

# Sum and delay Beam former for low SNR using Weiner filtering

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**Abstract**— Array Signal processing is a part of signal processing that uses sensors that are organized in patterns, or arrays to detect the signals and to determine the information about them. In this paper implementation of Sum and Delay Beam former in Lab- View in low SNR condition using Weiner filter is given. The result of simulation with simulated and real data is presented.

**Index Terms**—Beam forming, Sum and Delay, Simulated signals, Weiner filtering, Lab-View, Real Time signals.

## I. INTRODUCTION

Beam forming is the process of trying to concentrate the array to sounds coming from only one particular direction. The best way to not listen in one direction is to just steer all your energy towards listening in one direction. This is important concept because it is not just used for Array signal processing; it is used in many sonar systems [1] & [4].

## II. SUM AND DELAY BEAM FORMER

### A. Uniform Linear Array

Final stage when constructing any array, the design specifications should be determined by the properties of the signals that the array will detect [2]. All acoustic waves travel at speed 1500 m/s in underwater channel. The physical relationship describing acoustic waves is similar to that of light  $\lambda f = c$ . [5].The frequencies of signals that an array detects are important because they determine constraints on the spacing of the sensors. From study we found that fundamentally important condition for spacing between arrays is

$$d \leq c/2f_{max} \quad (1)$$

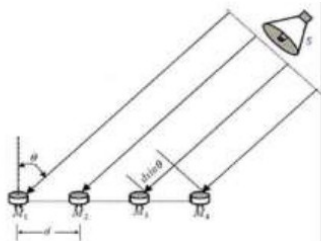


Figure1. The Uniform Linear Array

### B. Delay and Sum Beam former

The Sum and Delay beam former is based on the idea that if a uniform linear array is being used, then the output of each

Sensor will be the same, expect that each one will be delayed by a different amount [5]. So if the output of each sensor is delayed appropriately then we add all the outputs together the signal that was propagating through the array will reinforce, while noise will tend to cancel.

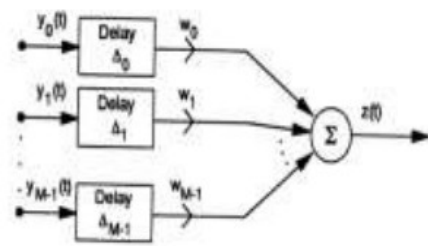


Figure 2. Sum and Delay Beam former.

## III. LAB-VIEW IMPLEMENTATION OF BEAM FORMING

### A. Delay and Sum approach simulation with simulated signals

In Delay and Sum beam forming approach appropriate amount of delays have to be introduced in the received signals, so that they can be combined to enhance each other. The Block Diagram of Sum and Delay beam former in Lab-View is shown in figure 3.

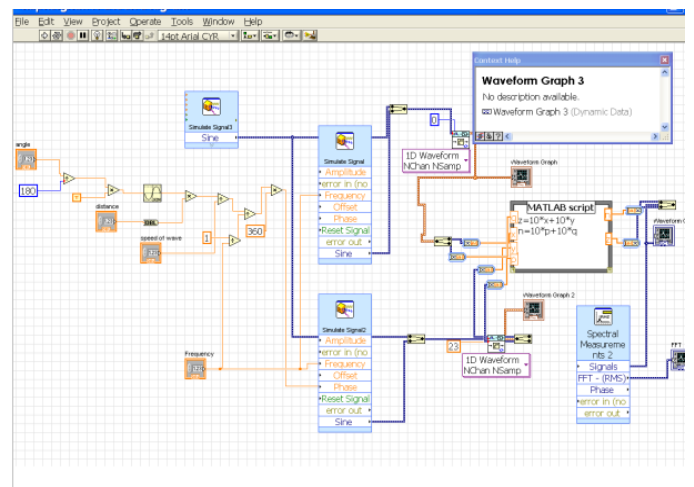


Figure 3. Lab- View block diagram of Sum and Delay Beam former.

B. Wiener filtering

The Wiener filter has a variety of applications in signal processing, image processing, control systems, and digital communications [6]. These applications generally fall into one of four main categories like system identification, deconvolution, noise reduction, signal detection. The Wiener filter solves the signal estimation problem for stationary signals. The filter was introduced by Norbert Wiener in the 1940's. A major contribution was the use of a statistical model for the estimated signal (the Bayesian approach!). The filter is optimal in the sense of the MMSE.

