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Optimizing the energy efficiency in MIMO system using singular value decomposition

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Abstract— **The notion of increasing the channel capacity by employing multiple antennas at transmitting and receiving ends yields the concept called MIMO (Multiple Input Multiple Output).The entire analysis of the MIMO channel is done by employing the channel matrix SVD (singular value decomposition) method, which mathematically transforms the channel into a set of parallel subchannels in a way that it can overcome rank deficiency and makes efficient use of the CSI (Channel State Information) at the transmitter and all subchannels are classified by their channel gain when the energy efficiency model is proposed. Based on the proposed subchannel grouping scheme the effective capacity of each grouped subchannels can be optimized. Therefore multichannel joint optimization problem is transformed into a multitarget single channel optimization problem. In addition, EEOPA algorithm is proposed to boost the energy-efficiency of MIMO system. Simulation results facilitate that proposed algorithm has advantage on improving the energy-efficiency and as well as effective capacity of MIMO system with QoS constraints. Hence optimizing the energy-efficiency in channels of MIMO system is done very efficiently under Qos statistical** ***exponent and average power constraint.***

Index Terms— **MIMO, SVD, Energy efficiency, Power allocation threshold, Qos exponent, Power constraint.**

I. INTRODUCTION

With the rapid development in information and communication technology the demand of different multimedia services and different internet supported applications on mobile devices requires a high speed data rate and good service of quality. This can be accomplished by multiple Antenna technology called MIMO. On the other hand MIMO can fulfill 3G & 4G demand and the MIMO diversity and MIMO multiplexing are the key factors to discuss and matter of concern is to achieve and support high speed data rate. MIMO multiplexing is a way to gain robustness and achievement in speed of data information.

With the increasing demand for improving the energy efficiency in multimedia communication system, various resource allocation schemes such as power allocation, bandwidth allocation and subchannel allocation have evolved. The energy efficiency is analyzed in fading channels, where the effective capacity was considered as a measure of the maximum throughput under certain statistical QoS constraints [1]. QoS driven power and rate adaptation scheme over wireless links was proposed for mobile wireless networks based on the effective capacity of the block fading channel model [2].

Furthermore, some QoS-driven power and rate adaptation schemes were proposed and analyzed for diversity and multiplexing systems by integrating information theory with the effective capacity [3].

In this paper, we further evaluate the energy-efficiency optimization for MIMO systems. Based on the matrix theory, numerous channel models have been proposed for MIMO communication, moreover, an expression for the marginal pdf (mpdf) of the ordered eigenvalues of complex noncentral Wishart matrices was derived to analyze the performance of singular value decomposition (SVD) in MIMO communication systems with Rician fading channels [4]. By using one of the most advanced technologies like Multiple-input–multiple-output (MIMO) can create independent parallel channels to transmit data streams, which improves spectral efficiency and system capacity without increasing the bandwidth requirement [8].

Practical MIMO systems involve multiple services, such as audio and video services, which are very sensitive to the delay parameter. These services have different QoS constraints. In view of this, the effective capacity of each subchannel mainly depends on the corresponding QoS constraint. By adopting a statistical QoS constraint, the effective capacity of each subchannel is calculated as the practical output in MIMO systems. Simulation results showed that multichannel communication systems can achieve both high throughput and stringent QoS at the same time. However, there has been research work addressing the problem of optimizing the energy efficiency under different QoS constraints in MIMO systems.

This paper is devoted to optimizing the energy-efficiency with statistical QoS constraints in MIMO systems. All subchannels in MIMO systems are first grouped by their channel gains. On this basis, a subchannel grouping scheme is developed to allocate the corresponding transmission power to each of the subchannels in different groups, which there by simplifies the multichannel optimization problem to a multitarget single-channel optimization problem.

