

BER Performance of OFDM using Iterative ICI reduction techniques

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Abstract— Cellular and Personal Communications Service (PCS) communications systems have historically been designed with voice traffic in mind. Voice can be characterized as relatively predictable, with each party talking about half the time in an interactive manner. Data traffic differs from voice as more unpredictable than voice traffic and data has very different requirement in terms of reliability. Data traffic encompasses a much different and wider range of services than voice. 4G (Fourth generation) is used to fulfill these requirements in much faster way and OFDM (orthogonal frequency division multiplexing) is much preferred technology used in 4G communications system. But the major problem of OFDM is frequency offset sensitivity between the transmitted and received signals, may be due to Doppler shift of the channel, or the difference between the transmitter and receiver local oscillator frequencies. This carrier frequency offset causes loss of orthogonality among sub-carriers and then signals carried by sub-carriers becomes dependent on each other, which leads to inter-carrier interference(ICI). Some methods of ICI reduction include frequency domain equalization like Operator perturbation technique (OPT), Parallel interference cancellation (PIC) and Gauss Seidel iteration technique. In this paper Performance of these techniques are analyzed .

Index Terms— OFDM, ICI, Gauss-Seidel, OPT, PIC.

I. INTRODUCTION

The OFDM system has been developed as a sensitive and powerful system against a multipath reception and is turned out to be quite useful for channels that present strong linear distortions. The basic principal of OFDM is to split the channel in to N number of sub channels and transmission of signal using orthogonal carriers, through these sub channels. It has been realized that the spectra of subcarriers are overlapping because they travel different paths from transmitter to receiver and reach the receiver not simultaneously but in succession and thus creating a path difference among themselves leading to existence of ICI. An Important issue in that context is the correct estimation of the channel at any given time. Similarly, oscillator frequency

offset and frequency drift can lead to ICI. Furthermore, the insufficient CP also leads to ICI, as the original symbols cannot be reconstructed by means of a one-tap equalizer alone. Irrespective of the reasons, ICI can lead to considerable performance degradation in many systems and must be combated. In this paper, some iterative inversion techniques for the general case of time-varying delay dispersive channels with insufficient CP like Operator perturbation technique (OPT), Parallel interference cancellation (PIC) and Gauss-Seidel Iterative reduction technique has been studied and analyzed their performance

II. OPERATOR PERTURBATION TECHNIQUE (OPT)

The conventional method to combat the effect of ICI and ISI is to adding the cyclic prefix (CP)[17]. The drawback of the CP (length G symbols) is that it reduces the spectral efficiency to $\frac{N}{N+G}$ [12]. This reduction can be eliminated only by omitting the CP. To improve the channel spectral efficiency with zero CP, the inversion of channel impulse matrix for ICI cancellation is done using Operator perturbation technique (OPT) [12], which reduces the Computational complexity also by the O (n²) operations per iteration.

A. OPT algorithm

OPT is an effective way to invert linear or non-linear operators, and is also called the Jacobi iteration [12]. It was originally introduced by J.C.Canon [12]. This is an iterative method for inversion of OFDM channel matrix H [17]. In standard OPT, a banded matrix \hat{H} consists of the diagonal and Q off-diagonals of matrix H as in is extracted and it is given as

For notational convenience we can write

$$Y = H X \quad (1)$$

$$X = H^{-1} Y \quad (2)$$

And approximate (1) by

$$Y = \hat{H} X - \epsilon \quad (3)$$

\hat{H} is the approximate operator whose inverse is easy to compute, ϵ is the deviation from the exact solution. E.g. \hat{H} could consist of the diagonal of H and zero off-diagonal elements. The solution of (2) is then found by the following iteration:

