Optimal Cooperative Beam forming Design for MIMO Decode-and-Forward Relay Channels

Srisuda Addepalli, Prof. Pragnesh Shah

Abstract — In this project, a transmit beam forming design for the Multiple-Input–Multiple-Output (MIMO) Decode-and-Forward (DF) half-duplex two-hop relay channels with a direct source destination link is taken into consideration. For this source, relay and destination nodes are set with multiple antennas; we formulated and solved the optimal beam forming vectors for source, relay and destination nodes jointly. In specific, we identify several unique properties of the optimal solutions through mathematical derivation, based on which we have developed a systematic approach to arrive at the optimal beam forming vectors for source, relay and destination nodes for different system configurations. We derive a low complexity explicit expression for the optimal beam forming vectors for some specific scenarios. Numerical results show that our anticipated beam forming design scheme can achieve the - optimal solution with low computational complexity for MIMO DF relay networks.

Index Terms— Beamforming and optimization, decode and forward, MIMO, relay, transmit

1. INTRODUCTION

Co-operative communications are the classic representation of communication network and are a graph with a set of nodes and edges. The nodes usually represent devices such as a router, a wireless access point, or a mobile telephone. The edges usually represent communication links or channels, for example: an optical fiber, a cable, or a wireless link. This work deals mainly with Rayleigh by fading wireless channels. Both the devices and the channels may have constraints on their operation. For example, a router might have limited processing power, a wireless phone has limited battery resources, the maximum transmission distance of an optical fiber is limited by several types of dispersion, and a wireless link can have rapid time variations arising from mobility and multi path propagation of signals. The purpose of a communication network is to enable the exchange of messages between its nodes. Due to the broadcast nature of wireless links, signal transmissions between two nodes may be received at the neighbor nodes. It has been understood in the information theory for over three decades that wireless communication from a source to destination can benefit from the cooperation of nodes that overheard the transmission, as These intermediate nodes may themselves generate transmissions based on the processing of the overheard signals. Let us consider the system depicted in Figure where one system node (source) is sending a message to another system node (destination). Due to the broadcast nature of the wireless link, this message is overheard by a third node of the network (relay). During the first phase of transmission, the source broadcasts the unitary message symbols to both the relay and destination using the power Es. The second phase consists of relay transmitting as a transformed version of its received signal to destination while source is silent. Note that two phases indicated are two independent transmissions. This may be achieved by using the orthogonal coding; e.g. using different time slots or different frequency carriers.

Cooperative strategies are amplify-and-forward (AF), classic multi-hop, compress-and-forward (CF), decode-and-forward (DF), multipath decode-and-forward (MDF).

The above strategies can be used for both wire line and wireless networks and they require with time more coordination. For example, consider a RC. AF and classic multi-hop do not necessarily require changes at the source or destination nodes, e.g., for multi-hop the relay can behave as if it is the destination or the source. AF does not necessarily require changes at the source but it does require some extra knowledge about the link capacities. DF requires changes at both the source and destination, and MDF requires additional changes at higher layers of the protocol stack. DF or MDF with network coding require even more changes at higher layers. Decode-and-forward (DF), where the relay decodes the received signals and then forwards the re-encoded information to the destination. Compared with the AF strategy, the DF strategy demands relay nodes with greater signal processing capabilities, but it outperforms the AF strategy especially when the source-relay channel is statistically better than the source-destination and relay-destination channels. In this paper, we consider half-duplex DF relaying systems, where the relay is only allowed to transmit and receive using orthogonal Time or frequency.

MIMO Process is the Multi-antenna MIMO (or Single user MIMO) technology has been developed and implemented in some standards, e.g., 802.11n products. SISO/SIMO/MISO are special cases of MIMO

Multiple-input and single-output (MISO) is a case when the receiver has a single antenna.

Single-input and multiple-output (SIMO) is a case when the transmitter has a single antenna.

Single-input and single-output (SISO) is a conventional radio system where neither the transmitter nor receiver has multiple antennas.

