

Review of Low-Pass FIR Filter Design Using Window Method

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Abstract—The aim of this paper is to design FIR filter of the order of 50 using various Window function Methods. This paper presents the importance of filter in signal processing. Digital filter are of two types (1) FIR (2) IIR. There are Various methods for Filter designing, this paper discuss the Window Method. Low pass FIR filter designing using Hamming, Hanning, Kaiser, Blackman, Tukey and Rectangular Window Function Methods are presented. Design of FIR filter is done in MATLAB by FDATool. Low pass filter is designed with Sampling frequency 5000 Hz and Cut-off frequency 1000 Hz. Magnitude and Phase Responses of low pass Filter using various window techniques are demonstrated. We found that Filter designing by window method is easy and fast.

Index Terms— FIR Digital Filter, Window Function, FDA tool in MATLAB

I. INTRODUCTION

A filter is essential part in signal processing. In signal processing there are many instances at which input signal contains unwanted noise which can affect the quality of the desired portion of the signal. So the objective of filtering is to extract the information from the signal and improve the quality of the signal [3]. It is the network which changes the amplitude, frequency, phase frequency characteristics of the signal in desired manner. A digital filter plays a very important role in DSP. Digital filters have characteristics such as linear phase response. If we compare digital filter with analog filter they are preferred in number of applications like speech processing, image processing and data compression [1]. The performance of digital filter does not vary with environmental conditions.

11. BASIC PRINCIPLE OF FIR FILTER

There are many kinds of digital filter and many ways to classify them [4]. There are two types of filters basically based on time domain characteristics.

- (1) Finite Impulse response
- (2) Infinite impulse response

FIR filters are Non-Recursive it means there is no feedback involved, IIR filter is Recursive it implies that feedback is present there [4], [13].

Based on frequency characteristics there are four types of digital filter Low-Pass Filter, High-Pass Filter, Band-Pass Filter and Band-Stop Filter [4].

Figure 1 shows the Magnitude characteristics of physically realizable filter. It is not essential that magnitude will be constant in the pass band of a filter. A small amount of ripple can be tolerated in the pass band. It is not necessary that filter response $|H(\omega)|$ is zero in the stop band. A small value of ripple in the stop band can be tolerated. A Band exists between the Pass-band and the Stop-band is called the Transition band. ω_p represents the pass band edge and ω_s represents the stop-band edge. $(\omega_s - \omega_p)$ defines the width of the transition band [17]. Bandwidth of the filter is defined by pass-band width. Pass-band ripple is represented by δ_1 and if there is ripple in the stop-band it is denoted by δ_2 .

when designing a FIR filter it is necessary to specify pass-band, stop-band, transition-band. In pass-band frequency must be passed unattenuated and in stop-band frequency must be passed attenuated.

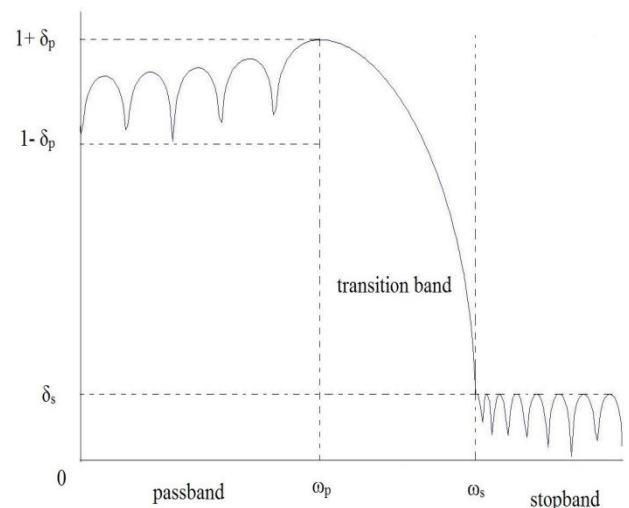


Fig.1 Magnitude characteristics of physically realizable Filter

Multipliers, delay elements and adders are the basic requirement in the FIR Filter design [11]. Figure 2 shows the Structure of FIR Filter of order N.

