

Sphere Decoder for Massive MIMO

Saranya.B

PG Student, Dept. of ECE, Periyar Maniammai University

Thanjavur, Tamilnadu, India

ABSTRACT- Modern wireless communication systems demanding high data rate operating in bandwidth deficient world is using Multiple Input Multiple Output (MIMO) antenna arrangement. MIMO arrangement helping achieving high data rates with a technique of special multiplexing saves bandwidth but needs efficient receivers with manageable hardware. The receivers employing linear equalizers or Decision Feedback equalizers in detection use less hardware but are of suboptimum. Optimum detectors are realized by Maximum Likelihood Detectors but not suitable for implementation due to the complexity. Therefore suboptimum detectors such as Viterbi Decoding, Sphere Decoding and Genetic Algorithms based detectors are suitable candidates. Here we compare the performance of sphere decoder with some other detector. The complexity and SER rate of sphere decoder is good when compare with other detectors used in MIMO receiver design. The performance of sphere decoder and maximum likelihood decoder is same but complexity is reduced in sphere decoder.

Keywords- Complexity, detector, MIMO system, SER, sphere decoder.

I. INTRODUCTION

The wireless communication is advanced and most vibrant areas and also the faster growing segment in the communication industry. Compared with wired communication this achieves long-range. In communication there is a demand for faster wireless access there is a need to change from the traditional Single Input Single Output (SISO) antenna systems into Multiple Input Multiple Output (MIMO) [1]. Using multiple antennas at transmitter and receiver achieving high data rates and minimum bit error rate compared to single input and single output. The multiple antenna system is used to increase channel capacity by the factor number of transmitted and receiver antenna without additional transmit power or spectral bandwidth. MIMO is essential for the

transmitters and receivers to have an accurate estimate of the channel state information (CSI). However, this is a challenging task in practice, especially for systems with a large number of transmit and receive antennas. The main objective in MIMO receiver design is to obtain low symbol error rates (SER) with acceptable computational complexity. But the receiver part of the MIMO system is complicated to design [11]. So, we have to choose detectors carefully. Because the Performance of some detectors is complicated so it increases complexity of MIMO system. Identifying a closest lattice point in the MIMO system is the major problem for the complexity. While searching through all possible combination it difficult to find correct transmitted signals. Many signal detectors are used in MIMO system [4]. In that Maximum likelihood detection is an optimal solution. The ML decoding is robust to channel estimation errors and is near optimal with respect to SER. The solution involves an exhaustive search through all possible transmitted signal vectors; this search has exponential complexity, which is undesirable in practical systems. Other sub-optimal algorithms such as MF, ZF and MMSE are practically considered with some disadvantage. Hence, we go to sphere decoder to implement the decoding. Sphere decoder gives near ML detection performance and lowers the computational complexity by limiting the search to the closest lattice point to the received signal within a sphere radius. Sphere decoder can restrict the search by drawing a circle around the received signal just small enough to enclose one signal point and eliminate the search of all points outside the circle.

In this paper we discuss about detector which are used in MIMO receiver system and compare the performance of detector's BER with SNR .Our simulation result show sphere decoder reduce complexity and gives good bit error rate. Section II describes the background information about detectors

of MIMO system. Section III discuss about the new proposed method. Finally simulation and results are discussed in Section IV.

II. RELATED WORKS

In this section, we discuss about the background information of detectors used in MIMO receiver. The goal of this topic has been to provide an overview of all detector used in MIMO receiver [15]. We are going to discuss which method is the best in practice. In signal detection method, the transmitted signals are assumed as interference signal except for desired data stream from the target transmit antenna. Here we minimized or nullified interference signal from other transmit antenna for detecting desired signal from target transmit antenna. Many signal detectors are used in MIMO system [4].

A. Zero Forcing

Zero Forcing is the linear algorithm used in communication system which applies the inverse of the frequency response of the channel. This Zero Forcing was proposed by Robert lucky. It applies the inverse of the channel frequency response of received signal, to restore the signal after the channel. In MIMO system it knowing the channel allows recovery of the two or more streams which will be received on the top of each other on each antenna. The name Zero Forcing corresponds to bring down the inter symbol interference (ISI) to zero in a noise free case. This will be useful when ISI is significant compared to noise.