Multiple-Input Multiple-Output (MIMO) is one of several forms of multiple antenna techniques available today designed to appreciably improve communication performance. The guarantee of higher data rates with increased spectral efficiency makes MIMO especially attractive in wireless communications where systems operate in high multipath environments. For this reason, wireless standards like 3GPP Long Term Evolution (LTE), IEEE 802.16 (adopted by the Wi MAX™ Forum) and WLAN 802.11n have recently adopted or are considering its use. All upcoming 4G wireless communication systems are expected to employ MIMO
technology. While spatial diversity and MIMO offers increased signal vigour or capacity improvements when operating in rich multipath environments (with suitable signal to noise and interference conditions), those benefits come at the cost of increased complexity and increased demands on processors. For the R&D engineer developing and integrating MIMO receivers, that translates into a key challenge: how to quickly and accurately test the receivers under real-world conditions and early enough in the design cycle to easily find and fix any problems. This capability is critical to maximize the receiver performance, minimize the design uncertainty and also reducing development cycle time.

A. Previous Work

In the earlier work on MIMO relay channels mainly focused on the network capacity from the information-theoretic perspective, recently some of these work is focused on the beam forming design for MIMO AF relay channels, while other work studies the beam forming design for MIMO DF relay channels. Most work on the DF MIMO relay beam forming assumed that no direct link between the source and the destination. Multiple pairs of source-destination nodes transmit information through multiple relay nodes. Relay beam forming was proposed to keep secrecy for the source-destinations information transmission. A more complex scenario was considered, where the source is deployed with multiple antennas. However, only the maximum-ratio transmission (MRT) beam forming technique was adopted at the source node. The joint source and relay beam forming design in two-hop DF MIMO relay channels with the consideration of source-destination direct link. The optimal beam forming to maximize the cut bound on system capacity was designed with deterministic CSI and statistical CSI, respectively. A numerical algorithm was designed to achieve an approximate optimal solution. A distributed beam forming technique for space-time coded dual-hop cooperative MIMO DF relay networks to minimize the pair-wise error probability. DF MIMO relay channels and formulated a precoding design with resource allocation problem. A linearly combined pre coding scheme for system rate maximization and presented a sub optimal solutions. Note that most of these work resort to complex numerical methods, often involving iterative algorithms, to calculate either exact or approximate optimal solutions due to the complexity of the formulated optimization problems.

B. Contributions

In this paper, we also consider the joint source-relay beam forming design for the three-node MIMO DF relay network with source-destination direct link. We assume that both the source and relay nodes are equipped with multiple antennas while the destination node is only deployed with single antenna. Such a transmission scenario is readily applicable to the down link transmission of a relay-enhanced cellular system where the base-station and the relay can accommodate multiple antennas but the mobile user equipment can only have a single antenna. First, we formulate an optimization problem on the joint source and relay beam forming design for the MIMO DF relay channel, which actually is a max-min fairness optimization problem. We first examine the properties of the optimal solutions. We effectively separate the phase angle design and real norm design problems for the optimal vectors. We also prove that the signal to noise ratio (SNR) of the MISO relay to destination channel can be considered as a concave function of the SNR of the MIMO source to relay channel.

For the first and the second cases, we derive explicitly expressions for the optimal solutions. For the third case, as it is hard to drive the unified explicit result, we further divide it into three different sub cases in terms of the number of antennas deployed at the source and relay nodes. For the scenario that two antennas are deployed at both the source and the relay nodes and single antenna is deployed at the destination node, i.e., 2:2:1 scenario, we derived the explicit expression of the optimal solution. For the scenario where $N_s > 1$ antennas are deployed at the source and only one antenna is equipped on the relay and the destination nodes i.e., $N_s:1:1$ scenario, we present a non-iterative numerical method to calculate the optimal solution. For the general $N_s:N_r:N_d$ scenario, we first fix the SNR of the source-relay link and design the optimal beam forming vector to maximize the SNR of source-destination link. Also for $N_s: N_r: N_d$ scenario, we present a non-iterative numerical method to calculate the optimal solution. Then, we design a bisection based algorithm to find the optimal solution. Finally, we arrive at an optimal beam forming design solution. Extensive simulation results show that our proposed solution can achieve the optimal beam forming design for the MIMO DF relay channel with low complexity.

II SYSTEM MODEL

Figure 1: System model of MIMO relay channel

Figure 2: Two-phase DF relay transmission.
OPTIMAL BEAMFORMING ALGORITHM

Case I: Consider a network model consisting of a source S, a relay R and a destination D, as shown in the figure. It is assumed that the direct link between S and D exists in the system and the relay R helps the information transmission from S to D. Multiple antennas are deployed at both S and R, and only one antenna is equipped at D. Half duplex mode is adopted so that R cannot transmit and receive signals at the same time. Therefore, each round of information transmission from S to D can be divided into two phases. In the first source phase, S broadcasts its information to both R and D, while in the second relay phase R decodes the received information and then forwards the decoded information to D. Thus, D can obtain the desired information by decoding the combined signals received over the above mentioned two phases.