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$$H = \begin{pmatrix} h_{1,1} & h_{1,2} & \dots & \dots & h_{1,n} \\ h_{2,1} & h_{2,2} & \dots & \dots & h_{2,n} \\ \cdot & \cdot & \dots & \dots & \cdot \\ \cdot & \cdot & \dots & \dots & \cdot \\ \cdot & \cdot & \dots & \dots & \cdot \\ h_{n,1} & h_{n,2} & \dots & \dots & h_{n,n} \end{pmatrix}$$

$$\hat{H} = \begin{pmatrix} h_{1,1} & 0 & 0 \dots & 0 \dots & 0 \\ h_{2,1} & h_{2,2} & 0 \dots & 0 \dots & 0 \\ h_{3,1} & h_{3,2} & h_{3,3} & 0 \dots & 0 \\ \cdot & \cdot & \dots & 0 \dots & 0 \\ h_{n,1} & h_{n,2} & \dots & \dots & h_{n,n} \end{pmatrix}$$

$$X_{(0)} = \hat{H}^{-1} \cdot Y \tag{4}$$

$$\epsilon_{(0)} = (\hat{H} - H) X_{(0)} \tag{5}$$

$$X_{(i+1)} = \hat{H}^{-1} \cdot (\epsilon_{(i)} + Y) \tag{6}$$

$$\epsilon_{(i+1)} = (\hat{H} - H) X_{(i+1)} \tag{7}$$

Where subscript i denote ith iteration, and equation (4) and (5) are initialization steps. Step (6) and (7) are repeated until reached a criterion. Substituting step (6) in step (7) gives:

$$X_{(i+1)} = X_{(i)} + \hat{H}^{-1}(Y - HX_{(i)}) \tag{8}$$

where $\hat{H}^{-1}(Y - HX_{(i)})$ can be represent as $e_{(x)}$ and $e_{(x)}$ is the error of old solution $X_{(i)}$. If $X_{(i)}$ converges to $X_{(\infty)}$, then

$$X_{(\infty)} = \hat{H}^{-1} \cdot (\epsilon_{(\infty)} + Y) \tag{9}$$

$$\epsilon_{(\infty)} = (\hat{H} - H) X_{(\infty)} \tag{10}$$

$$X_{(\infty)} = \hat{H}^{-1} \cdot ((\hat{H} - H) \cdot X_{(\infty)} + Y) \tag{11}$$

$$= X_{(\infty)} + \hat{H}^{-1} \cdot (Y - H \cdot X_{(\infty)}) \tag{12}$$

$$\Rightarrow 0 = \hat{H}^{-1} \cdot (Y - H \cdot X_{(\infty)})$$

This means that if $\det \{\hat{H}^{-1}\} \neq 0$, then we get the solution

$$Y - HX_{(\infty)} = 0 \tag{13}$$

and therefore, $X_{(\infty)} = X$. Step (7) of the iteration procedure requires a vector-matrix product that needs $O(n^2)$ operations. The sufficient condition for OPT to converge is that the matrix \hat{H} is strictly or irreducibly diagonally dominant, that is

within each row the absolute value of the diagonal term is greater than the sum of absolute values of other terms, given as:

$$|h_{i,i}| > \sum_{j \neq i} |h_{i,j}|$$

With the help of OPT technique, Channel matrix H inversion complexity reduces a lot. It requires $O(n^2)$ operations while a conventional equation solver based on matrix inversion requires $O(n^3)$ operations.

III. GAUSS-SEIDEL ITERATION TECHNIQUE

A similar method to Gauss-Seidel Iteration OPT is called the Gauss-Seidel iteration, which is named after German mathematicians Carl Friedrich Gauss and Philipp Ludwig von Seidel [11]. It is an improved version of OPT in respect of its memory efficiency [17], as it does not require additional memory to store $X(i)$. As with the OPT method, although Gauss-Seidel iteration can be applied to matrices with non-zero diagonals, convergence is guaranteed only when the main diagonal is strictly or irreducibly diagonally dominant. The main difference to Jacobi iteration is that when forming the sparse matrix, Gauss-Seidel iteration uses the lower triangle of the matrix H instead of using a banded matrix as in OPT.