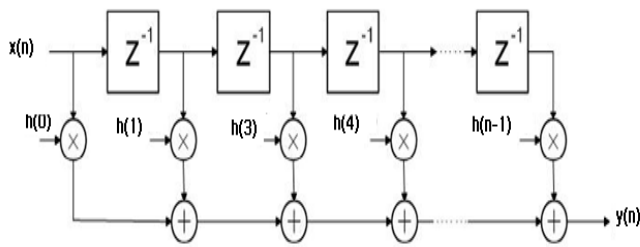


Fig.2 Structure of FIR Filter of order N

The input and the output signals for the IIR filter is given by [14]:-

$$Y(n) = \sum_{k=0}^{\infty} h(k)x(n-k) \quad (1)$$

The difference equation of an N order FIR filter is given by:

$$Y(n) = \sum_{k=0}^{N-1} h(k)x(n-k) \quad (2)$$

$x(n)$ is the input signal

$h(k)$ is the impulse response

K is the order of filter

$Y(n)$ is the output sequence of FIR filters

from equation (1) and (2) we see that, the impulse response of IIR Filter is of infinite duration while for FIR filter the impulse response is of finite duration [7]. Practically, the output of IIR filter using equation (1) is impossible. Because length of its impulse response is infinite. So equation (1) can be expressed in a recursive form as [2]

$$Y(n) = \sum_{k=0}^M b_k x(n-k) - \sum_{k=1}^N a_k y(n-k) \quad (3)$$

Where a_k and b_k represents the filter coefficients [7].

For FIR Filter, Transfer function can be obtained by [15][2]:

$$H(z) = \sum_{k=0}^{N-1} h(k) z^{-k} \quad (4)$$

For IIR Filter, Transfer function can be obtained by:

$$H(z) = \frac{\sum_{k=0}^M b_k z^{-k}}{1 + \sum_{k=1}^N a_k z^{-k}} \quad (5)$$

III. DESIGN METHODS FOR FIR FILTER

There are three common methods for filter design one is the window method [5] it truncates the Fourier series. Second is the frequency sampling technique [15] by using this method set of samples can be obtained in frequency domain. In Second method Frequency response of the filter is same as obtained at the sampling instant [8]. Third method is the optimal filter design method [16] this method is of two types (1) Discrete least squares method (2) Equiripple Method.

Throughout this paper we have discussed the window method. Designing FIR filters using window method is a direct approach.

It truncates Fourier series of the prescribed frequency response [13]. Window method is best because of its simple operation and it has ability to minimize Gibbs oscillations occur in the Fourier series method. The main advantage of window technique is that impulse response coefficients can be obtained easily instead of solving complex optimization problems.

Width of the Main lobe in window method should be narrow. The Amplitude of maximum side lobe should be minimum.

Desired frequency response $H(e^{j\omega})$ of any Digital Filter periodic in frequency can be expanded in Fourier series :-

$$H(e^{j\omega}) = \sum_{n=-\infty}^{\infty} h_d(n) e^{-j\omega n} \quad (6)$$

$$h_d(n) = \int_0^{2\pi} H(e^{j\omega}) e^{j\omega n} d\omega \quad (7)$$

Here, impulse response obtained is of infinite duration. Infinite impulse response can be converted into finite duration by truncating the infinite series at some point say $n=N-1$ to obtain an FIR filter of length N [9].

So, desired frequency response of FIR filter is obtained by modifying equation (6)

$$H(e^{j\omega}) = \sum_{n=0}^{N-1} h(n) e^{-j\omega n} \quad (8)$$

Window function is used in Fourier series method to truncate infinite duration impulse response.

The impulse response of non-causal FIR filter using a window function is given by:

$$h(n) = w(n) \cdot h_d(n) \quad (9)$$

Low Pass Filter is defined as a filter which allows to pass frequency components which is below the cut-off frequency, and rejects the frequency components which is above the cut-off frequency [6].