All channel state information (CSI) for each round of transmission is known at the transmitters, by using techniques such as channel training, feedback and channel reciprocity exploiting etc. As a result, S and R can configure their beam forming vectors accordingly to achieve the best transmission performance. Without loss of generality, the transmitted information from S and R can be represented by symbols $X_s$ and $X_r$, respectively. Assume that $N_s$ and $N_r$ antennas are deployed at S and R, respectively.

$$\log(\gamma_{D,S}) = \frac{1}{2}\log\left(1 + \frac{P_s}{\sigma_n^2}\right)$$

Thus, the complex baseband received symbol at D in the source phase is mathematically given by

$$\mathbf{y}_{D,S} = \mathbf{h}_{D,S}^T \mathbf{w}_S \mathbf{a}_S + \mathbf{n}_{D,S}$$

where $\mathbf{h}_{D,S}$ - Channel gain vector from source S to destination D

$$\mathbf{n}_{D,S}$$ - scalar additive Gaussian noise with unit variance,

It follows that the received signal to noiseratio (SNR) at D during the source phase is given by

$$\gamma_{D,S} = |\mathbf{h}_{D,S}^T \mathbf{w}_S|^2 \frac{P_s}{\sigma_n^2}$$

Meanwhile, the complex baseband signal vector received at N antennas of node R can be mathematically given by

$$\mathbf{y}_{R,S} = \mathbf{H}_{R,S} \mathbf{w}_S \mathbf{a}_S + \mathbf{n}_{D,R}$$

where

$$H_{R,S} = \{H_{ij}\}_{N_r \times N_s}$$ - denotes the channel gain matrix from source S to relay R

Applying singular value decomposition (SVD), we can rewrite $H_{R,S}$ as

$$H_{R,S} = \mathbf{U} \Lambda \mathbf{V}^H$$

where

$$\Lambda = \begin{pmatrix}
\lambda_1 & 0 & \ldots & 0 & \ldots & 0 \\
0 & \lambda_2 & \ldots & 0 & \ldots & 0 \\
\vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\
0 & 0 & \ldots & \lambda_{N_r} & \ldots & 0
\end{pmatrix}$$

Then the effective received SNR at R is

$$\gamma_{R,S} = 1 \frac{1}{2} |H_{R,S}^T \mathbf{w}_S|^2 \frac{P_s}{\sigma_n^2}$$

So, the achievable information rate from S to R is

$$C_R = \log_2 (1 + \gamma_{R,S})$$

Hence, the complex symbol received at D in the relay phase can be mathematically given by

$$\mathbf{y}_{D,S} = \mathbf{h}_{D,S}^T \mathbf{w}_S \mathbf{a}_S + \mathbf{n}_{D,S}$$

In the source information symbol phase Xs is the first multiplied with a beam forming vector $\mathbf{W}_s$ can be calculated according to the following equation

$$|\mathbf{w}_s|^2 = |w_{s,1}|^2 + |w_{s,2}|^2 + \ldots + |w_{s,N_s}|^2 = 1$$

Where $\mathbf{W}_s$ is the unit-norm vector.

Case II: In this project we consider the joint source-relay beam forming design for the three-node MIMO DF relay network with source-destination direct link. We assume that both the source, relay and destination nodes are equipped with multiple antennas.

MIMO-DF system model

![System model of the three-node MIMO relay channel where the source node, relay node and destination node are equipped with multiple antennas.](image)

We consider a three-node multicarrier MIMO communication System where the source node transmits information to the destination node with the aid of one relay node. The source, relay, and destination nodes are equipped with multiple antennas, respectively. To account for the practical half-duplex constraint that a node cannot transmit and receive at the same time within the same spectrum band, we assume that the source-relay and relay-destination channels are orthogonal.

