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An improvement in Transmit Diversity for MIMO SC-FDE

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***Abstract:*-Single-Carrier Frequency Domain Equalizer (SC-FDE) scheme has several advantages than multi-carrier scheme OFDM. SC-FDE avoids all the drawbacks of OFDM. This paper analyzes the diversity gain of multi-input-multi-output (MIMO) SC-FDE in frequency selective channels. This paper fully analyzes the Alamouti signaling scheme under linear equalizers such as Zero-forcing (ZF) and MMSE equalizers. The diversity gain is function of channel memory *v*, data block length *L*, data rate *R* and antenna configuration. Below a threshold rate full diversity is achieved, while at higher rate diversity decreases. Our analysis shows that the diversity of Alamouti MIMO SC-FDE is improved at higher rates i.e twice the diversity of conventional SISO scheme.**

Keywords: **- SC-FDE, MIMO, ZF, MMSE, equalization, diversity gain, Alamouti signaling**

I.INTRODUCTION

An alternative to OFDM to mitigate ISI is the use of Single-Carrier (SC) modulation combined with Frequency Domain Equalization (FDE) [1]. The complexity and performance of SC-FDE system is comparable, SC-FDE avoids the drawbacks of OFDM i.e large PAPR [2], [3] intolerance to amplifier nonlinearities, and high sensitivity to carrier frequency offsets (CFOs) [4].A single-Carrier scheme (SC-FDE) has lower computational complexity, due to its use of computationally efficient Fast Fourier Transform (FFT), than TD equalization, whose complexity grows exponentially with channel memory [5].

In this paper we analyze the performance of SC-FDE with MIMO. Basically the most prominent limitation of wireless communication is multi path fading due to which the capacity of the system is decreasing and the error rate is increasing. MIMO would be the most innovative approach in the modern wireless systems to improve the capacity with the highest superior quality [6]

A brief review of some existing schemes on diversity gain, first start with various block transmission schemes, that is uncoded OFDM is weak in symbol detection in frequency selective channel. It may not capture the full diversity of the ISI channel [8]. To mitigate this, motivation to achieve full diversity with error control coding, complex field coded (CFC)-OFDM introduced [8]. It achieves full diversity but due to the complexity of ML detection, motivation towards the study of linear equalizers such as Zero-Forcing (ZF) and MMSE equalizers. The first analysis of Diversity order of CFC-OFDM with linear equalizers was provided in [9]. Onggosanusi et al. [11] studied MMSE and zero-forcing (ZF) MIMO receivers. Mehana and Nosratinia considered the outage probability of linear equalizers [12] for Cyclic Delay Diversity (CDD). At high data rates the CDD diversity degenerates to the diversity of the SISO SC-FDE. It is known that SC-FDE transmit diversity for SISO is function of data rate and FFT size, and extended to MIMO. After that Alamouti scheme for SC-FDE is proposed by Dhahir [10].

In this paper we analyze the Alamouti signaling scheme to achieve the maximum diversity gain for two different linear equalizers. We proved that the diversity gain of Alamouti is twice the diversity of CDD, in that the diversity of MMSE is depend on channel memory, while the diversity of ZF Equalizer is twice only independent of channel memory

The rest of the paper is organized as follows. Section II will discuss about the System model, channel and some definitions are provided. Section III provides the proposed system, Alamouti signaling analysis for MMSE and ZF receivers. Section IV provides the simulation results and discussion finally some conclusion is given in Section V.

II.SYSTEM MODEL AND CHANNEL

The block diagram of an SC wireless communication system employing FD equalization is depicted in Figure 1. Each group of consecutive $\log_2^C$ information bits is mapped into a complex symbol belonging to a *C*-ary complex constellation.

