could get

$$\hat{h}_{LS} = (F^H X^H X F)^{-1} F^H X^H Y = F^{-1} X^{-1} Y \qquad (13)$$

Because $\hat{H} = F\hat{h}$, where $\hat{H}$ is the impulse response of the channel,

$$\hat{H}_{LS} = X^{-1} Y = \left[\frac{Y_0}{X_0}\frac{Y_1}{X_1} \cdots \frac{Y_{N-1}}{X_{N-1}}\right]^T \qquad (14)$$

The advantage of LS algorithm is its simplicity, because no consideration of noise and ICI. So, without using any knowledge of the statistics of the channels, the LS estimators are calculated with very low complexity, but obviously it suffers from a high MSE. LS method, in general, is utilized to get initial channel estimates at the pilot subcarriers, which are then further improved via different methods.

2) Minimum Mean Square Error (MMSE) Estimator

The MMSE estimator employs the second-order statistics of the channel conditions to minimize the MSE.Let us denote the error of channel estimation e as

$$e = H - \hat{H} \qquad (15)$$

Where H is actual channel estimation and $\hat{H}$is raw channel estimation, respectively, and the MSE of channel estimation is

$$\begin{aligned} E\{|e|^2\} &= E\left\{\left|H - \hat{H}\right|^2\right\} \\ &= E\left\{\left(H - \hat{H}\right)\left(H - \hat{H}\right)^H\right\} \end{aligned} \qquad (16)$$

where $E\{\ \}$ is the expectation.
Since the channel and AWGN are not correlated, we can rewrite (15) as

$$\hat{H}_{MMSE} = R_{HY} R_{YY}^{-1}\, Y \qquad (17)$$

Let us denote the auto-covariance matrixes of H, Y by R_{HH} , R_{YY}respectively, and cross covariance matrix between H and Y by R_{HY}. Let σ_N^2 is the noise-variance, since the channel and AWGN are not correlated, we could get

$$\begin{aligned} R_{HY} &= E\{HY^H\} \qquad (18) \\ &= E\{H(HX + W)^H\} \\ &= E\{HH^H X^H + HW^H\} \\ &= E\{HH^H\}X^H + 0 \\ &= R_{HH} X^H \end{aligned}$$

$$\begin{aligned} R_{YY} &= E\{YY^H\} \\ &= E\{(HX + W)(HX + W)^H\} \\ &= E\{HXH^H X^H + HXW + WH^H X^H + WW^H\} \qquad (19) \\ &= E\{HH^H\}XX^H + 0 + 0 + E\{WW^H\} \\ &= XR_{HH}X^H + \sigma_N^2 I_N \end{aligned}$$

If R_{HH} and $\sigma_N^2$are known to the receiver, channel impulse response could be calculated by MMSE estimator as below

$$\begin{aligned} \hat{H}_{MMSE} &= R_{HY} R_{YY}^{-1}\, Y \\ &= R_{HY} X^H (X R_{HH} X^H + \sigma_N^2 I_N)^{-1} X \hat{H}_{LS} \\ &= R_{HH}(R_{HH} + \sigma_N^2 (X^H X)^{-1})^{-1} \hat{H}_{LS} \end{aligned} \qquad (20)$$

The performance of MMSE estimator is much better than LS estimator, especially under the lower E_b/N_o. However, because of the required matrix inversions, the computation is very complex when the number of subcarriers of OFDM system increases. Therefore, an important drawback of the MMSE estimator can be the high computational complexity.

3) Linear Minimum Mean Square Error Estimator

From the $\hat{H}_{MMSE}$ in (20), we could find that the channel estimator need to get the inverse matrix of $R_{HH}X^H + \sigma_N^2(X^H X)^{-1}$. Because $(X^H X)^{-1}$are not the same in different OFDM symbols, its inverse matrix should be updated every time for the different OFDM symbols, which needs much computation. A simplification of MMSE estimator is to replace the $(X^H X)^{-1}$by its expectation $E\{(X^H X)^{-1}\}$, which means the average power of all subcarriers replace the instantaneous power of each subcarrier in order to reduce the computation, since matrix inversion of $R_{HH}X^H + \sigma_N^2(X^H X)^{-1}$ is no longer needed [10],[11]. Assuming the same signal constellation on all tones and equal probability on all constellation points, we get

$$E\{(X^H X)^{-1}\} = E\left\{\frac{1}{|X_k|^2}\right\} I \qquad (21)$$

where I is the identity matrix.
Let the average of SNR is

$$\overline{SNR} = \frac{E\{|X_k|^2\}}{\sigma_N^2} \qquad (22)$$

The term $\sigma_N^2(X^H X)^{-1}$ is approximated by $\frac{\beta}{\overline{SNR}} I$.
Where β is defined as

$$\beta = \frac{E\{|X_k|^2\}}{E\left\{\frac{1}{|X_k|^2}\right\}} \qquad (23)$$

Note that β is a constant depending only on the signal constellation.
For example, when 16QAM is used, the β is $\frac{17}{9}$
Then the modified MMSE can be written as

$$\hat{H}_{MMSE} = R_{HH}\left(R_{HH} + \frac{\beta}{\overline{SNR}} I\right)^{-1} \hat{H}_{LS} \qquad (24)$$

Therefore, if R_{HH} and SNR are known or fixed as known values, matrix inversion of $R_{HH} + \sigma_N^2(X^H X)^{-1}$is just needed to be calculated only once. But, because of consideration of influence of noise, the MSE of LMMSE is smaller than MMSE.