Let  $x(t)$  and  $y(t)$  be two zero mean, scalar-valued random processes, where  $y(t)$  is the measured process, and  $x(t)$  is to be estimated from  $y$ . In particular, consider the linear estimate of  $x(t)$  given by

$$\hat{x}(t) = \int_T h(t, \zeta) y(\zeta) d\zeta \quad (2)$$

In (1), the domain of integration,  $T$ , can be taken to be a finite or infinite continuum or discrete set of times. The integral sign should be viewed appropriately in the discrete case. We desire to find the filter,  $h(t)$ , that minimized the mean squared error

$$e(t) \hat{=} E | \hat{x}(t) - x(t) |^2 \quad (3)$$

The orthogonality principle [5] states that the optimal  $h(t)$  must satisfy the orthogonality condition

$$E\{[\hat{x}(t) - x(t)]y(t + \tau)\} = 0 \text{ for all } \tau \in T \quad (4)$$

Substituting (1) into (3), and using the linearity of the expectation operation yields

$$R_{xy}(t, t + \tau) = \int_T h(t, \zeta) R_{yy}(\zeta, t + \tau) d\zeta \text{ for all } \tau \in T \quad (5)$$

where we have defined the cross correlation function  $R_{xy}(t, t + \tau) = E[x(t)y(t + \tau)]$ . (6)

The solution of the integral equation, (4), is not an easy one, in general, since the optimal filter is time-varying. In the case where  $T = \{1, 2, \dots, n\}$   $x(t)$  and  $y(t)$  are simply elements of the  $n$ -dimensional random vectors,  $\mathbf{x}$  and  $\mathbf{y}$ . Similarly,  $h(t, \tau)$ ,  $R_{xy}(t, t + \tau)$ , and  $R_{yy}(t, t + \tau)$  are simply an elements of the  $n \times n$  matrices  $\mathbf{H}$ ,  $\mathbf{R}_{xy}$  and  $\mathbf{R}_{yy}$ , respectively. In this case, the system of equations (4) can be expressed simultaneously as

$$\mathbf{R}_{xy} = \mathbf{H} \mathbf{R}_{yy}.$$

If the inverse of  $\mathbf{R}_{yy}$  exists, then the optimal time-varying filter  $\mathbf{H} = \mathbf{R}_{xy} \mathbf{R}_{yy}^{-1}$ .

Our interest in this paper will be restricted to the case where  $x(t)$  and  $y(t)$  are jointly wide sense stationary (wss). In this case, (4) becomes

$$R_{xy}(\tau) = \int_T h(t, \zeta) R_{yy}(t + \tau - \zeta) d\zeta \text{ for all } \tau \in T. \quad (7)$$

In the case of finite dimensional random vectors, the wss condition implies that the correlation matrices in (5) will be Toeplitz [ref] in nature; that is, the value of the  $(i, j)$ th element will depend only on  $i - j$ . Even so, the solution,  $\mathbf{H}$ , of (5) will be time-varying. This follows, in part, from the fact that the inverse of a Toeplitz matrix is not, in general, Toeplitz. In the case where the integration region,  $T$ , extends over  $\pm\infty$  it can be shown (see Appendix A) that the time-varying filter becomes time-invariant, and in the frequency domain is given by

$$H(j\omega) = S_{xy}(\omega) / S_{yy}(\omega) \quad (8)$$

The mean squared error (mse) given by (2) becomes

$$e(t) \hat{=} E | x(t) - \hat{x}(t) |^2 = E\{[x(t) - \hat{x}(t)]x(t) - [x(t) - \hat{x}(t)]\hat{x}(t)\} \quad (9)$$

where the last equality (2) follows from the orthogonality condition (3). From the linearity of the expectation operator, it follows that the rightmost expectation can be expressed as

$$e(t) = R_{xx}(0) - \int_{-\infty}^{\infty} h(\tau) R_{xy}(-\tau) d\tau \quad (10)$$

Notice that the *mse* is time-invariant. To gain insight into the frequency dependence of the *mse* we can express (8) in terms of power spectral densities (*psd*'s):

$$e = \frac{1}{2\pi} \int_{-\infty}^{\infty} [ S_{xx}(\omega) - \frac{|S_{xy}(\omega)|^2}{S_{yy}(\omega)} ] d\omega \quad (11)$$

In the case of a signal plus uncorrelated noise, where the measurement process is  $y(t) = x(t) + u(t)$ , the *mse*, (10), becomes

$$e = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{S_{xx}(\omega) S_{uu}(\omega)}{S_{xx}(\omega) + S_{uu}(\omega)} d\omega = \frac{1}{2\pi} \int_{-\infty}^{\infty} H(\omega) S_{uu}(\omega) d\omega \quad (12)$$

Equation (11) states, in words, that the *mse* is the sum of error incurred at each frequency, and that this latter error is the noise power weighted by the Wiener filter value; which is the signal to signal-plus-noise ratio.

C. Result

Using the above explained algorithm a sum and delay beam former is implemented for SNR conditions and performance is evaluated. For a narrow band signal the time delay is equal to the phase delay [6]. Two sinusoidal signals have been simulated using simulate signal VI and an appropriate amount of delay as per the calculation of actual sensors has been introduced. Now these simulated signals were treated as actual sensor signal outputs. The waveforms to form been combined after insertion of proper delay using MATLAB script.