We consider a frequency selective quasi-static wireless fading channel. The equivalent baseband model for this Inter Symbol Interference (ISI) channel is given by a multipath model with $v+1$ paths and the channel follows a block fading model where the channel coefficients are independent complex Gaussian $\mathcal{CN}(0,\ 1)$ random variables that remain unchanged over the transmission data block of length L. To remove Inter-Block Interference (IBI) at receiver a Cyclic Prefix (CP) of length *v* is inserted. Received signals are contaminated with zero-mean unit variance complex additive white Gaussian noise (AWGN). The channel output is given by

$$y=\sqrt{\rho}Hx+n \qquad \text{.....(1)}$$

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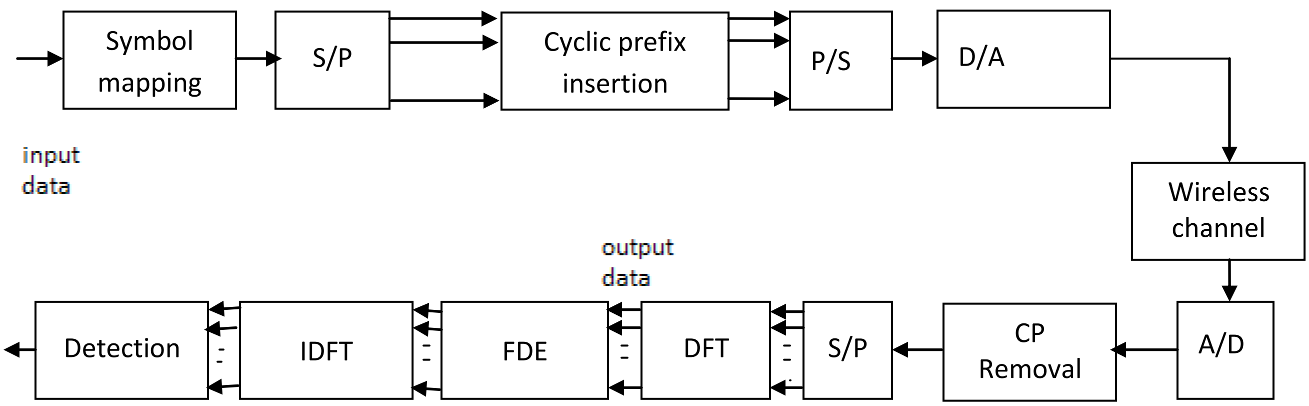

Fig. 1. Block Diagram of Single-carrier Frequency Domain Equalizer (SC-FDE)

Where x is the transmitted, y is the received and n is noise (AWGN) vectors given below. $\sqrt{\rho}$ is the transmitted signal power(SNR)

$$x = [x(1),.....x(L+v)]^T$$
$$y = [y(1),.........Y(L)]^T \qquad \text{.........} (2)$$
$$n = [n(1),.........n(N)]^T$$

The matrix **H** is the convolution matrix of the order of [L x $(L+v)$]. To remove IBI a CP is inserted so the transmitted vector can be expressed as $x = U_{CP}s$, s is the data extension block and U_{cp} is the CP unitary matrix

Hence the equation (1) can be modified as

$$y = \sqrt{\rho}HU_{cp}s + n = \sqrt{\rho}H_e s + n \qquad \text{...} (3)$$

The channel matrix H_e

$$H_e = \begin{bmatrix} h_0 & h_1 & \cdots & h_v & 0 & \cdots & 0 \\ 0 & h_0 & h_1 & \cdots & h_v & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & h_0 & h_1 & \cdots & h_v \end{bmatrix} \qquad \text{......} (4)$$

Where H_e is the eigen decomposition $H_e = Q^H \Lambda Q$, Q is the unitary DFT matrix. The diagonal elements of Λ are given by

$$\lambda_k = \sum_{i=0}^{v} h_i e^{-j\frac{2\pi i(k-1)}{L}} \quad for\ \ k = 1,....L. \qquad \text{.........}(5)$$

Assume perfect channel state information at the receiver, the linear ZF and MMSE equalizers are given by[13].

