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Direction of Departure (DOD) and Direction of Arrival Estimation in Bistatic MIMO Radar Using Covariance Recovery of Sparse Targets

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Abstract— **This letter propose a new method based on covariance sparse target recovery for Direction of Departure (DOD) and Direction of Arrival (DOA) in bistatic multi -input multi-output radar. The letter use this property that the number of target in the space is low. In this paper we recover the power of incident signal as a diagonal matrix which its diagonal element are corresponding to the impinging signals. We show that our method doesn't need the number of sources and also it is independent from the coherency between the signals. Simulation results demonstrate the effectiveness of our algorithm compared with other methods.**

Keywords- MIMO, Radar, DOA and DOD ,Sparse.

I. INTRODUCTION

Recently, there has been a noticeable interest in a new class of radar systems called multiple input, multiple output. MIMO radar system transmits orthogonal waveforms in each of antenna and extract them by match filters in the receiver It has been demonstrated that the MIMO radar have a more degrees of freedom for overcoming fading effects, enhancing spatial resolution, reducing radar cross section fluctuations, and also increase detection performance[1],[2]. Estimation method of angle estimation via rotational invariance technique (ESPRIT) algorithm adapted for DOA estimation in[3]. [4] Present another Esprit algorithm that has a lower complexity than [3], but they both have almost the same angle estimation performance. Reduced multiple signal classification (MUSIC) algorithm introduced in [5] to reduce 2D search in to 1D search. But the mentioned methods work when the number of targets is known and there isn't no coherency between the incident signals to the receive array. They also need more snapshots to approximate the covariance matrix, whereas number of sample in one coherent processing interval (CPI) is less than the number of snapshots that they need. Sparse signal representation has developed rapidly and has attracted a lot of attention in the last decade. Sparse representation methods exhibit a number of advantages over other DOA estimation methods. These include increased resolution and improved robustness to noise and limited number of snapshots. L1-SVD method [6] employs L1-norm to enforce sparsity and singular value decomposition to reduce complexity and sensitivity to noise, for source localization. But this method is very sensitive to the regularization parameter. [7] introduced another sparse based DOA technique which has a higher speed than[6], but the method has a high complexity because of calculating the inverse of matrix array covariance.

In this letter a new method for DOD and DOA estimation based on sparse covariance recovery of the impinging signal in bistatic radar system introduced, we assume that the incident signals to the receiving array are uncorrelated. And then we recover the covariance of these signal by using the L1- norm optimization problem. Because our method searches in one dimensional, it has a low complexity. Also it has a high performance, low complexity compared with [6].The proposed method can estimate the correlated incident signals because we don't recover the steering vectors by using signal subspace, therefor our method is independent form the coherency between the impinging signals. Simulation results show the efficiency of the method.

Notation: $(.)^T (.)^H$ denote transpose and conjugate, **respectively** $\|.\|_1$,$\|.\|_2$ and $\|.\|_f$ stand for L1-norm, L2-norm and Forbenius norm, respectively. $\otimes$ is Kronecker product; $E(.)$ is Expected value,

II. ANGLE ESTIMATION IN MIMO RADAR

A. Data model

Consider a bistatic MIMO radar with M half-wavelength spacing transmitting antennas and N half-wavelength spacing receiving antennas. Without loss of generality Assume that receiving and transmitting antennas are linear array (it is possible that they have a non-uniform array configuration, but in our method we need to know the array). It is also assumed that there are K targets (For simplicity we assume they are coherent). M different narrow-band wave form are emitted simultaneously , which have identical bandwidth and center frequency, but are temporally orthogonal, the output of the matched filters in baseband at the receiver can be expressed as[8].