C. Comb Type Pilot-Based Channel Estimation

In comb type pilot-based channel estimation, as shown in Fig. 2, for each transmitted OFDM symbol N_p , pilot signals are inserted into subcarriers with same interval in frequency from each other.

$$\begin{aligned} X(k) &= X(mN_f + l) \\ &= \begin{cases} x_{p(m)} & l = 0 \\ Data & l = 1,2,\dots,N_f - 1 \end{cases} \end{aligned} \qquad (25)$$

where $X(k)$is the information, including pilot and data, of all sub-carriers, $x_{p(m)}$is the value of m^th^ subcarrier pilot, N_f is the frequency interval of inserted pilot, respectively. If there are N subcarriers,

$$N_f = \frac{N}{N_p} \qquad (26)$$

According to the pilot position, frequency response of corresponding sub-channel is calculated in the receiver.

$$\hat{H}_p(m) = \frac{Y_p(mN_f)}{X_p(mN_f)} \qquad (27)$$

Where Y_p and X_p are output and input of pilot subcarrier, respectively.

When the pilot interval is shorter than coherent bandwidth, after the frequency response of pilot sub-channel is estimated, interpolation is used in frequency domain to get the channel estimation. Different interpolation methods will yield different accuracy.

1) *Piecewise Constant Interpolation*

Piecewise constant interpolation is the simplest method. The channel is estimated by previous pilot. And the channel estimation is given by,

$$\begin{gathered}\hat{H}(k) = \hat{H}(mN_f + l) = \hat{H}_p(m), \\ 0 \le l \le N_f\,, \;\; m = 0,1,\dots,N_p - 1\end{gathered} \qquad (28)$$

We could note that the channel estimation $\hat{H}(k)$ of all sub-carriers is determined by the estimation of mth pilot directly. As the simplest method, piecewise constant interpolation provides the worst performance.

2) *Linear Interpolation*

Linear interpolation performs better than the piecewise constant interpolation [12]. The channel estimation at the data subcarrier is obtained by estimation of response of two adjacent pilot sub-channels. But the precondition is linearity of transmitted functions of adjacent sub-channels. The linear interpolation is shown as below,

$$\begin{gathered}\hat{H}(k) = \hat{H}(mN_f + l) = \\ \left(\hat{H}_p(m+1) - \hat{H}_p(m)\right) + \frac{1}{N_f} + \hat{H}_p(m), \\ m = 0,1,\dots,N_p - 1\end{gathered} \qquad (29)$$

In (29), note that only two pilots are used in linear interpolation for channel estimation. Hence complexity of computation is simple. But performance is not necessarily satisfactory.

3) *Spline and Cubic Interpolation*

Spline and Cubic interpolations are done by using "interp1" function of MATLAB. Spline and Cubic interpolations produce a smooth and continuous polynomial fitted to given data points. Spline interpolations works better than linear interpolation for comb pilot arrangement.

IV. SIMULATION RESULTS

This section discusses the simulation setup and results of pilot-based channel estimation in OFDM system. Table (1) shows the OFDM system parameters

Table (1) OFDM system parameters

Bandwidth	1 MHz
Carrier frequency	1.9 GHz
Number of subcarriers	128
Number of symbols per subcarrier	100
Interval of subcarrier	7.8125 KHz
Number of multipath	5
Time delay of multipath	[0 2e-6 4e-6 8e-6 12e-6]
Cyclic prefix length	16
Modulation technique	16 QAM

Fig.3 andFig.4 show the comparison of Bit Error Rate (BER) performance of LS and LMMSE algorithms of block type pilot-based channel estimation for 16 QAM when Doppler frequency is 20 Hz and 120 Hz respectively. As SNR increases BER decreases for both cases. For a given SNR, LMMSE estimator shows better performance than LS estimator. The complexity of LMMSE estimators will be larger than LS estimators but give better performance in comparison to LS. It should be noticed that LMMSE estimators have been derived under assumption of known channel correlation and noise variance. In practice these quantities are either taken as fixed or estimated, possibly in an adaptive way. This will increase the estimator complexity but improve performance over LS estimators. It is also shown that as Doppler shift increases, BER increases for the same SNR as Block type pilots are sensitive to Doppler Shift.

