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Noise Removal in ECG Signal Using Savitzky-Golay Filter

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***Abstract*— ECG signal is affected by various types of noises like power line interference, Motion artifacts etc. ECG in presence of noise is very difficult to analyze and extract essential information correctly so to extract the information correctly it is necessary to filter out the noise present in the signal. To filter out the noise there are various filters are used. Savitzky and Golay gave a method for smoothing of data which is based on least-squares polynomial approximation. This involves fitting of a polynomial to an input samples set and then compute single point polynomial within the interval of approximation which means discrete convolution whose impulse response is fixed.**

***Index Terms*—Filter, Polynomial approximation, Smoothing**

I. Introduction

The ECG signal is usually corrupted by noise during the recording process. There are different types of noises which are present in the signal and their presence affects the signal characteristics and information. These may cause the loss and/or corruption of the information due to which it is difficult to observe and extract the parameters we are interested in. Savitzky and Golay gave a method for smoothing of data which is based on least-squares polynomial approximation. This involves fitting of a polynomial to an input samples set and then compute single point polynomial within the interval of approximation which means discrete convolution whose impulse response is fixed. Savitzky and Golay were trying to smooth noisy data of the chemical spectrum analyzers, and found out that least squares smoothing reduces noise and maintains the height and shape of waveform peaks. S-G filters can be used in smoothing noisy ECG data. Peak and shape preserving property of the S-G filters is has proved to be very efficient for accurate ECG processing.

II. PROCEDURE

The least squares polynomial smoothing[1] is shown below which has a sequence of samples x[n] of the signal. The moment[3] of the group of 2M+1 samples centered at n=0 were taken; coefficients of the polynomial were obtained by following equation.

$$p(n) = \sum_{k=0}^{n} n^k a_k \quad \text{......(1)}$$

The mean-squared error was minimized for the group of input centered at n=0, coefficients of the polynomial are given by equation.

$$\xi = \sum_{n=-M}^{M} (p(n) - x[n])^2$$

By putting value of p[n] in above equation we get

$$\xi = \sum_{n=-M}^{M} \left(\sum_{k=0}^{n} n^k a_k - x(n)\right)^2 \quad \text{......(2)}$$

To find the output at point n=0

$Y[0] = p(0) = a_0$

This means the output value is 0^{th} polynomial coefficient. Symmetry of the interval of approximation around the evaluation point is not important due which it becomes a filter having nonlinear phase, with the help of which sequences of the finite length can be smoothened. Shifting of the interval under which analysis is done by single sample to the right and out put at next sample. A new point the middle sample of the new 2M+1 sample block is now considered as origin, and on that point whole polynomial fitting process is repeated. For each sample whole process is repeated result of which produces a new out put value y[n]. Savitzky and Golay[3] gave that at each point, the output taken by performing sampling of the fitted polynomial is equal to a fixed linear combination of the local input set of samples; i.e. 2M+1 input samples in approximation interval are combined by a set of coefficients of fixed weight computed for a given order N of polynomial and interval of length 2M+1. Which means computation of the output can be done by discrete convolution of the form

$$y[n] = \sum_{m=-M}^{M} h[m]x[n-m] \quad \ldots\ldots\ldots\ldots\ldots(3)$$

Or

$$y[n] = \sum_{m=n-M}^{n+M} h[n-m]x[m] \quad \ldots\ldots\ldots\ldots\ldots(3')$$

The single finite impulse response in sampling interval 2M+1 corresponding to the least square polynomial fitting is by calculating the polynomial coefficients by differentiating[2] the equation (2) with respect to each of the N+1 coefficients a_i and equating it to 0. We get the following equation for i=0, 1, 2, 3… N

$$\frac{\partial \xi}{\partial ai} = \sum_{n=-M}^{M} 2n^i \left(\sum_{k=0}^{N} n^k a_k - x[n]\right) = 0$$

Interchange the order of summation we get

$$\frac{\partial \xi}{\partial ai} = \sum_{k=0}^{N}\left(\sum_{n=-M}^{M} n^{i+k}\right)a_k$$

The above equation can be written as

$$\frac{\partial \xi}{\partial ai} = \sum_{n=-M}^{M} n^i x[n] \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4)$$

i=0, 1, 2, 3 ….N

A matrix A of size (2M+1) by (N+1) is defined as

$A = \{\alpha_{n,i}\}$ where

$\alpha_{n,i} = n^i \quad -M \le n \le M$

i=0, 1, 2, 3…..N

The matrix is called as design matrix for polynomial approximation. The transpose of matrix A is given by $A^T = \{\alpha_{i,n}\}$ and there is a product matrix B, and $B = A^T A$ which is an

(N+1) × (N+1) matrix.

The input vector can be defined as $x=[x[-M],\ldots\ldots.x[-1],x[0],\ldots..x[M]]^T$

and the vector polynomial coefficient matrix

$a=[a_0,a_1\ldots\ldots\ldots.a_N]^T$

The equation (5) can be expressed in matrix form

$$Ba = A^T Aa = A^T x$$

The above equation can be solved and written as

$$A = (A^T A)^{-1} A^T x = Hx$$

The row number 0 of (N+1) × (2M+1) matrix

$H = (A^T A)^{-1} A^T$ represents the output y[0] for sample centered at n=0.

For finding out the impulse response of the equivalent LTI system is first compute matrix H. then output will be given by the definition of matrix multiplication as

$$Y[0] = \sum_{m=-M}^{M} x[m]h_{0,m}$$

Where $h_{i,m}$ denotes the elements of matrix H and $h_{0,m}$ denotes elements of the row no 0.