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II. SYSTEM MODEL

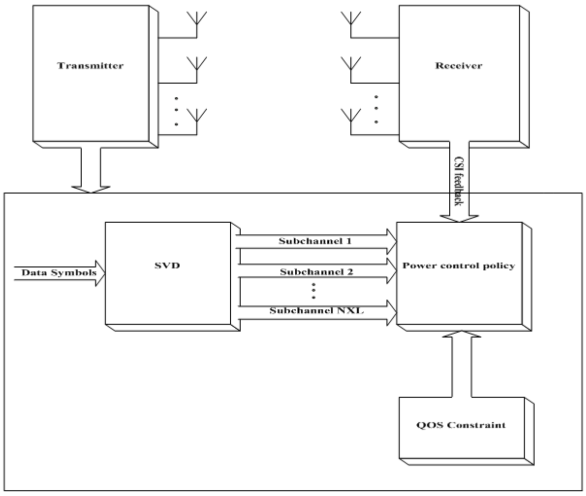

Fig.1: MIMO system model

Consider a MIMO system as shown in Fig.1. It has $N_r \times N_t$ antenna matrix, where N_r is the number of receive antennas and N_t is the number of transmit antennas.
A MIMO system is usually described as follows:

$$Y = Hx + n \tag{1}$$

Let C denote the complex space, where $y \in C^{Nr}$ is the received signal vector, $x \in C^{Nt}$ is the transmitted signal vector, $H \in C^{Nr \times Nt}$ is the channel matrix and $n \in C^{Nr}$ is additive white gaussian noise. Assume $E\{nn^H\} = I^{Nr \times Nr}$, where $E\{\cdot\}$ denotes the expectation operator.

First, the upper-layer packets are divided into frames at the data link layer from the transmit antennas, which forms the data symbols and into bit streams at the physical layer. The frame duration is denoted by T_f, which is assumed to be less than the fading coherence time. Assume discrete-time channel experience block fading. Based on this assumption, the channel gain is invariant within the frame duration T_f but varies independently from one another. Making such an assumption is mainly based on the following reasons. First, the effective capacity in a block fading channel only depends on marginal statistics of a service process. Second, more importantly, there exists a simple and efficient approach to convert the power adaptation policy obtained in block-fading channels to that over correlated fading channels. Further assume that given the transmit power, the specific multichannel transmission scheme and an instantaneous channel gain can achieve the Shannon capacity. Based on the above two assumptions, for each given power-control policy, the resulting effective capacity reaches its maximum for all channel realizations.

SVD splits the channel matrix M ($N = \min(N_t, N_r)$) at each subcarrier into parallel single-input–single-output (SISO) channels i.e subchannels. It also provides power allocation to each subchannel and also total power gain of the channel matrix with the singular values that leads to full capacity utilization of the MIMO system. Then it removes noises and liner dependent elements by using only the most important singular values. The perfect channel state information (CSI) is available and perfectly estimated at the receiver and its availability is facilitated through CSI feedback to the transmitter without delay.

Furthermore, an average transmission power constraint P is configured for each subchannel in the MIMO system. With this average transmission power constraint, transmitters are able to perform power control adaptively according to the feedback CSI and QoS constraints. Thus the energy efficiency in the MIMO system can be optimized.

III. ENERGY-EFFICIENCY MODELING OF MIMO SYSTEM

By applying SVD method to the channel matrix $\boldsymbol{H} = \boldsymbol{U\tau V}^{T}$, where U and V are Identity matrices and τ is Diagonal matrix. Diagonal matrix has non-zero entries only along its diagonal. Then diagonal elements are arranged in the descending order of their magnitude by decomposition which is the form of diag $(\tau_1, \tau_2 \ldots\ldots\ldots, \tau_p)$ is the singular values of channel matrix commonly known as subchannel gain set. Hence channel at each data symbol is decomposed into parallel SISO subchannels by the SVD method.

