

# Denoising of Images Using Empirical Wavelet Transform and Gram Schmidt Orthogonalisation

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**Abstract**— Empirical Wavelet is a type of wavelet that is adapted to the processed signal. This is an adaptive signal decomposition method, which decomposes the signal or image into a number of segments known as modes. These modes correspond to the AM –FM components of that signal. This Wavelet transform is applicable for denoising, decompression etc. Denoising is used to remove the noise content from the signal and to recover the original signal. While performing denoising using EWT it is observed that the segmented modes are not orthogonal. In order to improve the PSNR values and to make the modes orthogonal Gram Schmidt orthogonalisation procedure is used.

**Index Terms**— AM-FM components, Denoising, Empirical, Wavelet.

## I. INTRODUCTION

In signal processing, time frequency analysis means analyze a signal in both time and frequency domain simultaneously. Signal analysis in adaptive manner is very useful for signal processing. Generally we can represent a signal as a linear combination of basis functions. In methods like wavelet and Fourier transform these basis functions are derived independently, but in adaptive techniques these functions are derived from the information contained in the signal. Jerome Gilles proposed a new approach to build adaptive wavelets that is known as Empirical Wavelet Transform. This method is able to separate the Nonlinear and Non-stationary part of the signal. These components have a compact support Fourier spectrum.

These techniques are applicable for signal denoising. Denoising is a technique that is used to remove noise content from the signal and to reconstruct the original signal. In the field of signal processing denoising is still a challenging problem. So many methods are there to remove noise content from the signal and to recover the original signal. Each of these methods has their own advantages and limitations. Wavelet transform analysis has been widely used for the purpose of denoising. Traditional denoising schemes are based on linear methods. That is not suitable for nonlinear and

non-stationary signals. To perform signal denoising in nonlinear and non-stationary signals an adaptive signal denoising method using Empirical wavelet transform and Gramschmidt orthogonalisation is proposed in this paper. Gramschmidt orthogonalisation is used because the obtained modes are not orthogonal. The usage of orthogonalisation increases the PSNR values.

## II. EMPIRICAL WAVELET TRANSFORM

Empirical Wavelet Transform is an adaptive method using this method it is able to separate the nonlinear and non-stationary part of the signal. This method is effective for find out the amplitude modulated – frequency modulated that means AM –FM component of a signal. This AM – FM components have a Fourier spectrum and that spectrum is centered around a specific frequency.

In this method first we take the Fourier spectrum of the image and this whole spectrum is divided into a number of segments known as modes. This segmentation provides adaptability to this wavelet transform. Therefore the segmentation of Fourier spectrum is an important task.

The representation of Empirical Wavelet Transform is similar to that of classical wavelet Transform. This wavelet also contains an approximation coefficient and a detail coefficient. [1]

The detail coefficients can be represented as,[1]

$$W_f^s(n, t) = \langle f, \psi_n \rangle = \int \overline{f(\tau)} \psi_n(\tau-t) d\tau = (\widehat{f}(\omega) \psi_n^-(\omega))^v$$

And the approximation coefficients by[1]

$$W_f^s(0, t) = \langle f, \phi_1 \rangle = \int \overline{f(\tau)} \phi_1(\tau-t) d\tau = (\widehat{f}(\omega) \phi_1^-(\omega))^v$$

Where  $\psi_n^-(\omega)$  and  $\phi_1^-(\omega)$  are defined by[1]

$$\Phi_n^-(\omega) = \begin{cases} 1 & \text{if } |\omega| \leq (1-\gamma)\omega_n \\ \cos\left[\frac{\pi}{2}\beta\left(\frac{1}{2\gamma}(|\omega|-(1-\gamma))\right)\right] & \text{if } (1-\gamma)\omega_n \leq |\omega| \leq (1+\gamma)\omega_n \\ 0 & \text{otherwise} \end{cases}$$

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$$\Psi_n^{-1}(\omega) = \begin{cases} 1 & \text{if } (1+\gamma)\omega_n \leq |\omega| \leq (1-\gamma)\omega_{n+1} \\ \cos\left[\frac{\pi}{2}\beta\left(\frac{1}{2\gamma}\left(|\omega| - (1-\gamma)\omega_{n+1}\right)\right)\right] & \text{if } (1-\gamma)\omega_{n+1} \leq |\omega| \leq (1+\gamma)\omega_{n+1} \\ \sin\left[\frac{\pi}{2}\beta\left(\frac{1}{2\gamma}\left(|\omega| - (1-\gamma)\omega_n\right)\right)\right] & \text{if } (1-\gamma)\omega_n \leq |\omega| \leq (1+\gamma)\omega_n \\ 0 & \text{otherwise} \end{cases}$$

The reconstructed signal can be represented as, [1]

$$f(t) = W_f^{-1}(0, t) * \phi_1(t) + \sum_{n=1}^N W_f^{-1}(n, t) * \psi_n(t) \\ = (W_f^{-1}(0, \omega) \phi_1^{-1}(\omega) + \sum_{n=1}^N W_f^{-1}(n, \omega) \psi_n^{-1}(\omega)) v$$

### III. GRAM-SCHMIDT ORTHOGONALISATION

Orthogonalization is the process of finding a set of orthogonal vectors that span a particular subspace. In mathematics, the Gram–Schmidt process is a method for orthonormalising a set of vectors in an inner product space, most commonly the Euclidean space  $R^n$ . The Gram–Schmidt process takes a finite, linearly independent set  $S = \{v_1, \dots, v_k\}$  for  $k \leq n$  and generates an orthogonal set  $S' = \{u_1, \dots, u_k\}$  that spans the same  $k$ -dimensional subspace of  $R^n$  as  $S$ .