B. Gauss-Seidel algorithm

In this iteration method, the channel matrix H decomposed into three sub-matrixes noted as

$$H = D + E + F = D(I + L + U) \tag{14}$$

Where E, L is strict lower triangular matrixes and F, U is strict upper triangular matrixes and D is a diagonal matrix. For the data vector $X = [X_0, X_1 \dots X_{N-1}]$, Jacobi type iteration is used to evaluate $X_{(k+1)}$, for already evaluated $X_i^{(k+1)} \dots X_s^{(k+1)}$ evaluation could be done on the sth row of $LX^{(k+1)}$ in matrix notation

$$X^{(k+1)} = LX^{(k+1)} + UX^{(k)} + D^{-1}Y \tag{15}$$

$$X^{(k+1)} = (I - L)^{-1} (UX^{(k)} + D^{-1}Y) \tag{16}$$

The iteration matrix $(I - L)^{-1}U$ is called Gauss Seidel iteration matrix [13]. The algorithm can be summarized in these steps:

- Initial guess of the unknown Data vector $X^{(0)} = 0$.
- Using the equation to compare $X^{(k+1)}$. Once $X_i^{(k+1)}, i = 0, 1, \dots, N-1$ in each iterative step as soon as the part of the new data vector have been calculated.
- After each iteration, examine if the errors are less than tolerance

$$\epsilon_i = \frac{|X_i^{(k+1)} - X_i^{(k)}|}{|X_i^{(k+1)}|}, i = 0, 1, \dots, N-1$$
- If the error is not smaller than the tolerance then continue the iteration.

The main idea of this algorithm is to update the data vector $X^{(k)}$ in each iterative vector step as soon as the part of the new data vector $X^{(k+1)}$ have been calculated, therefore ,the convergence of the algorithm can be accelerated effectively.

IV. PARALLEL INTERFERENCE CANCELLATION TECHNIQUE(PIC)

The Jacobi Iteration and Gauss-Seidel iteration uses complete linear operations to achieve interference cancellation[17]. Non-linear operation can also be incorporated with linear operations to achieve improved accuracy. Parallel Interference cancellation (PIC) and Serial Interference Cancellation (SIC) are two of this kind. PIC uses a similar approach to OPT, where the cancellation process takes place in a symbol wise way for OFDM systems.

C. PIC algorithm

For initialization, calculate a first estimate,

$$X_i^{(0)} = (\text{diag } H)^{-1} \cdot Y$$

Here, diagH is the diagonal of H only and can be inverted with computational effort O(n). The interference $I^{(n)}$ in step n is

$$I^{(n)} = HX^{(n-1)} - \text{diag}H X^{(n-1)} \tag{17}$$

where we omitted the block index sub i for notational convenience. The updated interference-cancelled received symbol vector $Y^{(n)}$ is simply

$$Y^{(n)} = Y - I^{(n)} \tag{18}$$

Hence the updated estimate of the transmitted symbol on subcarrier k is

$$X_k^{(n)} = Y_k^{(n)} / H_{kk} \tag{19}$$

where we equalize the interference-cancelled received symbol by the channel response on the frequency of subcarrier k. This algorithm is called "parallel"[14], because Eqs. (18) and (19) can be applied to all channels $k = 0 \dots n - 1$ simultaneously by a vector-matrix calculus. Due to the vector matrix multiplication, the computational complexity of the algorithm is $O(n^2)$. Before entering the next round of iterations; the estimated symbols can be processed in a nonlinear fashion. In the simplest case, they could be forced to the nearest symbol of the used alphabet. More generally

$$\hat{X}_k^{(n)} = f(X_k^{(n)}) \tag{20}$$

Where $f(.)$ is the nonlinear decision function, like for BPSK [2] hyperbolic tangent function,

$$X_k^{(n)} = \tanh(c \cdot X_k^{(n)})$$

because far-off estimates are forced to ± 1 and thus reduced in their impact on the next cancellation stage, while estimates with small amplitudes are more or less unchanged. The factor c in the hyperbolic tangent function controls the slope near zero. Small c (e.g., $c < 1$) are appropriate if the estimates are not yet reliable. Large c ($c > 10$) are appropriate for accurate

estimates, e.g., if the signal-to interference ratio (SIR), is large. That is the case for good channels with small delay spread [14]. Performance can be further improved if c is increased from one iteration step to the next. In the beginning, decisions are still unreliable so that small c is appropriate. At the end of the iteration, the decision function approximates the signum function, and all decisions are in the set $\{\pm 1\}$. This technique stems from an optimization technique called "simulated annealing"[14] and avoids the convergence of the MSE to a local minimum. The error vector and the MSE in the receiver are defined as

$$E = Y - H \cdot \hat{X}_i$$

$$MSE = \|E\|^2$$

The iteration stops after the MSE falls below a predefined threshold or the number of iterations reaches a predefined maximum number.