Frequency response for low pass filter is given by:

$$H_{LP}(e^{j\omega}) = \begin{cases} 1 & 0 \leq \omega \leq \omega_c \\ 0 & \omega_c \leq \omega \leq \pi \end{cases} \quad (10)$$

Impulse response for ideal low pass filter can be obtained by:

$$h_{LP}(n) = \frac{\sin \omega_c n}{\pi} \quad -\infty < n < \infty \quad (11)$$

IV. WINDOW FUNCTION METHOD FOR FIR FILTER DESIGN

Window function can be divided into two parts: fixed window function and adjustable window function [12]. Hanning, Hamming, Blackman and rectangular window are the type of fixed window function. Kaiser, Tukey window is a type of adjustable window function. Adjustable window function is best because it has control parameter.

Parameters used for FIR Low-Pass Filter to obtain Magnitude and Phase Response:

Filter order = 50

Sampling Frequency = 5000 Hz

Cut-off Frequency = 1000 Hz

Filter Type=Low-Pass

Hanning window

The Hanning window is also known as a raised cosine window [10]. The Hanning window has one advantage that it reduces the side lobes. It has good frequency resolution over rectangular window.

The Hanning Window function can be defined mathematically as:

$$W = \begin{cases} 0.5 \left(1 - \frac{\cos 2\pi n}{M-1} \right), & \text{from } 0 \text{ to } n-1 \\ 0, & \text{elsewhere} \end{cases}$$

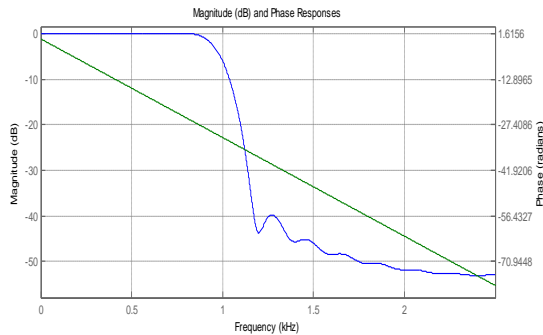


Fig. 3 Magnitude and phase response of Hanning window

Hamming window

The Hamming window is just like the Hanning window, it is also known as a raised cosine window. Hamming window exhibits characteristic which is similar to the Hanning window. It suppresses the side lobe. [10]

Mathematically, the Hamming window function can be written as:

$$W(n) = \begin{cases} 0.54 - 0.46 \cos\left(\frac{2\pi n}{N}\right), & \text{from } 0 \text{ to } n-1 \\ 0, & \text{elsewhere} \end{cases}$$

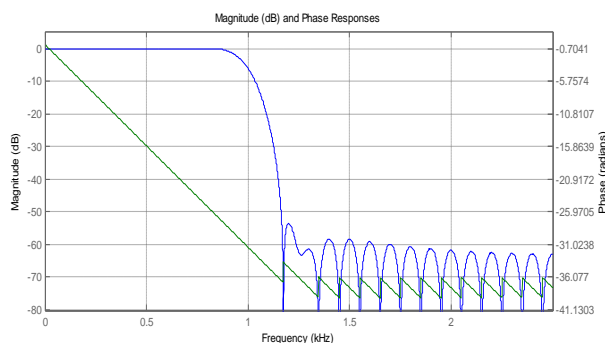


Fig. 4 Magnitude and phase response of Hamming window

Blackman window

The Blackman window is just like Hanning and Hamming windows. If we compare Blackman window with other windows there is better improvement in stop band attenuation [10].

The Blackman window function can be written as:

$$W(n) = \begin{cases} 0.42 - 0.5 \cos\left(\frac{2\pi n}{N}\right) + 0.08 \cos\left(\frac{4\pi n}{N}\right), & \text{from } 0 \text{ to } n-1 \\ 0 & \text{elsewhere} \end{cases}$$

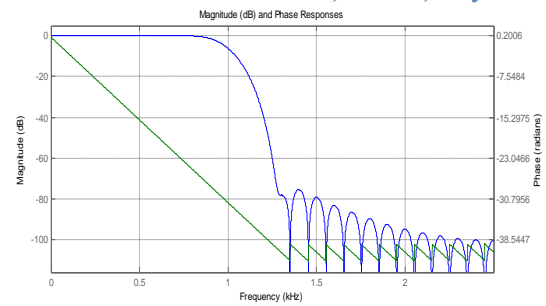


Fig. 5 Magnitude and phase response of Blackman window

Kaiser window

The Kaiser Window function can be written as [10] -:

$$W(n) = \begin{cases} I_0(\beta \sqrt{1 - (2(n+1)(N+1))}) & \text{from } 0 \text{ to } n-1 \\ 0 & \text{elsewhere} \end{cases}$$