In this paper, the effective capacity of each subchannel is taken as the data rate with a certain QoS constraint. So the total effective capacity of N x L subchannels is taken as the system output and the total transmission power allocated to N x L subchannels is treated as the system input. As a consequence, the energy efficiency of MIMO systems is given by [2]:

$$\eta = \frac{C_{total}(\theta)}{E\{P_{total}\}} = \frac{\sum_{k=1}^{M}\sum_{l=1}^{N} C_{eff}(\theta)_{k,l}}{E\{P_{total}\}} \tag{2}$$

Where $C_{eff}(\theta)_{k,\,l}$ $(k = 1, 2, \ldots, M,\ l = 1, 2, \ldots, N)$ is the effective capacity of the k^{th} sub channel over the l^{th} symbol, and $E\{P_{total}\}$ is expectation of the total transmission power allocated to all N x L sub channels.

A. QoS Statistical exponent Guarantees:

θ is the QoS statistical exponent, which indicates the exponential decay rate, the queue length process $Q(t)$ converges in distribution to a random variable (r.v.).

$$\theta = -\lim_{t \to \infty} \log\left(\frac{P(\theta \geq t)}{t}\right) \tag{3}$$

It states that smaller θ corresponds to a slower decay rate, which implies that the system corresponds to a looser QoS guarantee, while a larger θ leads to a faster decay rate, which means that a more stringent QoS requirement can be supported. Due to its relationship with statistical QoS provisioning, θ is called the QoS exponent.

B. The Effective Capacity:

Now the effective capacity of each subchannel depends on the corresponding QoS constraint. The effective capacity is defined as the maximum constant arrival rate that a wireless channel can sustain while satisfying statistical QoS constraints. The effective capacity can be defined as follows.

$$C_{eff}(\theta) = -\frac{1}{\theta}\log\left(E\left(e^{-\theta R}\right)\right) \tag{4}$$

$$R = T_f B \log_2(1 + \mu(\theta,\tau)\tau) \tag{5}$$

where R denotes the bit rate within the frame duration, τ denotes the subchannel gain, and μ (θ, τ) denotes the transmission power allocated to a subchannel.
After SVD of channel matrices, N x L parallel subchannels is accessed. The channel gain over each of these N x L parallel subchannels follows mpdf. Assuming as the mpdf of channel gain over the k^{th} ($k = 1, 2, \ldots M$) subchannel at the l^{th} ($l = 1, 2, \ldots, N$) subcarrier, then the effective capacity $C_{eff}(\theta)_{k,l}$ over the k^{th} subchannel at the lth subcarrier is derived as

$$C_{eff}(\theta)_{k,l} = -\frac{1}{\theta} \log \left(\int_0^\infty e^{-\theta T_f B \log_2\left(1+\mu_{k,l}(\theta,\tau)\tau\right)} p\tau_{k,l}(\tau)d\tau\right) \tag{6}$$

where $\mu_{k,l}(\theta, \tau)$ is the transmission power allocated to the k^{th} subchannel at the l^{th} subcarrier.

An average transmission power constraint P over each subchannel is a function of not only the instantaneous SNR but also on the QoS exponent θ is defined as

$$P = \int_0^\infty \mu_{k,l}(\theta,\tau) p\tau_{k,l}(\tau)d\tau \tag{7}$$

The expectation of transmission power **E** {P_{total}} is given by

$$E\{P_{total}\} = P \text{ x } N \text{ x } L \tag{8}$$

By substituting (7) and (8) in (2), the energy efficiency model as given as

$$\eta = \frac{\sum_{k=1}^{M}\sum_{l=1}^{N} -\frac{1}{\theta}\log\left(\int_0^\infty e^{-\theta T_f B\log_2\left(1+\mu_{k,l}(\theta,\tau)\tau\right)} p\tau_{k,l}(\tau)d\tau\right)}{P \text{x} N \text{ x} L} \tag{9}$$