V. SIMULATION & RESULTS

D. Simulation Parameters:

Simulation parameters chosen for the model of OFDM transceiver are listed in Table 1.0. Simulation is carried out Frequency selective Rayleigh channel using BPSK modulation technique:

Table 1.0

OFDM Parameter	Value
FFT size (nFFT)	64
Number of used subcarriers(nDSC)	52
FFT Sampling frequency	20MHz
Subcarrier spacing	312.5kHz
Cyclic prefix duration (T _{cp})	0.8μs
Data symbol duration (T _d)	3.2μs
Total Symbol duration (T _s)	4μs

Simulation results are plotted for BER performance of different iterative methods considering absence and presence of OPT, PIC, and Gauss - Seidel methods.

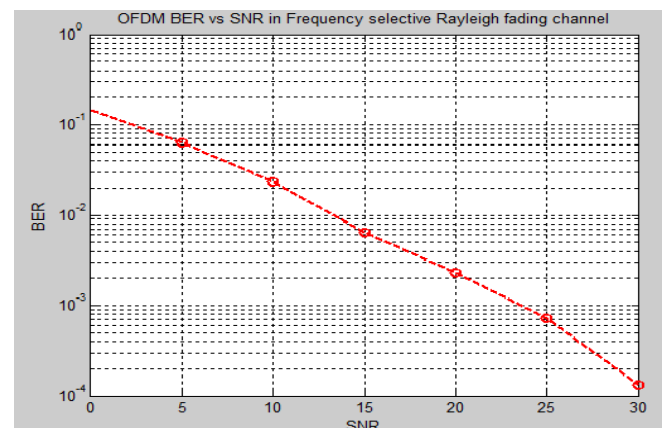


Figure 1: BER vs SNR for OFDM

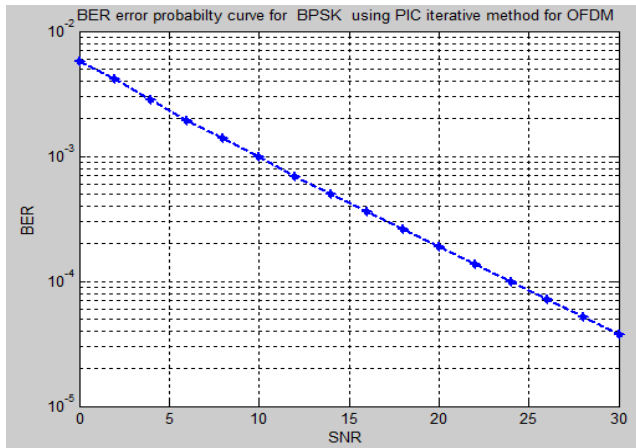


Figure 2: BER vs SNR for PIC

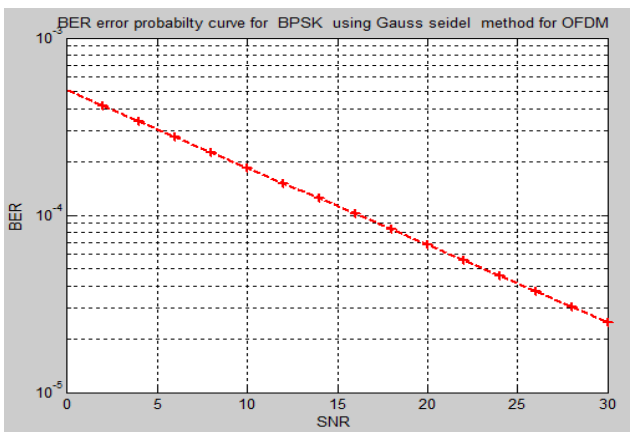


Figure 3: BER vs SNR for Gauss Seidel

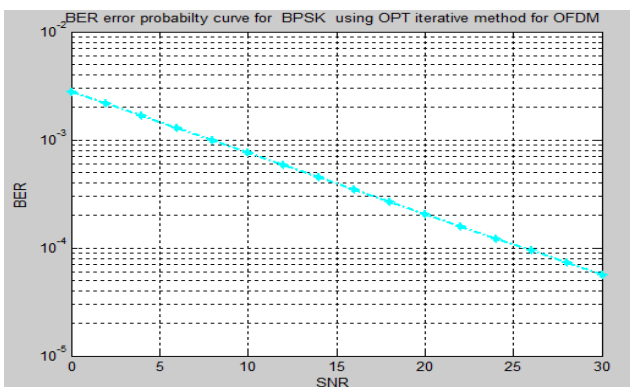


Figure 4: BER vs SNR for OPT

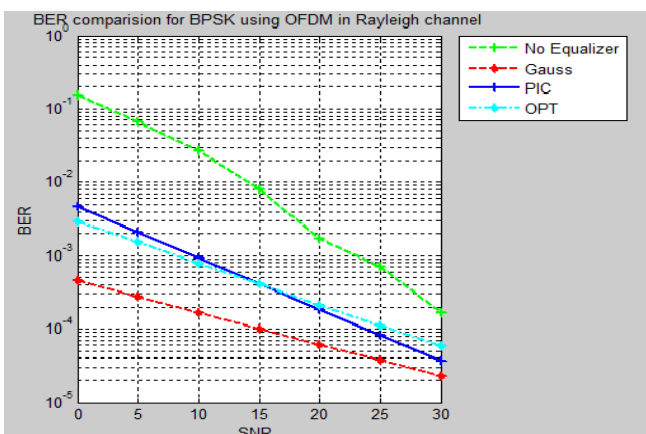


Figure 5: Comparison between PIC, OPT and Gauss

E. Simulation Results:

Comparison results of different iterative ICI cancellation and no equalizer in Rayleigh fading channels are shown in the form of Figure 5 as well as table 2.0.