$$W = \left(c\rho^{-1}I + \mathrm{H}^H\mathrm{H}\right)^{-1}\mathrm{H}^H \qquad \text{.............} (6)$$

Where c=1 for MMSE and c=0 for ZF equalizer.
The diversity is given by

$$d \stackrel{\Delta}{=} -\lim_{\rho\to\infty} \frac{\log \mathrm{P}_e}{\log \rho} \qquad \text{................} (7)$$

And the outage diversity is

$$d_{out} \stackrel{\Delta}{=} -\lim_{\rho\to\infty} \frac{\log \mathrm{P}_{out}}{\log \rho} \qquad \text{..................} (8)$$

Where P_e is the pair wise error probability (PEP), P_{out} is the outage probability $\mathrm{P}_{out} = \mathrm{P}(I(x,y) < R)$, where $I(x, y)$ is the mutual information between x and y and R is the target rate

III .PROPOSED SCHEME

The proposed transmit diversity scheme is Alamouti signaling method, which provides larger diversity gain. Alamouti presents a remarkably simple scheme to achieve transmit diversity, for an array of two elements, without any loss of bandwidth and the transmit power is equally divided among those antennas. The scheme transmits two symbols over two time periods and no bandwidth is wasted. This is the beauty of Alamouti's scheme. In the simplest case, the model supports a 2 x 1 system and can be extended to 2 x N system.

We consider Single-Carrier block transmission over frequency- selective channel with memory v .Each data block length L and a CP of length v is inserted at the beginning of each transmit block to eliminate

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IBI.Denote the n^{th} symbol transmitted from antenna i is $x_i(n)$. Pairs of length N blocks $x_1(n)$ and $x_2(n)$ are generated by an information source.

The code words transmitted by the two antennas over the two consecutive symbol periods k, $k+1$ can be written as

$$X \cong \begin{bmatrix} x[k] & x[k+1] \\ -x^*[k+1] & x^*[k] \end{bmatrix} \quad \text{........(9)}$$

$$\begin{array}{cc} \uparrow & \uparrow \\ \text{Antenna1} & \text{Antenna2} \end{array}$$

here $(.)^*$ denotes complex conjugate. The CP is added to make all channel matrices circulant and total transmitted power is equally divided among the antennas.

The signals observed at the receiver in the two consecutive symbol periods are given by

$$\begin{aligned} y[1] &= \sqrt{\rho}h_1 x[1] + \sqrt{\rho}h_2 x[2] + n[1], \\ y[2] &= -\sqrt{\rho}h_1 x^*[2] + \sqrt{\rho}h_2 x^*[1] + n[2] \end{aligned} \quad \text{....(10)}$$

Where h_k is the channel coefficient and $n[k]$ is the AWGN. A DFT is then applied to diagonalize the channel is as follows

$$Y[k] = \sqrt{\rho}\Lambda_k X[k] + \sqrt{\rho}\Lambda_{k+1} X[k+1] + N[k]$$

k=0, 2, 4……
……………. (11)

Where Y[k], X[k]and N[k] are the DFT vectors of y[k], x[k]and n[k] and Λ_k are diagonal matrices. Assume the channels are fixed over two consecutive blocks

$$Y \triangleq \begin{pmatrix} Y(k) \\ Y(k+1) \end{pmatrix} = \begin{pmatrix} \Lambda_1 & \Lambda_2 \\ \Lambda_2^* & -\Lambda_1^* \end{pmatrix} \begin{pmatrix} \sqrt{\rho}X(1) \\ \sqrt{\rho}X^*(2) \end{pmatrix} + \begin{pmatrix} N(k) \\ N^*(k+1) \end{pmatrix} \quad \text{……………. (12)}$$

Multiply on both sides by the orthogonal matrix Λ^* then the above equation transformed to

$$\tilde{Y} \triangleq \Lambda^* Y = \begin{pmatrix} \tilde{\Lambda} & 0 \\ 0 & \tilde{\Lambda} \end{pmatrix} \begin{pmatrix} \sqrt{\rho}X(1) \\ \sqrt{\rho}X^*(2) \end{pmatrix} + \tilde{N} \quad \text{........(13)}$$