$$\mathbf{y}(t) = [\mathbf{a}_r(\theta_1) \otimes \mathbf{a}_t(\phi_1), \mathbf{a}_r(\theta_2) \otimes \mathbf{a}_t(\phi_2), \ldots, \mathbf{a}_r(\theta_K) \otimes \mathbf{a}_t(\phi_K)]\mathbf{s}(t) + \mathbf{n}(t) \quad (1)$$

$$\begin{aligned} \mathbf{a}_t(\theta_i) &= [1, \exp(-j2\pi(d/\lambda)\sin(\theta_i), \ldots, \exp(-j2\pi(M-1)(d/\lambda)\sin(\theta_i)] \\ \mathbf{a}_r(\phi_i) &= [1, \exp(-j2\pi(d/\lambda)\sin(\phi_i), \ldots, \exp(-j2\pi(N-1)(d/\lambda)\sin(\phi_i)] \end{aligned} \quad (2)$$

Where $\mathbf{y}(t)$ is a $MN \times 1$ vector; $\mathbf{a}_t(\theta_i)$, $\mathbf{a}_r(\phi_i)$ are transmit steering vector for the θ_i and receive steering vector for ϕ_i ,($i = 1, 2, \ldots, K$), that are constant in each CPI; λ is the Wavelength of incident signals; $\mathbf{S}(t) = [s_1(t) \ s_2(t) \ \ldots \ s_K(t)]^T$ is a column vector consisting of the phases and amplitudes of the K narrow band coherent signals sources at time t , and $s_k(t) = \alpha_k \exp(j2\pi f_k t)$ whit f_k being Doppler frequency and α_k the amplitude; $\mathbf{n}(t)$ is the received additive white Gaussian noise.

III. DOA AND DOD ESTMATION BASED ON SPARSE SIGNAL RECOVERY

A. *Sparse representation of incident signals*

We start to formulate the DOA and DOD estimation problem for bistatic MIMO radar as a sparse representation:

$$\mathbf{x} = \mathbf{A}(\theta, \phi)\mathbf{S} + \mathbf{n} \quad (3)$$

Where $\mathbf{y}$ is a $MN \times 1$ vector (Single Snapshot problem); $\mathbf{n}$ is the Gaussian noise, $\mathbf{A}$ is an over-complete basis matrix, corresponds to the receive steering vector and transmit steering vector which express as follow:

$$\begin{aligned} \mathbf{A}(\theta, \phi) = [&\mathbf{a}_t(\tilde{\theta}_1) \otimes \mathbf{a}_r(\tilde{\phi}_1), \mathbf{a}_t(\tilde{\theta}_1) \otimes \mathbf{a}_r(\tilde{\phi}_2), \\ &\ldots, \mathbf{a}_t(\tilde{\theta}_1) \otimes \mathbf{a}_r(\tilde{\phi}_P), \ldots, \mathbf{a}_t(\tilde{\theta}_Q) \otimes \mathbf{a}_r(\tilde{\phi}_1), \\ &\ldots, \mathbf{a}_t(\tilde{\theta}_Q) \otimes \mathbf{a}_r(\tilde{\phi}_P)] \end{aligned} \quad (4)$$

The set $\{(\tilde{\theta}_i, \tilde{\phi}_j), i = 1, 2, \ldots, Q \, and \ j = 1, 2, \ldots, P\}$ is a sampling grid of all potential DODs and DOAs in spatial domain, in(equation) $\mathbf{S}$ is a $QP \times 1$ sparse signal(reflected from the all potential DODs and DOAs) vector whose k th is nonzero if the source come from the ($floor(PQ/k)^{\circ}, (\text{mod}(PQ, k)^{\circ})$ and zero otherwise. This property could be used to improve the angle estimation in collocated MIMO radar.in general $Q, P >> K$. It is obvious that $\mathbf{A}$ is a known steering matrix and does not depend on incident DODs and DOAs. Therefore estimate of the angles of arrival can be calculated with finding the sparse vector $\mathbf{S}$ [6]. (Notice that this method is not adopted for MIMO radar but it developed it generally):

$$\begin{aligned} &\min \|\mathbf{S}\|_1 \\ &\text{subject to} \|\mathbf{y} - \mathbf{A}\mathbf{S}\|_2^2 \le \varepsilon^2 \end{aligned} \quad (5)$$

But this method has a high computational complexity, when the number of snapshots increase. To reduce the complexity [7] proposed a method based on Sparse representation of array Covariance, which has a lower complexity when the number of snapshots increase. To reduce the complexity and using the performance of multiple snapshots, we find the covariance of incident signals as a diagonal matrix.