Table 2.0

SNR db	BER			
	No equalizer	OPT	PIC	Gauss
5	6.6e ⁻⁰²	1.7e ⁻⁰³	2.3 e ⁻⁰³	2.9 e ⁻⁰⁴
10	2.7 e ⁻⁰²	8.9 e ⁻⁰³	9.8 e ⁻⁰⁴	1.8 e ⁻⁰⁴
15	7.9 e ⁻⁰²	4.4 e ⁻⁰⁴	4.2 e ⁻⁰⁴	1.1 e ⁻⁰⁴
20	1.7e ⁻⁰³	2.3 e ⁻⁰⁴	1.9 e ⁻⁰⁴	6.5e ⁻⁰⁵
25	7.0e ⁻⁰⁴	1.2 e ⁻⁰⁴	8.9 e ⁻⁰⁵	3.9 e ⁻⁰⁵
30	1.7 e ⁻⁰⁴	6.0 e ⁻⁰⁵	3.9 e ⁻⁰⁵	2.4 e ⁻⁰⁵

VI. CONCLUSION

In this Paper, different ICI reduction methods are analyzed using the iterative methods. ICI originates from dispersive channels, pulse shaping, or a missing cyclic prefix and has to be equalized for reliably data transmission. Compared to direct matrix inversion techniques that requires O(n³) operations, iterative interference cancellation only needs O(n²) and time variant channels also can be equalized in iterative methods. While Gauss-Seidel method is no more difficult to use than the Jacobi method, and it often requires fewer iterations to produce the same degree of accuracy. For example only five iterations of the Gauss-Seidel method, one can achieved the same accuracy as was obtained with seven iterations of the Jacobi method. As shown in the Table 2.0, Gauss-seidel outperforms better than others for the same no. of iterations.

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