Where $\tilde{\Lambda} \triangleq \Lambda_1^H \Lambda_1 + \Lambda_2^H \Lambda_2$ is a N X N diagonal matrix whose element i is $|\Lambda_1(i,i)|^2 + |\Lambda_2(i,i)|^2$ i.e., the received signal incorporates order-2 diversity

Diversity Analysis of MMSE Receiver:

The received signal for two blocks is given by

$$Y = \sqrt{\rho}\Lambda X + N \quad \text{…………………(14)}$$

Assume perfect channel state information at receiver, The MMSE equalizer is give by

$$W_{MMSE} = (\Lambda^H \Lambda + \rho^{-1} I)^{-1} \Lambda^H \quad \text{………………(15)}$$

Performing the equalization process followed by IDFT given

$$\begin{aligned} \tilde{y}_1 &\triangleq \sqrt{\rho} Q^H (\Lambda^H \Lambda + \rho^{-1} I)^{-1} \Lambda^H \Lambda Q x_1 + \tilde{n}_1 \\ \tilde{y}_2 &\triangleq \sqrt{\rho} Q^H (\Lambda^H \Lambda + \rho^{-1} I)^{-1} \Lambda^H \Lambda Q x_2 + \tilde{n}_2 \end{aligned} \quad \text{........(16)}$$

Where Q is the unitary Discrete Fourier Transform matrix with elements

$$Q(m,n) = \frac{1}{\sqrt{L}} \exp[-j\frac{2\pi}{L}(m-1)(n-1)], \quad m=n=1,..L \quad (17)$$

The SINR of the MMSE for detecting the symbol k of the vector x_1 and x_2 are denoted by $\gamma_{1,k}$ *and* $\gamma_{2,k}$ and given by

$$\gamma_{1,k} = \gamma_{2,k} \triangleq \gamma_k = \frac{\rho}{[(\rho^{-1} I + Q^H \tilde{\Lambda} Q)^{-1}]kk} - 1 \quad \text{...(18)}$$

The $Q^H \tilde{\Lambda} Q$ matrix is circulant so all γ_k are equal. The outage expressions are as follows due to the equalizer structure, the effective mutual information between x and $\tilde{y}$ is equal to the sum of the mutual information of their components

$$\begin{aligned} I_{MMSE}(x;\tilde{y}) &= \sum_{k=1}^{L} \log(1+\gamma_k) \\ &= L\log(1+\gamma_k) \end{aligned} \quad \text{...(19)}$$

By substituting eq (18) in eq(19)

$$I_{MMSE}(x;\tilde{y}) = -L\log\left(\frac{1}{L}\sum_{k=1}^{L}\frac{1}{1+\rho\tilde{\lambda}_k}\right) \quad \text{.........(20)}$$

Where the eigenvalues $\tilde{\lambda}_k$ are the diagonal elements of $\tilde{\Lambda}$ and are given by $\tilde{\lambda}_k = |\lambda_{1,k}|^2 + |\lambda_{2,k}|^2$ where $\lambda_{i,k}$ are the eigenvalues of the channel $\mathbf{H_i}$

The N x N diagonal matrix Λ is given by

$$\Lambda_{k,k} = diag(\lambda_{1,1}, ...\lambda_{N,N}) = \begin{bmatrix} \lambda_{1,1} \cdots 0 \\ \vdots \quad \ddots \quad \vdots \\ 0 \cdots \quad \lambda_{N,N} \end{bmatrix} \quad \text{...... (21)}$$

k=1,….., L.