To efficiently exploit the system hardware, the relay node uses the same antennas to transmit and receive signals. Due to its merit of simplicity, a linear non regenerative strategy is applied at the relay node to process and forward the received signal.
\[
\mathbf{Y}_{D,R} = \mathbf{h}_{D,R}^T \mathbf{w}_r \mathbf{a} + \mathbf{n}_{D,R}
\]  
(8)

\(\mathbf{h}_{D,R}\) denotes the channel gain vector from relay to destination. The received SNR at D in the relay phase is given by

\[
\gamma_{D,R} = \left| \mathbf{h}_{D,R}^T \mathbf{w}_r \right|^2 \mathbf{P}_r
\]  
(9)

After that, D combines the signal received over the S-D link in the source phase and the signal received from R in the relay phase using the MRC method. Thus the achievable information transmission rate at D in the relay phase is

\[
C_D = \log_2 \left( 1 + \gamma_{D,S} + \gamma_{D,R} \right)
\]  
(10)

Since it has been well established that the maximum information rate from source S to destination D over the half-duplex DF relay channel is bounded by \(C_R\) and \(C_D\), the total achievable information transmission rate at D from S over the MIMO relay channel can be given by

\[
C_{DF} = \frac{1}{2} \left( \min \{ C_R, C_D \} \right)
\]

\[
= \frac{1}{2} \left( \min \{ \log_2 (1 + \gamma_{R,S}), \log_2 (1 + \gamma_{D,S} + \gamma_{D,R}) \} \right)
\]
(11)

where the pre-log parameter, \(1/2\), actually captures the time-division feature of the half-duplex relay system.

### III. PROBLEM FORMULATION

The optimization problem is formulated and mathematically it can be given as the following.

We need to jointly optimize the beamforming vectors both at S and R to maximize the achievable rate \(C_{DF}\) for MIMO DF relay channels. Based on the system model that is given in the section -2, the optimization problem can be mathematically given by

\[
\begin{align*}
\max_{w_S, w_r} & \quad \frac{1}{2} \log_2 \left( 1 + \min \left( \gamma_{R,S}, \gamma_{D,S} + \gamma_{D,R} \right) \right) \\
\text{s.t.} & \quad \mathbf{w}_S \mathbf{H}_{R,S} \mathbf{w}_S \geq \mathbf{t} \\
& \quad \mathbf{w}_r \mathbf{H}_{D,R} \mathbf{w}_r \geq \mathbf{t} \\
& \quad \mathbf{w}_S \mathbf{H}_{R,S} \mathbf{w}_S \mathbf{H}_{R,S} \geq \mathbf{t}
\end{align*}
\]
(12)

Since the function

\[
f(x) = \frac{1}{2} \log_2 (1 + x)
\]

monotonically increases with the variable \(x\) and \(f(x) > 0\) always holds for \(x > 0\), the problem in (12) then can be refined to be a more concise version as

\[
\begin{align*}
\max_{w_S, w_r} & \quad \min \left( \gamma_{R,S}, \gamma_{D,S} + \gamma_{D,R} \right) \\
\text{s.t.} & \quad \mathbf{w}_S \mathbf{H}_{R,S} \mathbf{w}_S \geq \mathbf{t} \\
& \quad \mathbf{w}_r \mathbf{H}_{D,R} \mathbf{w}_r \geq \mathbf{t} \\
& \quad \mathbf{w}_S \mathbf{H}_{R,S} \mathbf{w}_S \mathbf{H}_{R,S} \geq \mathbf{t}
\end{align*}
\]
(13)
rank(W) = 1  \quad (21)

By using the rank-1 constraint, i.e., so-called SDR-relaxation, the problem in (SDR) then is relaxed to be convex SDP problem. According to the SDP problem should have a rank-one solution and solved using some numerical methods. However, numerical methods can not present insightful information for better understanding the effects.

of beamforming on system performance. The computational complexity of the SDR approach is high, especially when the number of transmit antennas \( N_s \) and \( N_r \) are large.