IV. OPTIMIZATION OF ENERGY-EFFICIENCY IN MIMO SYSTEM

In MIMO systems, channel gain depends on the distribution of eigenvalues of Hermitian channel matrix $\mathbf{HH}^*$, where the elements of **H** are complex valued with real and imaginary parts by a normal distribution N (0, 1/2) with mean value of 0 and variance value of 1/2. [4]

Based on the results of SVD channel matrix, subchannels at each subcarrier are sorted in a descending order of channel gains. Starting from the joint pdf of eigenvalues of the Wishart channel matrix, the subchannel gain derived from mpdf of subchannels ordered at the k^{th} position in the descending order of channel gains is derived. Further, all subchannels at L subcarrier are grouped based on their mpdfs. Then sub channel grouping results to gives a close form solution to optimize the energy efficiency of MIMO systems.

Optimization Solution of Energy Efficiency:

In order to maximize the energy efficiency of MIMO systems with statistical QoS constraints, the optimization problem can be obtained as

$$\eta_{opt} = max\left\{\frac{\sum_{k=1}^{M}\sum_{l=1}^{N} -\frac{1}{\theta}\log\left(\int_0^\infty e^{-\theta T_f B\log_2\left(1+\mu_{k,l}(\theta,\tau)\tau\right)} p\tau_{k,l}(\tau)d\tau\right)}{P x \ N \ x \ L}\right\}$$
$$= \frac{max\left\{\sum_{k=1}^{M}\sum_{l=1}^{N} -\frac{1}{\theta}\log\left(\int_0^\infty e^{-\theta T_f B\log_2\left(1+\mu_{k,l}(\theta,\tau)\tau\right)} p\tau_{k,l}(\tau)d\tau\right)\right\}}{P x \ N \ x \ L} \tag{10}$$

where η_{opt} is the optimized energy efficiency. From the above equation (10) the energy efficiency of MIMO systems mainly depends on transmission power-allocation results $\mu_{k,l}(\theta, \tau)$ over N x L subchannels. Now, the optimization problem in (10) is a multichannel optimization problem, which is necessary to obtain a closed-form solution in mathematics.

In most recent studies on MIMO systems, the energy-efficiency optimization problem is solved by a single-channel optimization model [3]. How to change the multichannel energy-efficiency optimization problem into the single channel energy-efficiency optimization problem and to derive a closed-form solution are great challenges in this paper.

The optimized transmission power allocation $\mu_{opt}(\theta, \tau)$ is given as follows [4]

$$\mu_{opt}(\theta,\tau) = \begin{cases} \dfrac{1}{\beta\frac{1}{\alpha+1}\tau\frac{\alpha}{\alpha+1}} - \dfrac{1}{\tau} & \tau \geq \beta \\ 0 & \tau < \beta \end{cases} \tag{11}$$

where β is the transmission power-allocation threshold over a subchannel and α is the normalized QoS exponent.
An average transmission power constraint P is determined for each sub channel. Thus, the transmission power-allocation threshold of each sub channel must satisfy the following constraint:

$$\int_{\beta_{k,l}}^{\infty}\left(\frac{1}{\beta_{k,l}\frac{1}{\alpha+1}\tau\frac{\alpha}{\alpha+1}} - \frac{1}{\tau}\right) p\,\tau_{k,l}(\tau)d\tau \leqslant P \tag{12}$$

Therefore sub channel grouping is carried out as follows

1. Sub channels are sorted in descending order of sub channel gains at each subcarrier:
$$\tau_{1,l} \geq \tau_{2,l} \geq \cdots \geq \tau_{M,l} \geq 0$$
where $l = 1, 2 \ldots N.$
2. For n = 1: N, select the sub channels with the same order position into different channel groups at different subcarriers.
3. Repeat steps 1 and 2 for all symbols.
4. M groups with the same order position subchannels are achieved.