A. *Proposed Method*

The array covariance matrix (3) is given by:

$$\mathbf{R} = E\{\mathbf{y}(t)\mathbf{y}(t)^H\} = \mathbf{A}(\theta, \phi)\mathbf{R}_s\mathbf{A}^H(\theta, \phi) \quad (6)$$

Where $\mathbf{R}_s$ is the $(PQ \times PQ)$ covariance of overcomplete vector, $\mathbf{S}$,. When the impinging signals to the array are uncorrelated, $\mathbf{R}_s$ is a diagonal matrix. We define $\Gamma = diag(\mathbf{R}_s)$ as a vector, which kth is nonzero if source come from the ($floor(PQ/k)^{\circ}, (\text{mod}(PQ, k)^{\circ})$ and zero otherwise. To find the sparse vector Γ , we change the optimization problem (5) into a l-norm problem as follow:

$$\begin{aligned} &\min \ \|G\|_1 \\ &\text{subject to } \|\hat{\mathbf{R}} - \ \mathbf{A}(q,f)\mathrm{L}\mathbf{A}^H(q,f)\|_f^2 < e \end{aligned} \quad (7)$$

where L is a diagonal matrix which its elements on diagonal constitutes vector, G . The ε is the maximum acceptable error which play an important role in the performance of angle of arrival estimation. A small ε emphasizes the role of l_2 norm, which may produce many spurious peaks in spatial spectrum. A large ε emphasizes the role of l_1 norm, which may cause wrong DOD and DOA estimation. The ε is selected so that the probability $\varepsilon^2 \le \|\mathbf{n}\|_2^2$ is small. Because we assumed that $\mathbf{n}$ is Gaussian and its elements are i.i.d, the distribution of $\|\mathbf{n}\|_2^2$ is approximately χ^2 with MN degrees of freedom. We can see the selection of ε needs the knowledge of noise statistic. Other way to choose the ε is to use an optimization package called SeDuMi [9]. Because distribution χ^2 is a good approximation of $\|\mathbf{n}\|_2^2$ for nominal to high Signal to Noise Ratio (SNR).For simplicity we formulate (7) in SOC optimization problem[10] as follow:

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$$\min \; z + g¡$$

subject to: (8)

$$\left\|\hat{\mathbf{R}} - \mathbf{A}(q,f)\mathrm{L}\mathbf{A}^{H}(q,f)\right\|_{f}^{2} < z, \; and \; \|\mathrm{G}\|_{1} < ¡$$

where $\hat{\mathbf{R}} = (1/L)\sum_{i=1}^{L} y(t_i)y(t_i)^H$, the parameter g is the regularization parameter. To choose the appropriate regularization parameter we search on some plausible values for g . Finally the minimum of (5) can be obtained from optimization software such as [10].optimization problem in (8) search for the proper sparse vector G, which is only is dependent on size G, therefore unlike L1-SVD method formulation of our method doesn't change when we use the multiple snapshots for proper DOD and DOA estimation and therefore its complexity is independent from the number of the snapshots.