In order to solve the beamforming problem in (20) more efficiently and to get better insights into the beamforming design for MIMO DF relay channels, we adopt the following alternative approach based on the properties of the optimal solution. Note that our approach will lead to much lower complexity without requiring any relaxation. Since the matrix \( V = V_1V_2 \ldots V_{N_s} \), in the following equation

\[
H_{R,S} = U \Lambda V^H
\]  \quad (22)

is a unitary matrix of full rank, for an arbitrary beamforming vector \( W \) with \( W_S \|| 2 = 1 \) there exists a unique complex vector \( W = (w_1, w_2, \ldots w_{N_s})^T \) satisfying

\[
W_S = Vw = (V_1w_1 + V_2w_2 + \ldots + V_{N_s}w_{N_s})^T
\]  \quad (23)

Noting that \( W = V^H W \) which is also a unit-norm complex vector. By substituting (23) into (14) and (15) it follows that the received

SNR \( \gamma_{D,S}^{'} \) and \( \gamma_{R,S}^{'} \) can be re-expressed as

\[
\gamma_{D,S}^{'} = \|h_{D,S}^T w_s\|^2 P_s = \|h_{D,S}^T Vw\|^2 = \|h_{D,S}^T V_1 w_1 + h_{D,S}^T V_2 w_2 + \ldots \|^2 P_s
\]

\[
= (\sum_{i=1}^{N_s} |h_{D,S}^T V_i w_i|)^2 P_s
\]  \quad (24)

And

\[
\gamma_{R,S}^{'} = \|H_{R,S} w_s\|^2 P_s = \|U \Lambda V^H Vw\|^2 P_s = \|U \Lambda \| V^H U^H w\|^2 P_s = \|W^H \Lambda^H U^H w P_s = \|\Lambda w\|^2 P_s
\]

\[
= (\sum_{i=1}^{N_s} |w_i|)^2 P_s
\]  \quad (25)

respectively. Note that \( \lambda_i = 0 \) for \( i = N+1 \ldots N_s \). Therefore, to seek the optimal beamforming vector \( W_s^{opt} \) for the optimization problem (8) is equivalent to find the corresponding optimal vector \( W^* \) for the following optimization problem,

\[
\max_{W_s} \min_{y_{R,S}^{',},y_{D,S}^{'}} \{y_{R,S}^{',} + y_{D,S}^{'},R_s\}
\]

s.t. \( \|W_s\|^2 = 1 \) \quad (26)

Once the optimal \( W^* \) is obtained, the optimal beamforming vector \( W_s^* \) can be calculated in terms of (23).

Furthermore, by observation of (24) and (25), we find that the phase angle of each element of \( W \) only affects \( \gamma_{D,S}^{'} \) while \( \gamma_{R,S}^{'} \) is just determined by the amplitudes of the elements of vector \( W \), which indicates that to solve the optimization problem in (26), the phase angles and amplitudes of the vector \( W \) can be separately designed.

Now, let us begin to discuss how to design the phase angles and amplitudes of the vector \( W \). Clearly, the optimal phase angle must satisfy that for a given set of amplitudes of \( w \), the maximal \( \gamma_{D,S}^{'} \) can be achieved, while the optimal amplitude must satisfy that with the optimal phase angle, it can lead to the optimal solution of problem (15).

**LEMMA:1**

The optimal phase angle \( \angle W^* = \angle W^*1, \angle W^*2, \ldots \angle W^* N_s \) for the values of \( W \) cophases the product As \( h_{D,S}^Tv_i \) for \( i = 1, 2, \ldots, N_s \), it follows that

\[
\begin{align*}
\angle W^*_1 &= \theta - \angle(h_{D,S}^Tv_1) + 2k_1\pi \\
\angle W^*_2 &= \theta - \angle(h_{D,S}^Tv_2) + 2k_2\pi \\
&\vdots \\
\angle W^*_N_s &= \theta - \angle(h_{D,S}^Tv_{N_s}) + 2k_N_s\pi
\end{align*}
\]

and \( \gamma_{D,S}^{'} \) depending on the amplitudes of \( W^i \) then it is given by \( \gamma_{D,S} = \)

\[
(\sum_{i=1}^{N_s} |h_{D,S}^Tv_i| |w_i|^2) P_s
\]

\[
(\sum_{i=1}^{N_s} |h_{D,S}^Tv_i| |w_i|)^2 P_s
\]  \quad (28)

Where \( \angle(h_{D,S}^Tv_i) \) represents the phase angle of the vector \( h_{D,S}^Tv_i \) \( i = {1, 2, \ldots, N_s} \), \( \theta \) can be constant phase value and \( k_1, k_2, \ldots, k_{N_s} \in \mathbb{Z} \) with \( \mathbb{Z} \) being set of all integers.