Based on the above proposed subchannel grouping scheme, effective capacity can be optimized for each grouped subchannel according to their mpdfs. So it solves the multichannel joint optimization problem into a multitarget single-channel optimization, which significantly reduces the complexity of optimized energy-efficiency.

$$\int_{\beta_n}^{\infty}\left(\frac{1}{\beta_n\frac{1}{\alpha+1}\tau\frac{\alpha}{\alpha+1}} - \frac{1}{\tau}\right) x \left(\int_{M-1} \ldots \int p(\tau_1,\tau_2,\ldots\tau_M)\, d\tau_i d\tau_{i+1} \ldots d\tau_j\right) x\, d\tau \leqslant P,$$

$$1 \leqslant i < j \leqslant M \text{ and } i \neq n, j \neq n \tag{13}$$

where β_n $(1< n< M)$ is the transmission power-allocation threshold of the n^{th}-group sub channels.

Based on the transmission power-allocation threshold for individual grouped sub channels in (13), then the optimized transmission power allocation for the nth group subchannels is defined as follows:

$$\mu_{opt_n}(\theta,\tau) = \begin{cases} \dfrac{1}{\beta_n\frac{1}{\alpha+1}\tau\frac{\alpha}{\alpha+1}} - \dfrac{1}{\tau} & \tau \geq \beta_n \\ 0 & \tau < \beta_n \end{cases} \tag{14}$$

Hence the solution for optimizing the energy efficiency in MIMO systems with Qos constriants is determined as

$$\eta_{opt} = \frac{\sum_{n=1}^{M} -\frac{L}{\theta}\log\left(\int_0^\infty e^{-\theta T_f B\log_2\left(1+\mu_{opt\_n}(\theta,\tau)\tau\right)} p\tau_n(\tau)d\tau\right)}{P \ x \ N \ x \ L} \tag{15}$$

$$= -\frac{1}{\theta \text{ X P x L}}\sum_{n=1}^{M}\log \text{x}\left(\int_0^\infty e^{-\theta T_f B\log_2\left(1+\mu_{opt\_n}(\theta,\tau)\tau\right)} p\tau_n(\tau)d\tau\right) \tag{16}$$

FLOWCHART:

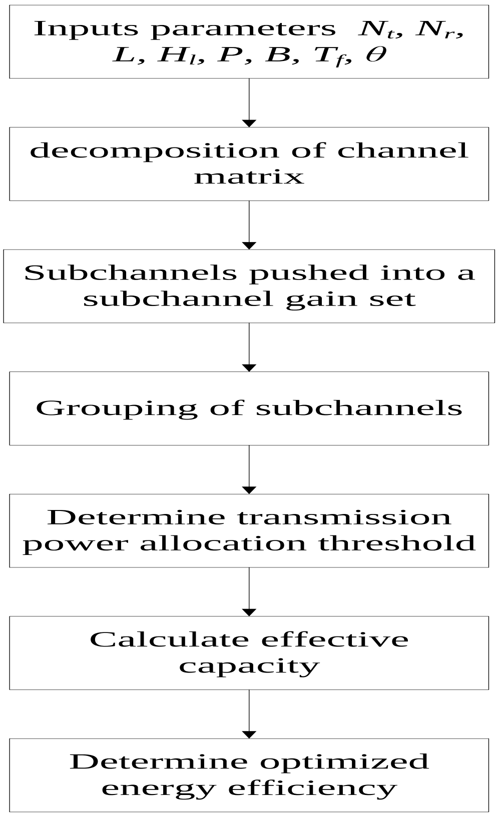

fig2**:** Flowchart of EEOPA (energy-efficiency optimized power allocation Algorithm)

V. SIMULATION RESULTS

The effective capacity and the energy efficiency can be evaluated in MIMO systems by configuring three typical scenarios with different antenna numbers in Figs. 2 and 3: 1) $M_t = 2$ & $M_r = 2$2) $M_t = 3$ & $M_r = 2$ and 3) $M_t = 4$ & $M_r = 4$.