### IV. SIMULATION RESULTS



Fig. 1 Noisy Image

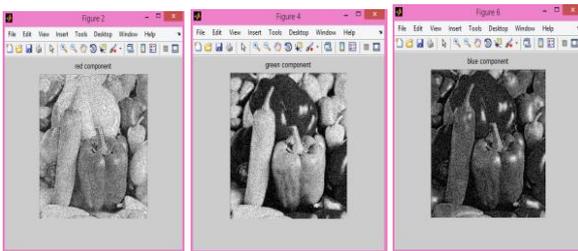


Fig. 2 Separated Red, Green, Blue Components

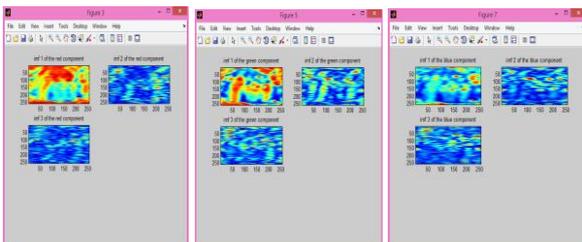


Fig. 3 Separated IMFs of each Component



Fig.4Denoised Image

Table.1 PSNR Values before orthogonalisation

IMAGE	GAUSSIAN NOISE	POISSON NOISE	SALT & PEPPER NOISE
	44.9121	45.0675	45.2036
	45.0596	45.4462	45.5701
	42.2097	42.2698	42.2269
	40.4456	40.5639	40.4924
	40.7602	40.8629	40.6020

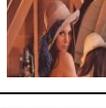
Table .2 MSE Values Before orthogonalisation

IMAGE	GAUSSIAN NOISE	POISSON NOISE	SALT & PEPPE R NOISE
	2.0983	2.0246	1.9621
	2.0282	1.8555	1.8033
	3.9094	3.8557	3.8939
	5.8684	5.7107	5.8055
	5.4583	5.3308	5.6608

Table.3 PSNR Values after applying orthogonalisation

IMAGE	GAUSSIAN NOISE	POISSON NOISE	SALT & PEPPER NOISE
	58.0780	61.2655	52.5747
	59.5816	64.2662	53.3773
	60.0396	60.1761	55.6316
	61.0629	57.9265	55.7523
	58.8686	57.9397	56.8745

Table.4 MSE Values after orthogonalisation

IMAGE	GAUSSIAN NOISE	POISSON NOISE	SALT & PEPPER NOISE
	0.1472	0.0692	0.5442
	0.1031	0.0341	0.4494
	0.0925	0.0896	0.2629
	0.0726	0.1525	0.2555
	0.1220	0.1521	3.5961

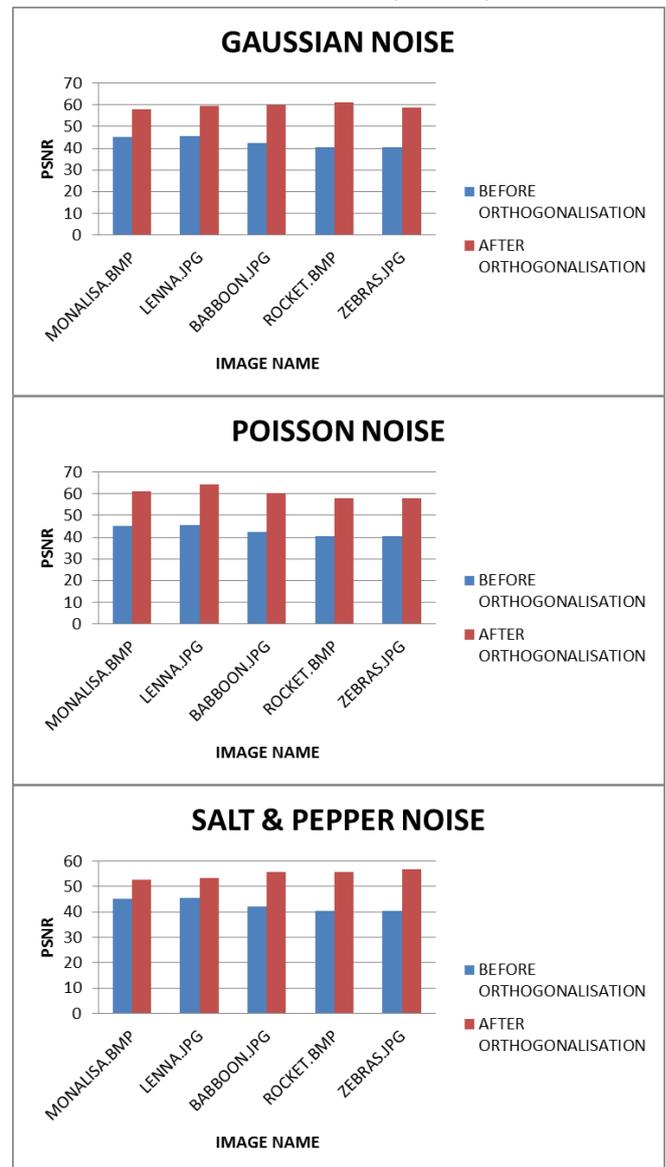


Fig 4. Comparison graph shows the PSNR values for different Noises Before and after orthogonalisation

The above table shows the PSNR and MSE values for different images before and after applying orthogonalisation. From the table and comparison graph it is clear that Empirical wavelet transform with an orthogonalisation gives the better result.

## V. CONCLUSION

Empirical Wavelet Transform is an adaptive data decomposition method. Using this we can perform various operations such as denoising, decompression etc. Experimental results shows that empirical wavelet transform with gram Schmidt orthogonalisation improves the PSNR values to a range of about 50 or 60.

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