**PROOF:**

Lemma 1 can be proved by recalling the inequality of complex numbers that

\[
|x_1 + x_2 + \ldots + x_n| \leq |x_1| + |x_2| + \ldots + |x_n|
\]

\( i \in \{1, 2, \ldots, n\} \) and noting that the equality holds if and only if the phases of all the complexes are with the same value or
with $2k\pi$ ($k$ is an integer) difference. By using this inequality, Lemma 1 is easily proved. Equipped with Lemma 1, our beamforming vector design can be transferred into seeking the absolute value of the elements of vector $\gamma$. Thus, the optimization problem of (26) can be equivalently transformed into

$$\max_{\alpha_i, \beta_i, \gamma} \left\{ \sum_{i=1}^{N_S} \alpha_i^2 P_s \left( \sum_{i=1}^{N_S} \beta_i \sqrt{\alpha_i} \right)^2 P_s + \gamma^*_{D,R} \right\}$$

s.t. \(|W_1|^2 + |W_2|^2 + \ldots + |W_{N_s}|^2 = 1\) (29)

For notation conciseness in the following derivation, we define $\alpha_i = |w_i|^2$ and $\beta_i = |h_{D,S}^T v_i|^2$.

Thus, the optimization problem in (29) is simplified into

$$\max_{\alpha_i} \text{MIN} \left\{ \sum_{i=1}^{N_S} \alpha_i^2 P_s \left( \sum_{i=1}^{N_S} \beta_i \sqrt{\alpha_i} \right)^2 P_s + \gamma^*_{D,R} \right\}$$

s.t. $\sum_{i=1}^{N_S} \alpha_i = 1$ (30)

IV. OPTIMAL BEAMFORMING DESIGN

Case I:
Optimal Beamforming Design for 2:2:1 Scenario

The reasons are, firstly, the deployment of two antennas is more realistic than deploying two more antennas in many practical cases. Especially on some size-limited devices such as notebook PC. Secondly, the MIMO scenario also applies to the situation that only two antennas are selected to transmit signals at the source and relay nodes and 2 X 2 MIMO channel has attracted much attention. Especially in the later versions, more antennas are suggested to be employed, deploying two antennas is commonly considered as the most practical antenna choice strategy due to the larger quantity of time-frequency resource consumption of channel estimation and feedback they produce. Especially for the channels with fast variation and large Doppler frequency spread which makes channel estimation more complex and resource consumptive. Hence, in this subsection, we will give out the close-form result for the overall optimal beamforming design for the scenario. The closed-form solution at hand, the beamforming design is significantly simplified. In the scenario, Lemma 1 is still valid, which presents the optimal phase design, and in this case the optimization problem is reduced to

$$\gamma_{D,S} = \left( |h_{D,S}^T v_1|^2 |W_1| \right. \left. + |h_{D,S}^T v_2|^2 |W_2|^2 \ldots + |h_{D,S}^T v_{N_s}|^2 |W_{N_s}|^2 \right)^2 P_s$$

Where $V$ unitary matrix of full rank.

$$\max_{\alpha_1, \alpha_2} \min \left\{ \alpha_1^2 \alpha_1 + \alpha_2^2 \alpha_2 \left( \beta_1 \sqrt{\alpha_1} + \beta_2 \sqrt{\alpha_2} \right)^2 + \gamma^*_{D,R} \right\}$$

s.t. $\alpha_1 + \alpha_2 = 1$, $\alpha_1, \alpha_2 \geq 0$. (31)

Then if the condition satisfies the flow will come to 2:2:1 scenario. Then for 2:2:1 scenario the alpha value can be calculated according to the following equations

$$x = \frac{1}{\lambda_1 - \lambda_2^2} \left( \beta_1 \sqrt{\lambda_1^2 P_s + \beta_2 \lambda_2^2 P_s} + \gamma^*_{D,R} \right)$$

According to the flow diagram the optimal beamforming method is carried out.

![Fig 4: illustration of the mapping from $\gamma_{RS}$ to $\gamma^*_{D,S}$](image)

The points $x_E, y, y_E$, and $y_T$ can be calculated from the following equations:

$$X_T = \lambda_1^2 P_s$$

$$y_T = \beta_1^2 P_s + \|h_{D,R}\|^2 P_t$$

and

$$X_E = \left( \sum_{i=1}^{\gamma_{RS}} \beta_i^2 \right) P_s$$

$$y_E = \sum_{i=1}^{\gamma_{RS}} \beta_i^2 P_s \|h_{D,R}\|^2 P_t$$

(34)

Then the condition satisfies the flow will come to 2:2:1 scenario.
Where

\[
A = \frac{\lambda_1^2 - \lambda_2^2 - \beta_1^2 + \beta_2^2}{2 \beta_1 \beta_2} \\
B = \frac{\lambda_1^2 P_S - \beta_2^2 P_S - \rho_D R}{2 \beta_1 \beta_2}
\]

If the solutions are all non-negative, only the one which maximizes \(\lambda_1^2 \alpha_1 + \lambda_2^2 \alpha_2\) should be selected as the final optimal solution.