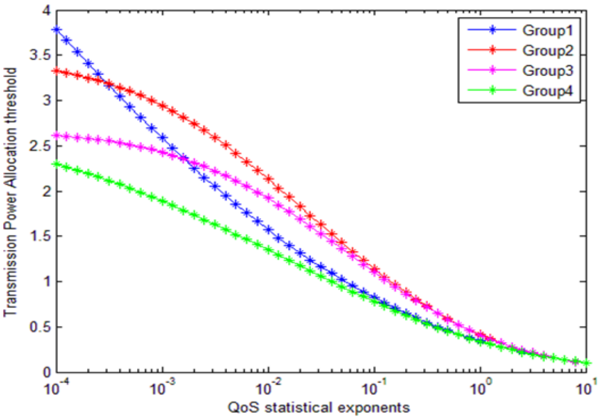

Fig.3. Transmission power-allocation threshold βn w.r.t each grouped subchannels considering different QoS statistical exponents θ for 4x4.

As the subchannels are sorted by the descending order of subchannel gains, then subchannel gain of subchannel groups decreases with the increase in group indexes, when the QoS exponent $\theta \leq 10^{-3}$. When the QoS exponent $\theta > 10^{-3}$, the transmission power-allocation threshold βn starts to decrease with the increase in subchannel gains in subchannel groups and also channel matrix is associated with a unique non-zero singular values (Eigen values) and the channel gain i.e columns of H are along the same direction.

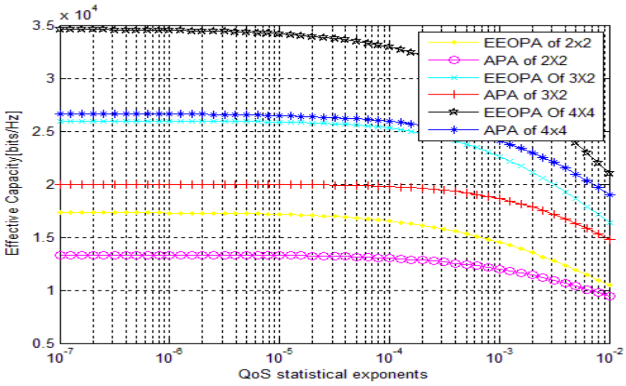

Fig.4. Effective capacity Ctotal θ of the EEOPA and APA algorithms w.r.t the QoS statistical exponent θ considering different scenarios.

The effective capacity of the MIMO channel is assessed by the rank of H matrix with only the non- zero Eigen mode and also on its condition number of the channel matrix H i.e, the total power gain is distributed among the channel which facilitate larger capacity in the high SNR regime. Hence by comparing the EEOPA and APA algorithm from the graph, the effective capacity for different antenna combinations is progressively increased with the Qos statistical exponent.

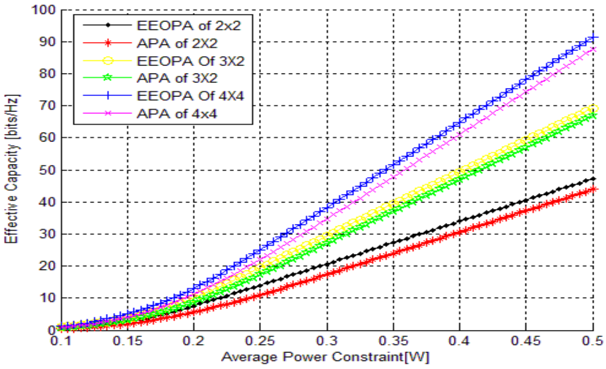

Fig.5. Effective capacity Ctotal θ of the EEOPA and APA algorithms w.r.t the average power constraint P considering different scenarios.

The effective capacity increases with the increase in the average power constraint. Since the channel gain of parallel subchannels follows the mpdf which mainly depends on the singular values of channel matrix. Hence by comparing the EEOPA and APA algorithm for different antenna combinations from the graph, the effective capacity is exponentially increased from zero, when the power constraint is fixed as p = 0.1.