The outage probability of the MMSE receiver is given by

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$$P_{out} = \mathrm{P}\left(\frac{1}{L} I_{MMSE} < R\right)$$
$$= \mathrm{P}\left(\sum_{k=1}^{L} \frac{1}{1+\rho\tilde{\lambda}_k} > L2^{-R}\right) \quad \text{........ (22)}$$

The outage analysis is as follows:

Consider the case $L=v+1$.Let $\tilde{\alpha}_k \cong -\frac{\log \tilde{\lambda}_k}{\log \rho}$, we have

$$\frac{1}{1+\rho\tilde{\lambda}_k} = \frac{1}{1+\rho^{1-\tilde{\alpha}_k}} \quad \text{....................(23)}$$

At high SNR the second term $\frac{1}{1+\rho^{1-\tilde{\alpha}_k}}$ is either zero or one,i.e

$$\lim_{\rho\to\infty} \frac{1}{1+\rho\tilde{\lambda}_k} = \begin{cases} \rho^{\tilde{\alpha}_k - 1} & \tilde{\alpha}_k < 1 \\ 1 & \tilde{\alpha}_k > 1 \end{cases} \quad \text{..... (24)}$$

So the outage probability is

$$P_{out} = \mathrm{P}\left(\sum_{k=1}^{L} \frac{1}{(1+\rho\tilde{\lambda}_k)} > L2^{-R}\right)$$
$$= \mathrm{P}\left(\sum_{\tilde{\alpha}_K > 1} 1 > L2^{-R}\right)$$
$$= \rho^{-M(\lfloor L2^{-R} \rfloor + 1)} \quad \text{............ (25)}$$

Consider the next MISO Alamouti signaling under two transmission scenarios with block length *L1, L2* where *L2 > L1*for which the block length *L1=v+1* with the different eigenvalues of channel $\mathbf{H_1}$ and $\mathbf{H_2}$.
Thus the diversity of 2 x 1 Alamouti scheme for MMSE receiver is

$$d = \begin{cases} 2(v+1) & R \le \log \frac{L}{v} \\ 2\left(\lfloor 2^{-R} L \rfloor + 1\right) & R > \log \frac{L}{v} \end{cases} \quad \text{.........(26)}$$

Where *L* is data block length

The result shows that the Alamouti provides twice the diversity of SISO-SC-FDE. If the analyses for N > 1 receive antennas the outage probability will depend on eigenvalues, below shows the diversity order for different antenna configurations

Table 1. Comparison of Antenna configuration

Antenna Configuration	**Max. Array Gain**	**Max. Diversity Gain (Order)**
SISO	1	1
SIMO	M_R	M_R
MISO	M_T	M_T
MIMO		$M_R M_T$

In a 2 x N Quasi-static frequency-selective channel with memory *v*, using Alamouti signaling the diversity if MMSE linear receiver is

$$d = \begin{cases} 2N(v+1) & R \le \log \frac{L}{v} \\ 2N\left(\lfloor 2^{-R} L \rfloor + 1\right) & R > \log \frac{L}{v} \end{cases} \quad \text{................(27)}$$

Zero-Forcing Receiver
Here we analyze the linear zero-forcing equalization for Alamouti transmission
The received signal for two blocks is in DFT form

$$Y = \sqrt{\rho}\Lambda X + N \quad \text{..................... (28)}$$

The ZF Equalizer from eq(6) by putting *c*=0 is given by

$$W_{ZF} = (\Lambda^H \Lambda)^{-1} \Lambda^H \quad \text{.............................(29)}$$

The SINR of the ZF detected symbols are given by

$$\gamma_{1,k} = \gamma_{2,k} \triangleq \gamma_k = \frac{\rho}{[(Q^H \tilde{\Lambda} Q)^{-1}]kk} \quad \text{..... (30)}$$

The mutual information is given by

$$I_{MMSE}(x;\tilde{y}) = \sum_{k=1}^{L} \log(1+\gamma_k)$$
$$= L\log(1+\gamma_k) \quad \text{.................. (31)}$$

The outage probability is given that

$$P_{out} = \mathbf{P}\left(\sum_{k=1}^{L} \frac{1}{\rho\tilde{\lambda}_k} > \frac{L}{2^{-R}-1}\right) \quad \text{... (32)}$$