Once find out the alpha value by using lemma 1 the beamforming matrix \(w^{*}\) can be calculated.

Later by using the beamforming matrix \(w^{*}\), the capacity can be calculated. Then while coming to \(Ns:1:1\) scenario the \(W^{*}\) matrix can be calculated. Once the \(W^{*}\) value can be calculated the achievable information rates (capacity) can be calculated according to the following system model equations.

\[
W_S^{*} = \mu_1 e_1 + \mu_2 e_2
\]

Figure 5: Optimal beamforming design

Case II:
Optimal beamforming design for multiple antennas at destination:

Zero-forcing precoding

In wireless communications, zero-forcing (or Null-Steering) precoding is a method of spatial signal processing by which the multiple antenna transmitter can null multiuser interference signals. To get Regularized zero-forcing precoding is enhanced by considering the impact on a background noise and unknown user interference, where it can be emphasized in the result of interference signal nulling.

The Null-Steering is a method of beamforming for narrowband signals where we want to have compensating delays of receiving signals from a specific source at different elements of the antenna array. To make use of the antenna arrays, we need to sum and average the signals coming to different elements, but this is the only possible when delays are equal. Otherwise we first need to compensate the delays and then to sum them up. To achieve this, we add the weighted version of the signals with appropriate weight values. We do this in such a way that the frequency domain output of weighted sum produces a zero result. This method is called null steering.

If the transmitter knows the downlink channel state information perfectly, ZF-preceding can achieve the system capacity when the number of users is large. And with limited channel state information at the transmitter the performance of ZF-preceding decreases depending on the accuracy of CSIT. ZF-preceding requires the significant feedback overhead with respect to signal-to-noise-ratio so as to achieve the full multiplexing gain. Multiuser interferences remain as usual since they cannot be nulled with beams generated by the imperfect CSIT.

Mathematical description
In a multiple antenna downlink system which comprises $N_t$ transmit antenna access point (AP) and $K$ single receive antenna users, the received signal of user $k$ is described as

$$y_k = h_k^T X + n_k, \quad k = 1, 2, \ldots, K$$

where $X = \sum_{i=1}^{K} s_i p_i w_i$ is the vector of transmitted symbols, $n_k$ is the noise signal, $h_k$ is the channel vector for user $k$, and $w_i$ is the linear precoding vector. The equation can be written as

$$y_k = h_k^T \sum_{i=1}^{K} s_i p_i w_i + n_k = h_k^T s_k p_k w_k + n_k, \quad k = 1, 2, \ldots, K$$

For comparing, we need to describe the received signal model for multiple antenna uplink systems. The uplink system with a $N_r$ receiver antenna access point (AP) and $K$ single transmit antenna users, the received signal at the AP is described as

$$y = \sum_{i=1}^{K} s_i h_i + n$$

where $s_i$ is the transmitted signal of user $i$, $n$ is the $N_r \times 1$ noise vector, $h_i$ is the $N_r \times 1$ channel vector.

The amount of the feedback should be quantified by resource required to maintain at least a given throughput performance gap between zero-forcing with perfect feedback that should be with a limited feedback, i.e.,

$$\Delta R = R_{ZF} - R_{FB} \leq \log_2 g$$

The required feedback bits of a spatially uncorrelated channel should be scaled according to SNR of the downlink channel, which is

$$B = (M - 1) \log_2 \rho_{b,m} - (M - 1) \log_2 (g - 1)$$

where $M$ is the number of transmit antennas, $\rho_{b,m}$ is the SNR of the downlink channel.