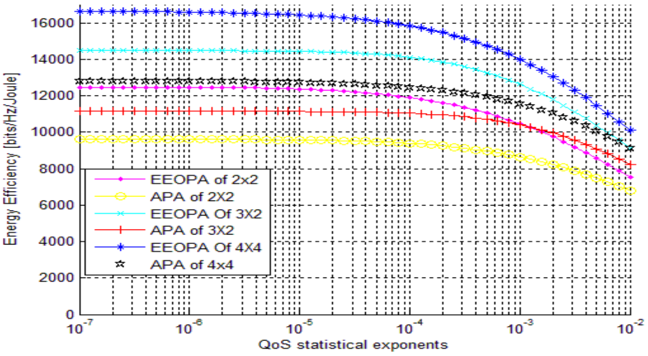

Fig.6. Energy efficiency η of the EEOPA and APA algorithms w.r.t the QoS statistical exponent θ considering different scenarios.

To optimize the energy efficiency, transmission power-allocation threshold is the main parameter of MIMO system. The energy efficiency decreases with the increase in the QoS statistical exponent θ, which means efficiency mainly depends on the transmission power-allocation threshold over the subchannels. Thus the channel spatial multiplexing can improve the energy efficiency.

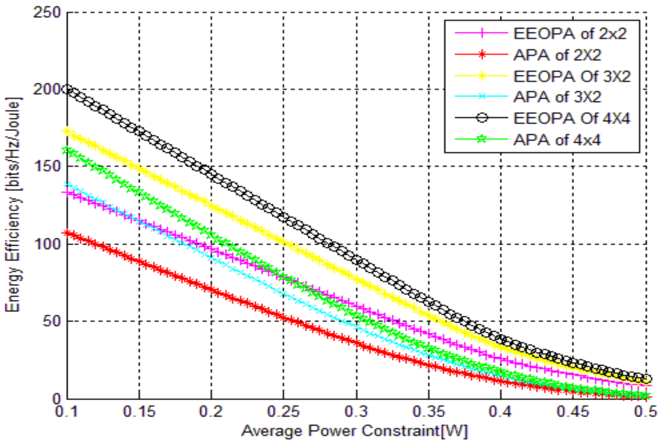

Fig.7. Energy efficiency η of the EEOPA and APA algorithms w.r.t the average power constraint P considering different scenarios.

The energy efficiency decreases with the increase in the average transmission power constraint P, as the transmission power-allocation threshold decreases with the increase in the average power constraint P for any grouped subchannels .That means, the energy efficiency η increases with the increase in the subchannel gains in grouped subchannels when the average power constraint $P \leq 0.15$. While when the $P > 0.15$, the the energy efficiency η starts to decrease with the increase in subchannel gains in grouped subchannels.

VI. CONCLUSION

In this paper, the idea of increasing the channel capacity by employing multiple antennas at transmitting and receiving ends yields the concept called MIMO (Multiple Input Multiple Output). By employing the channel matrix SVD (singular value decomposition) technique, which is superior to the other schemes (such as V-BLAST) in a way that it can overcome rank deficiency and makes efficient use of the CSI (Channel State Information) at the transmitter and all subchannels are classified by the their channel gain when the energy efficiency model is proposed to MIMO system with QoS constraints. Based on the proposed subchannel grouping scheme the effective capacity of each grouped subchannels can be optimized. Therefore multichannel joint optimization problem is transformed into a multitarget single channel optimization problem. In addition, EEOPA algorithm is proposed to improve the energy-efficiency of MIMO system. By comparing with the traditional APA algorithm, simulation results facilitate that proposed algorithm has advantage on improving the energy-efficiency and as well as effective capacity of MIMO system with QoS constraints. Thus the tradeoff between effective capacity and energy-efficiency was analyzed. Hence optimizing the energy-efficiency is done very efficiently under Qos statistical exponent and average power constraint.

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