It has to be noticed that it generates same coefficients and does not depend on the signal vector, then x could be set as a unit impulse centered in the interval $-M \le n \le M$, and can be used for solving for all coefficients of the polynomial. And these polynomial coefficients are denoted by ã this coefficients are not equal to the coefficients generated by the local approximation in the given interval implicitly. The impulse response can be obtained by doing evaluation of the corresponding polynomial in interval $-M \le n \le M$.

The ã is given by

$$\tilde{a} = (A^T A)^{-1} A^T d$$

where $d=[0,0,0,0\ldots\ldots\ldots0,1,0,0\ldots\ldots.0,0]^T$ is a (2M+1) ×1 matrix and then A^T is given as

$$A^T = \begin{bmatrix} (-M)^0 \ldots. & (-1)^0 \ldots\ldots & (M)^0 \\ \vdots & \vdots & \vdots \\ (-M)^N \ldots.. & (-1)^N \ldots.. & M^N \end{bmatrix}$$

Then for input d, it gives $A^T d$ which is a column matrix of size (N+1) × 1 given as

$$A^T d = [1, 0 \ldots\ldots\ldots0, 0]^T$$

And $(A^T A)^{-1}$ is given by following matrix

$$(A^T A)^{-1} = \begin{bmatrix} \tilde{a}0 & \tilde{a}1 & \cdots & \tilde{a}N \\ \vdots & & \vdots & \vdots \\ \tilde{a}N & \bullet & \cdots & \bullet \end{bmatrix}$$

The polynomial evaluated at the integers of matrix H formed by above matrix multiplication is denoted by $\tilde{u}(n)$ in the interval $-M \le n \le M$ given by following equation.

$$\tilde{u}(n) = \sum \tilde{a}_k n^k \qquad -M \le n \le M$$

Hence impulse response of filter is given by

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$$h[n] = h_{0,n} = \tilde{u}(n)$$

By convolving the impulse response with input in eq. (1) we get the output signal.

III. RESULT

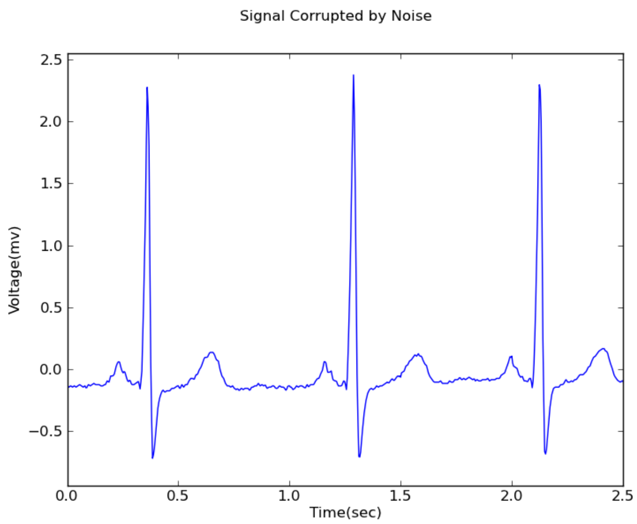

Figure 1: Signal corrupted by noise.

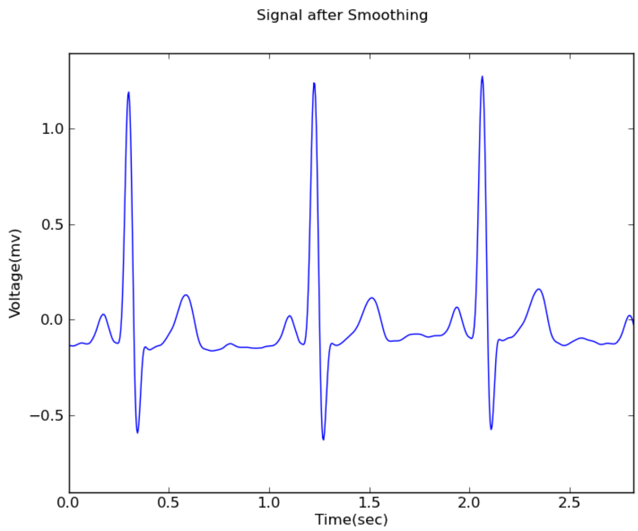

Figure 2: Signal after Smoothing.

IV. CONCLUSION

Diseases of heart are increasing in today's world and it is becoming the main cause of death and the ECG is the most important tool to diagnose the heart problems and its cost is also low and easily available. But ECG signal is contaminated by many types of noises which affects the diagnosis and gives the incorrect information. Many types of filter were developed to eliminate the noise present in ECG and smoothing. Savitzky-Golay is used here the S-G filter removes noise and smooth the signal without much loss of information and signal characteristics and originality. The parameters of S-G filter are the frame size and polynomial degree and whole performance is dependent on these parameters. Here the effects of change of these parameters are studied. The results came from the experiment shows what degree and frame size gives efficient smoothing.

V. REFERENCES

[1]Peter A. Gorry, "General lest-squares smoothing and differentiation by the convolution (Savitzky-Golay) method", Analytical Chemistry., vol.62,pp. 570-573, Mar. 15 1990

[2]Savitzky-Golay Smoothing Filters WH Press, SA Teukolsky - Computers in Physics, 1990 - scitation.aip.org

[3] What is a savitzky-golay filter?
RW Schafer - Signal Processing Magazine, IEEE, 2011 - ieeexplore.ieee.org

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