B. Complexity Analysis

The main computational cost of the proposed method is solving (7) or (8), Complexity of the proposed method is higher than subspace methods such as RD-MUSIC and ESPRIT algorithm where the main complexity is in calculating the array covariance matrix and its EVD. If we use the L sample for angle estimation, complexity of L1-SVD is $O(L^3(QP)^3)$, which shows that complexity of our method is much lower than L1-SVD. L1-SRACV need to calculate the $\mathbf{R}^{-1/2}$, eigenvalue decomposition (EVD) and $O(L^3(QP)^3)$ which show L1- SRACV is much high than our complexity, also this method need to estimate the noise variance. Although complexity of the proposed method is higher than subspace methods such RD-MUSIC, but it is has a higher resolution, higher accuracy, much robust for correlated impinging signals and doesn't need to calculate the number of incident signals,(which is a critical parameter for subspace methods).

IV. SIMULATION RESULTS

Here we consider a some simulations to show the effectiveness of the proposed method for estimationg the Direction of Departure and Direction of Arrival in MONO and BI static MIMO radar with collocated antennas. Our method is compared with the RD-MUSIC algorithm[5] (we didn't consider the ESPRIT algorithm, because its performance is near to the MUSIC). Monte Carlo simulations are carried out to evaluate the performance of angle estimation targets using the proposed method. Define root mean squared error (RMSE) of the angles as $(1/K)\sum_{k=1}^{K}\sqrt{E[(\hat{\theta}_k-\theta_k)^2+(\hat{\phi}_k-\phi_k)^2]}$ to assess the estimation performance. All the numerical results are obtained from 500 independent trial.

In first simulation we consider a MIMO bistatic radar system, with ULA in which the array distance is 0.5λ, and $M=6, N=8$,the number of snapshot is $L=1$. The input signal-to-noise ratio (SNR) is defined as $10\log_{10}(\sigma_k^2/\sigma_n^2)$ where $\sigma_1^2 = \ldots = \sigma_k^2$ .There are three non-coherent targets with their angles correspondingly distributed as $(\theta_1,\phi_1)=(10^\circ,45^\circ)$, $(\theta_2,\phi_2)=(25^\circ,35^\circ)$ and $(\theta_3,\phi_3)=(35^\circ,11^\circ)$ respectively. $(\alpha_1,\alpha_2,\alpha_3)=(1,1,1)$ Figure 2 introduce the excellent performance of the proposed method when we only have a single snapshots in SNR=5dB. it is clear that the subspace method is weak for estimation the DOD and DOA in the MIMO radar. Although the RD-MUSIC has a lower complexity than our method but we must to consider the limitation of number of snapshots in MIMO radar.

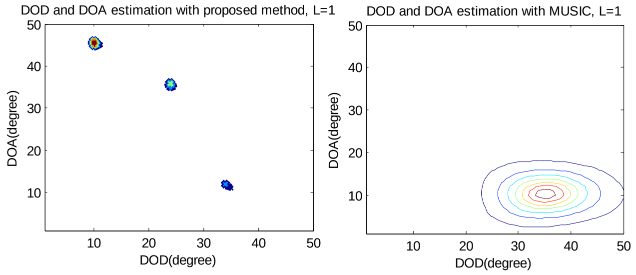

Figure 1. Comparison of our method with RD-MUSIC for bistatic MIMO radar with non-coherent targets.

Second numerical result is shown in figure 3.configration of MIMO is same as first simulation, in this simulation we assume there are three targets, which two targets of them are coherent with correlation coefficient 0.9, DODs and DOAs are same as previous simulation that the target in $(\theta_1,\phi_1)=(10^\circ,45^\circ)$, $(\theta_2,\phi_2)=(25^\circ,35^\circ)$ are coherent. For the MUSIC algorithm number of snapshots is L=20. As can be seen proposed method extremely estimates the DODs and DOAs, While MUSIC failed.

For better understanding we have done third simulation for comparison of proposed method with RD-MUSIC in MONO static MIMO radar. for our method L=1 and for the MUSIC L=20; SNR is 5dB, there are three targets ($\theta_1=10^\circ,\theta_2=24^\circ,\theta_3=39^\circ$) which targets in $\theta_2=24^\circ$ and $\theta_3=39^\circ$ are coherent with correlation coefficient 0.9. Figure 4 show the performance of proposed method in comparison with RD-MUSIC algorithm. Also we can see the resolution of proposed method is higher than RD-MUSIC.