x is the transmitted data over H channel now we get y received data

$$y = Hx \quad (1)$$

Using zero forcing detector in the MIMO receiver part we get the estimated transmitted data is

$$\hat{X} = Hx \left(\frac{1}{H} \right) \quad (2)$$

$\left(\frac{1}{H} \right)$ or H^{-1} is the channel inverse. The simplest way of calculating inverse is by means of QR factorization, $H = QR$. It can also be calculated in a more stable way and it avoids inverting the upper triangular matrix R. Here zero forcing detectors are used to find the transmitted data from the received signal. In reality ZF does not work in most of the application for the following reason:

1. If the channel has finite length, the impulse response of ZF needs to be infinity long. At some frequency they received signal may be weak. To compensate, the magnitude of the ZF grow very large. If any noise is added after the channel growed a large factor and then it destroys the overall SNR
2. If channel have zeros in its frequency response that cannot inverted at all.
3. The ZF removes all ISI and it is ideal when the channel is noiseless. However, when the channel is noisy, the ZF will amplify the noise greatly at frequencies f where the channel response $H(j2\pi f)$ has a small magnitude (near zero of the channel) in the attempt to invert the channel completely .

B. Minimum Mean Square Error Detector

Minimum Mean Square error does not usually eliminate ISI completely but instead minimizes the total power of the noise and ISI component in the output. The MMSE is used to minimize $E(x^2)$. It is used to reduce error signal. A MMSE estimator is a method in which it minimizes the mean square error (MSE), which is a universal measure of estimator quality. The same problem they are discussed above in zero forcing is addressed in MMSE also. Because, MMSE is the small modification in the ZF denominator of the channel frequency.

Let us assume that x be an unknown random variable and R be a known random variable, then

$$R = HX + n \quad (3)$$

An estimator $X(R)$ is any function of the measurement y, and its mean square error is given by

$$MSE = E \{ (\hat{X} - X)^2 \} \quad (4)$$

Where the expectation is taken over both X and R. The MMSE always performs better than the ZF equalizer and is of the same complications of implementation.

C. Maximum Likelihood

The maximum likelihood detector for a MIMO receiver operates by comparing the received signal vector with all possible noiseless received signals in the receiver side searches across all possible combinations, and tries to solve the inter channel interference (ICI) caused by transmitting from all antennas simultaneously, on the same frequency. Under certain assumptions, this receiver achieves

optimal performance in the sense of maximizing the probability of correct data detection. Maximum-likelihood (ML) detection for high order MIMO systems face a major challenge in computational complexity that grows exponentially with the number of transmitted and received antennas and depends only on the spectral efficiency [4]. This limits the practicality of these systems from an implementation point of view, because it's impossible to implement for large array sizes and high order digital modulation schemes particularly for mobile battery-operated devices. This reality motivated researchers to consider other suboptimal approaches for MIMO decoding, such as Zero Forcing (ZF), Minimum Mean Square Error (MMSE). The computational complexity of ML is equal to the complexity of single-input multiple-output (SIMO) systems. The model for the generic multiple-input multiple-output (MIMO) system can be written as

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n} \quad (5)$$

where \mathbf{x} denotes the transmitted signal vector of dimension $N \times 1$ and \mathbf{y} denotes the noisy received signal vector of dimension $P \times 1$; \mathbf{H} is the channel matrix of dimension $P \times N$ represents a vector of independent Gaussian noise. When the transmitted symbols are uniformly distributed, the optimum decoder (in the sense of minimizing SER) is the maximum likelihood (ML) decoder. The ML detector calculates the squared distance \mathbf{d} between the received vector \mathbf{y} and every possible signal constellation \mathbf{X} :

$$\mathbf{d} = \|\mathbf{y} - \mathbf{H}\mathbf{x}\|^2 \quad (6)$$

At a receiver, a detector forms an estimate of the transmitted symbol $\hat{\mathbf{x}}$. The optimal detector minimizes the average probability of error, i.e., it minimizes $P(\hat{\mathbf{x}} \neq \mathbf{x})$.

$$\min \|\mathbf{y} - \mathbf{H}\mathbf{x}\|^2 \quad (7)$$

Where the minimization over all the points in the constellation. Given the "skewed" lattice $\mathbf{H}\mathbf{x}$, find the "closest" lattice point to a given dimensional vector \mathbf{x} . The closest lattice point search problem is known to be, in general, of exponential complexity. The basic idea is to specify in advance the number of constellation points to be considered when calculating Euclidean distance metrics for each transmit antenna [12]. The ML decoding technique is the optimal decoding technique achieving the best performance in terms of Symbol Error Rate

(SER).[8] ML decoder uses an exhaustive search algorithm where all possible codeword are checked, and the one with minimum distance is selected as the final codeword.