To get the feedback of $B$ bits through the uplink channel, and the throughput performance of the uplink should be larger than or equal to 'B'.

$$b_{FB} \log_2(1 + \rho_{FB}) \geq B$$

where $b = \Omega_{FB} T_{FB}$ is the feedback resource consisted by multiplying the feedback frequency resource and the frequency temporal resource subsequently and $\rho_{FB}$ is SNR of the feedback channel. Then, the required feedback resource to satisfy

$$\Delta R \leq \log_2 g$$

is

$$b_{FB} \geq \frac{\log_2(1 + \rho_{FB})}{(M - 1) \log_2 \rho_{b,m} - (M - 1) \log_2 (g - 1)} \log_2(1 + \rho_{FB})$$

There it is differently from the feedback bits case, the required feedback resource is a function of both downlink and uplink channel conditions. It is reasonable to include the uplink channel status in the calculation of the feedback resource since the uplink channel status determines the capacity, i.e., bits/second per unit frequency band (Hz), of the feedback link. Considered when SNR of the downlink channel is high. Then, the feedback resource will be only proportional to the number of transmit antennas

$$b_{FB,\text{min}} = \lim_{\rho_{FB}\to\infty} \frac{(M - 1) \log_2 \rho_{b,m} - (M - 1) \log_2 (g - 1)}{\log_2(1 + \rho_{FB})}$$

From the above equation that the feedback resource ($b_{FB}$) is not necessary to scale according to SNR of the downlink channel, which is almost does not match to the case of the feedback bits.

V. SIMULATION RESULTS

In this section, we show the effectiveness of our proposed optimal beamforming design for MIMO DF relay channels through numerical examples. Without loss of generality, we assume $P_s=P_r=P$, in all simulations. For comparison, we consider optimal beamforming method for multiple antennas at the destination also.
From the above figure 6 we can see that the optimal beamforming design for 2:2:4 scenario achieves the maximum achievable information rates when compared to 2:2:1 scenario.

From the above figure 7 we can see that the optimal beamforming design for 2:4:4 scenario achieves the maximum achievable information rates when compared to 2:4:1 scenario.

From the above figure 8 we can see that the optimal beamforming design for 4:1:4 scenario achieves the maximum achievable information rates when compared to 4:1:1 scenario.

From the above figure 9 we can see that the optimal beamforming design for 4:2:4 scenario achieves the maximum achievable information rates when compared to 4:2:1 scenario.

From the above figure we can see that the optimal beamforming design for 4:4:4 scenario achieves the maximum achievable information rates when compared to 4:4:1 scenario.

From the above figure we can see that the optimal beamforming design for 4:4:4 scenario achieves the maximum achievable information rates when compared to 4:4:1 scenario.

It is shown that more antennas cause a higher computational complexity for the scheme. However, it shows that our scheme achieves much lower complexity compared with the BLP scheme. Especially with the increment of , the reduced complexity becomes much more notable.

VI. CONCLUSION

In this work, we considered the beamforming design for MIMO DF relay channels, where the source node, relay node and destination nodes are equipped with multiple antennas. It will be experimented by using different scenarios. From the 2:2:4 scenario, this scenario achieves the maximum information rate other than the remaining scenarios. We developed an efficient scheme to solve the optimization problem and determine the optimal beamforming vector for MIMO DF relay networks. Unlike previous work on beamforming design for MIMO DF channels, our beamforming design was based on the exact capacity formulation, which can achieve high accuracy. As an ongoing effort, we can extend to the beamforming design to the scenario where we can implement this MIMO concept using some other pre-coder designs in future work and also we can implement the different scenarios practically in future work.
ACKNOWLEDGMENT

I would like to express my special thanks of gratitude to my guide (Prof. Pragnesh Shah) as well as our principal (Dr. R. I. K. Moorthy) who gave me the golden opportunity to do this wonderful project on the topic (Optimal Cooperative Beamforming Design for MIMO Decode-and-Forward Relay Channels), which also helped me in doing a lot of Research and I came to know about so many new things I am really thankful to them.

REFERENCES


Srisudha Addepalli received the Bachelor’s degree from the college of J.A. Institute of Information Technology, Madras University, Chennai, India in 2004. From October 2004 to August 2011, she worked as Assistant Professor at the Department of Electronics& Communication Engineering, Acharya Nagarjuna University, Andhra Pradesh, India. She is currently doing M.E. at PIIT with Electronics, Mumbai.

Prof. Pragnesh Shah is working as professor in Pillai Institute of Information Technologies, Mumbai University, Mumbai, India from 2004 with Electronics department, India.