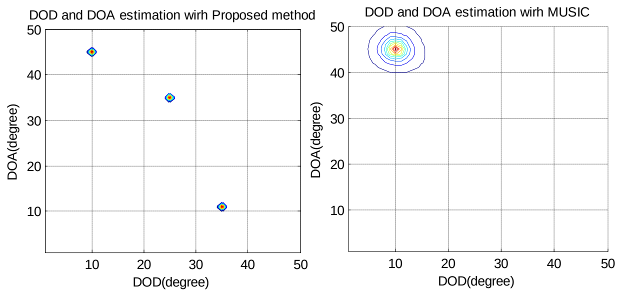

Figure 2.comparison of our method with RD-MUSIC for bistatic MIMO radar with coherent targets

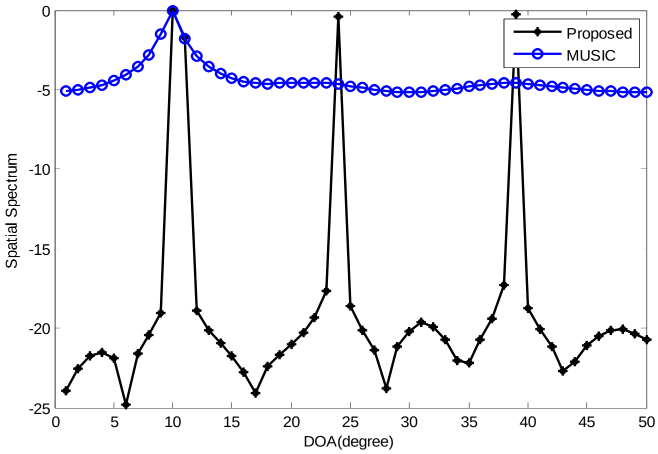

Figure 3.comparison of our method with RD-MUSIC for mono-static MIMO radar with coherent targets

Figure 4 show the average RMSE of angle of arrival estimation between different targets with different SNR, respectively for mono-static MIMO radar. DOAs of target is same as Figure 4, but there is not coherency between the sources. In this figure, we can find that the proposed method can estimate the angle of targets with good performance with only one snapshot.

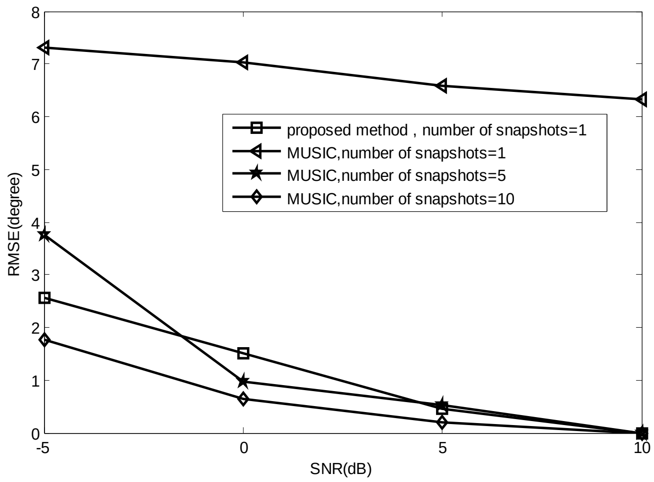

Figure 4.RMSE of DOAs versus SNR

V. CONCLUSION

In this paper we have discussed the joint DOD and DOA estimation in bistatic and mono-static MIMO radar system. We proposed an angle of arrival estimation method based on sparse signal representation, we introduced two case: single snapshot and multiple snapshots. Our method has a more complexity than subspace method for angle of arrival estimation, the proposed algorithm has a high performance, and work very well when the number of snapshots is low (especially L=1). In other way simulation results verified the effectiveness of proposed method in the presence of coherent targets.

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