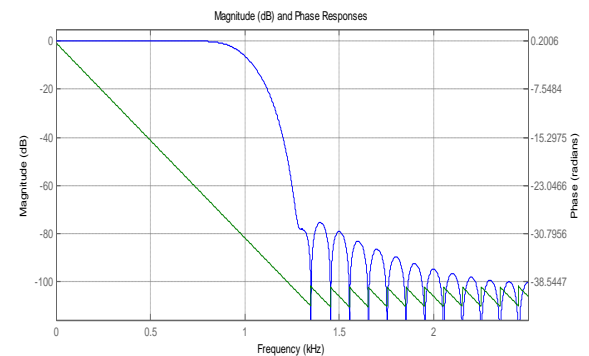


Fig. 6 Magnitude and phase response of Kaiser Window

Tukey Window

The Tukey window is also called the tapered cosine Window. The Tukey Window is defined as:

$$W_t(n) = \begin{cases} \frac{1}{2} \left[1 + \cos \left(\pi \left(\frac{2n}{\alpha(M-1)} - 1 \right) \right) \right], & 0 \leq n \leq \frac{\alpha(M-1)}{2} \\ 1, & \frac{\alpha(M-1)}{2} \leq n \leq (M-1) \left(1 - \frac{\alpha}{2} \right) \\ \frac{1}{2} \left[1 + \cos \left(\pi \left(\frac{2n}{\alpha(M-1)} - \frac{2}{\alpha} + 1 \right) \right) \right], & (M-1) \left(1 - \frac{\alpha}{2} \right) \leq n \leq (M-1) \end{cases}$$

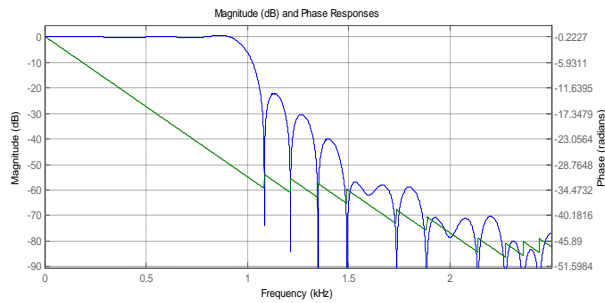


Fig. 7 Magnitude and phase response of Hanning window

Rectangular Window Function

The weighting function for the Rectangular window is given by

$$\omega_R(n) = \begin{cases} 1, & \text{for } n \leq \frac{M-1}{2} \\ 0, & \text{otherwise} \end{cases}$$

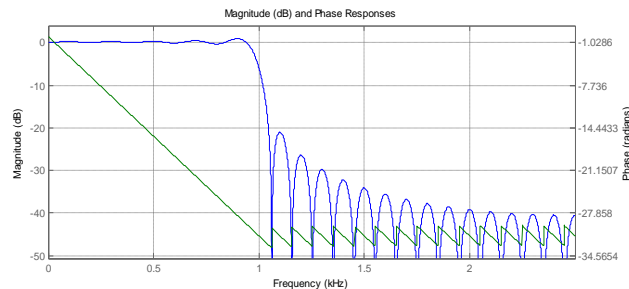


Fig. 8 Magnitude and phase response of Rectangular window

V. FIR FILTER DESIGN USING FDA TOOL

FDA TOOL is a special filter designing analytical tool in the MATLAB signal processing toolbox. It has simple and convenient operation. It is mostly used for Filter designing. With the help of FDA tool, we can design digital FIR or IIR filter very fast by simply specifying the filter parameters.

VI. CONCLUSION

This paper discusses the Window method. The major advantages of Window technique is its simplicity and easy to understand. Using FDA tool in Matlab, We can design Digital filter very easily.

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