Let $\tilde{\lambda}_k = \rho^{-\alpha_k}$ so we get the outage probability is

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$$P_{out} = \mathrm{P}\left(\sum_{k=1}^{L} \rho^{(\alpha_k - 1)} > \frac{L}{2^R - 1}\right)$$
$$= \mathrm{P}\left(\max_k \{\alpha_k - 1\} > 0\right)$$
$$= \rho^{-1} \qquad \text{............ (33)}$$

Where $\tilde{\lambda}_k = |\lambda_{1,k}|^2 + |\lambda_{2,k}|^2$ and which obey Gamma distribution. $\lambda_{i,k}$ is the eigenvalues of channel $\mathbf{H_i}$.

The diversity of 2 x 1 Alamouti scheme under ZF SC-FDE is only d_{ZF}=2 independent of data block length L, data rate R.

IV.SIMULATION RESULTS

The simulation results Fig.2 shows Energy bit versus BER for Alamouti transmission scheme for block length L=5 and above the threshold data rates R=5. The diversity of ZF receiver is **2** independent of rates(R), whereas the diversity of MMSE is greater than or equal to two.

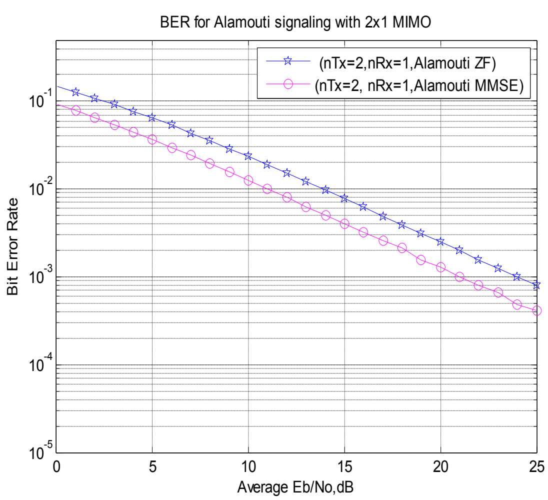

Fig .2. Bit Error Rate of linear Zero-forcing and MMSE receiver

Fig .3 compares the performance of SC-FDE for various transmit diversity schemes like Cyclic-Delay Diversity, Alamouti signaling and theoretical second order diversity for MISO selective channel. The CDD cyclic prefix system achieves full diversity for rates below a threshold level and in Alamouti also. But diversity in Alamouti is double the diversity of CDD cyclic prefix.

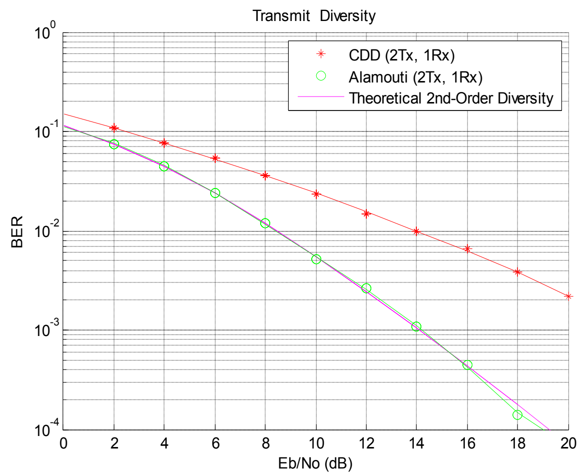

Fig .3. Comparisons of single-carrier transmission transmit diversity schemes, CDD and Alamouti schemes

V.CONCLUSION

In this paper we analyzed the diversity of single-carrier Cyclic-prefix block transmission with frequency-domain linear equalization. We characterized the common transmit diversity scheme Alamouti. The diversity gain achieved for proposed one is increases at higher threshold data rates. We performed the diversity for proposed scheme under linear equalizer.

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