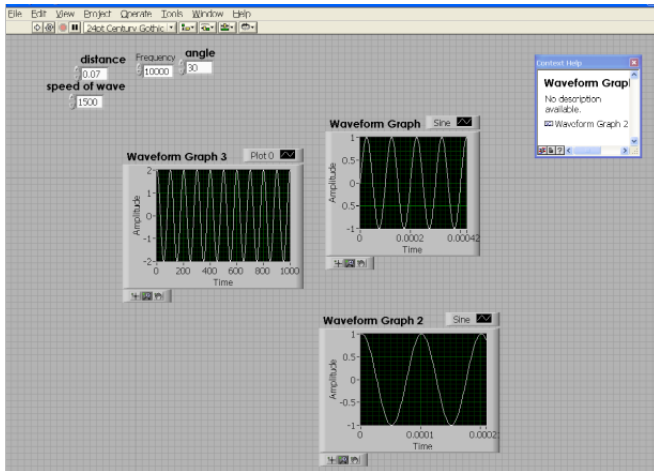


Figure 4. Result of Simulation of Sum and Delay beam former with simulated signals

D. Lab-View Implementation of Beam Forming with Real time signals

Real time signal testing of the algorithm developed in Lab-View is carried out in under water lab. The electrosonika projector is used as a signal source from different angles. The receiver array consists of two D/140/H hydrophones placed at a distance of 2.5cm to avoid spatial aliasing for 30 KHz signal frequency.

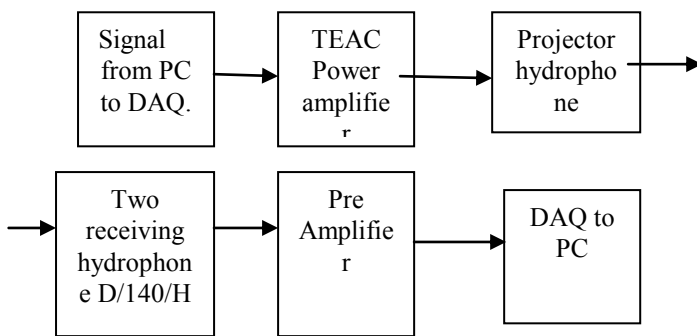


Figure 5. Diagram for acquisition of signals of two hydrophones

According to set up signal at some specific angle from broad side of the array is transmitted. The signal is generated using signal simulation VI in Lab-View and fed to the TEAC power amplifier through data acquisition card. The amplified

signal is now fed to the projector which is electrosonika in this case. The sinusoidal signal of 30 KHz frequency is generated using simulate signal VI and fed to the DAQ using virtual DAQ VI as shown in figure 6. The signal from DAQ was fed to power amplifier, which sends the signal to the transmitter through matching network as shown in figure 5. The transmitter is placed at a particular angle from the broad side of the receiver array in tank. The signals from the two receivers of array have been acquired using the same DAQ card. The calculated amount of delay has been inserted in the appropriate the wave and they were combined using MATLAB script as shown in figure 6.

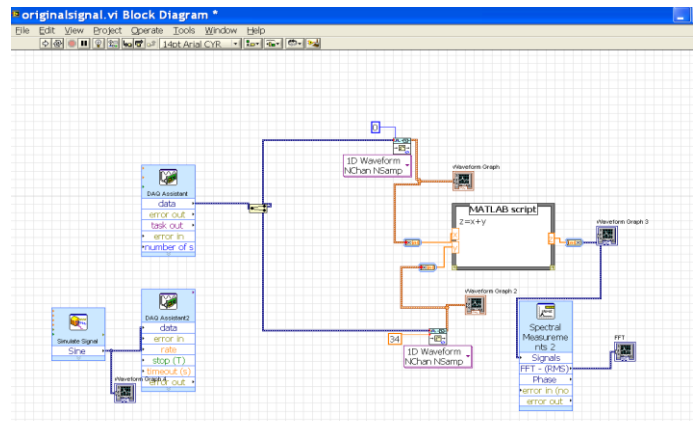


Figure 6. Delay and sum beam former block diagram for real time signals

E. Results of simulation

The result of simulation of real time signal is shown in figure 7. The 0.6 volt and 0.4 volt signal have been acquired from receiver array and the waveform shown in graph 4 having 1 volt of signal is the combined signal after insertion of proper amount of delay. The FFT plot shows the FFT of combined signal. There is one peak at 30 KHz frequency in FFT plot.

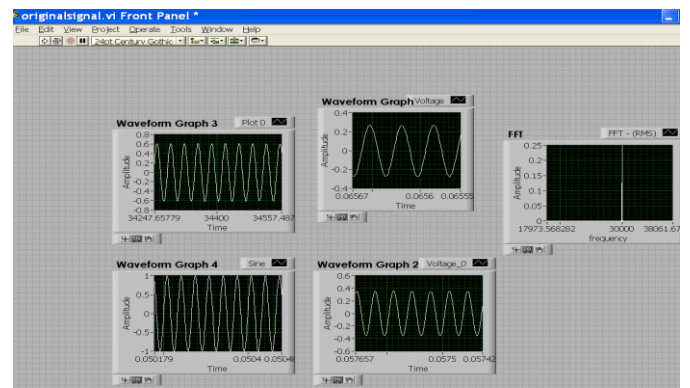


Figure 7. Result of simulation of sum and delay beam former with real signals

#### IV. CONCLUSION

Basic Theory of Linear Array is studied and constraints are pointed out that are useful for implementation of Algorithms, Finally two sinusoidal signals of different phase have been simulated using simulate signal VI in Lab-View, and for Real Time data and results are presented.

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