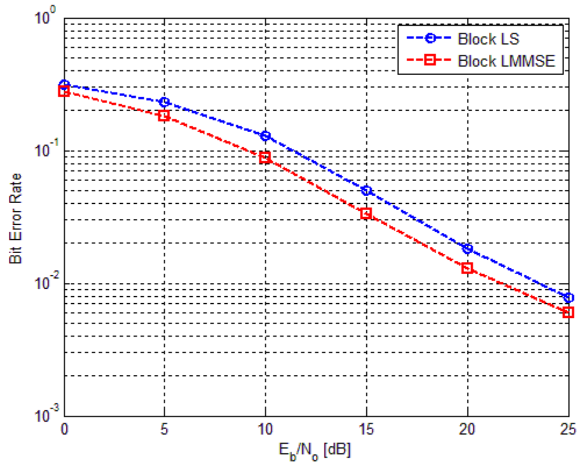

Fig.3 Comparison of LS and LMMSE for block-type pilot with 20 Hz Doppler shift.

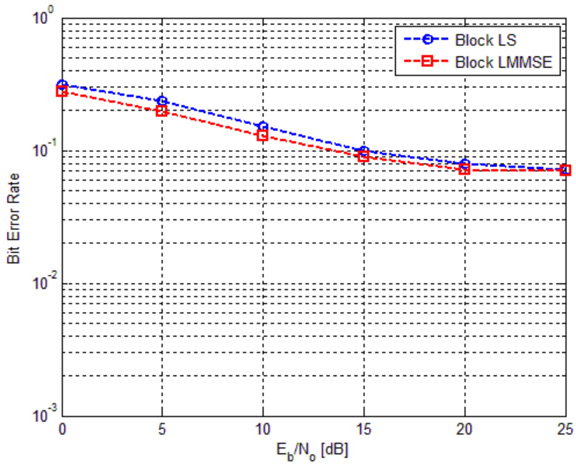

Fig. 4 Comparison of LS and LMMSE for block-type pilot with 120 Hz Doppler shift.

Fig.5andFig.6 show the comparison of different interpolations of comb-type pilot-based channel estimation with LS estimator with different number of pilots for 16QAM modulation technique. As the increase of polynomial orders, performance has significant improvement. But, at the same time, computation also will be much more complex. On the other hand, BER decreases when SNR is increasing. It is also clear to see the improvement of performance with more pilots. Because increasing the number of sub-channels for the pilot is equivalent to reduce the interval between the channels, so that the correlation between two adjacent pilots increases, and estimated values of sub-channel characteristics by interpolation are more accurate. Therefore, estimated performance could be improved. However, the increasing of number of sub-channels for pilot decreases the spectrum efficiency.

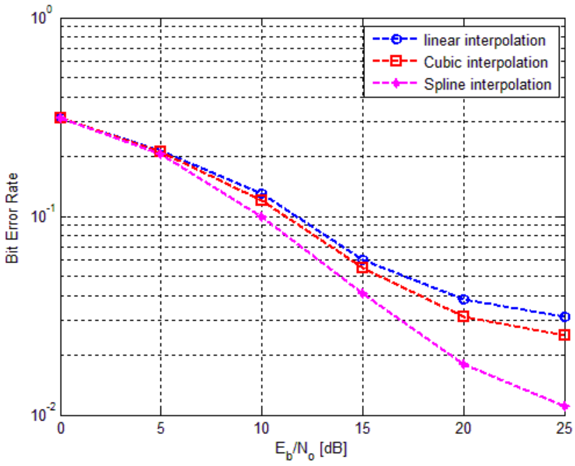

Fig. 5 Comparison of comb-type with 64pilots.

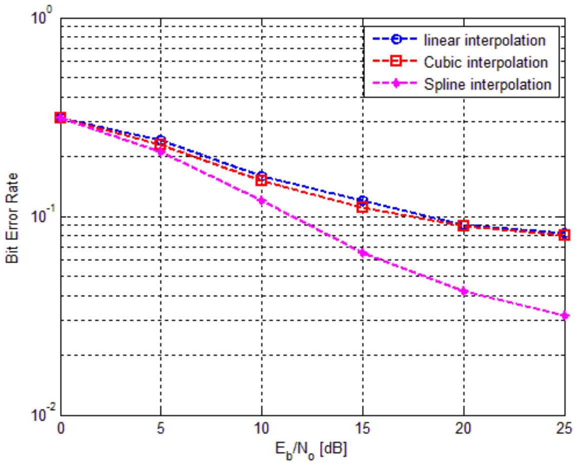

Fig.6 Comparison of comb-type with 32 pilots

V. CONCLUSION

In this paper, pilot-based channel estimation of OFDM system is discussed in detail. Focus has been placed on two types of pilot, block-type and comb-type. First the wireless multipath channel effect is analyzed theoretically, and the multipath fading channel model is given. Then, introduced the OFDM wireless communication technology. After that, the focus on pilot-based channel estimation of OFDM is discussed.

In slow fading environment, block type pilot-based channel estimation in OFDM system has better performance. LS estimator yields the worst performance but with the simplest complexity. Based on LS algorithm, LMMSE is analyzed and it shows better performance compared with LS but with more computation complexity.

In fast fading environment, comb-type pilot-based channel estimation in OFDM system has better performance. Linear interpolation, spline and cubic interpolations are discussed. The simulation results show that the performance becomes better as the increasing order of polynomial for interpolation. But the complexity also increases.

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