D.Sphere Decoder

Sphere Decoding is a new type of decoding technique which aims to reduce the computational complexity of the decoding technique. In case of a sphere decoder, the received signal is compared to the closest lattice point, since each codeword is represented by a lattice point. The number of lattice points scanned in a sphere decoder depends on the initial radius of the sphere. The correctness of the codeword is in turn dependent on the SNR of the system. The search can easily be restricted by drawing a circle around the received signal. So the search allows only those codeword to be checked that happen to fall within the sphere. All the remaining codeword outside the sphere are not taken into consideration for decoding. The radius must be chosen in such a way that the value should cover the lattice. The initial radius selected plays a critical role in identifying the correct point in the lattice. Ideally, the noise variance of the system is found and the initial radius of the sphere is adjusted according to the Signal to Noise Ratio. This entails the sphere decoder to find at least a single point inside the sphere. [10] The main idea of the Sphere Decoder is to reduce the computational complexity of the maximum likelihood detector by only searching over only the noiseless received signals that lie within a sphere of radius R around the received signal.

$$\hat{\mathbf{x}} = \operatorname{argmin}_{\mathbf{x} \in \mathbf{x}^M} \|\hat{\mathbf{y}} - \mathbf{H}\mathbf{x}\|^2 \leq R^2 \quad (8)$$

BLOCK DIAGRAM

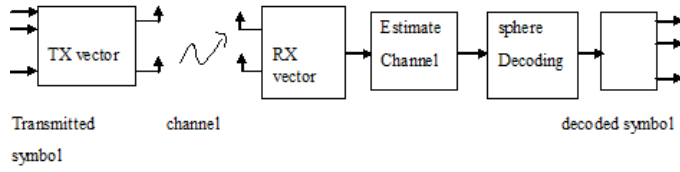


Fig.1 2 × 2 MIMO transceiver architecture with sphere decoder

Based on the performance of MIMO system we construct the above block diagram. In this transmitter part of the block diagram bits are send and using modulation techniques the bits are converted into symbols. Then the transmitted symbols are sending via multiple antennas at the transmitter. This symbol through over the wireless channel and reaches the receiver via receiving antenna. The output of the receiver having noise with input signal. Using detector we find the transmitted signal. Here we are choosing sphere decoder. It searches the exact transmitted symbol within the sphere radius. The symbol in outside of the radius is discarded. so the complexity is reduced using sphere decoder After the receiver the performance is based on the detectors. Finally we get the transmitted bit.

III PROPOSED WORK

SIMULATION SYSTEM

The following figure explains about the process of our project. Here the bits are sending from the transmitter part to receiver part via wireless channel. Consider the bits are X . It may be X_1, X_2, \dots, X_n based on the data rate. These bits are sending to the Modulated symbol block. In this block the bits are converted into the symbols. Each symbol consist of few bits presented it based on the Modulation technique. After that the symbols are transmitted via multiple antennas present at the transmitted side. This antenna sends the symbols to the next block channel. Now we get the output form Hs . Here some noises are added with the output so we get $Hs+n$. here n is the Gaussian noise. This reaches the receiver side of the multiple antennas. Then this antenna sends the received symbols to the next section. Here the transmitted symbols are estimated with the help of the Euclidean distance of received symbols, channel and transmitted symbols.

$$\hat{X} = \|y - Hs\|^2 \quad (9)$$

Using the above equation we calculate the estimated values. Now sphere decoder going to find the accurate transmitted symbols with low complexity. From the estimated value we consider only the value within the sphere. So number of searching node is decreases. Compared to other methods the complexity is reduced here. Finally the symbols are converted into bit as we transmitted [3].

The sphere decoder is developed on two stages. Firstly a pre processing stage computes the QR factorization of the channel matrix, H and after this a search stage finds the estimation of transmitted symbol \hat{x} .

$$H = Q \begin{bmatrix} R \\ 0 \end{bmatrix} \quad (10)$$

$$\hat{X} = \text{argmin } x \in x^M |y - Hx|^2 \quad (11)$$

Apply equation 10 into 11

$$\begin{aligned} &= \text{argmin } x \in x^M |y - Q \begin{bmatrix} R \\ 0 \end{bmatrix} x|^2 \\ &= \text{argmin } x \in x^M |[Q^T]y - \begin{bmatrix} R \\ 0 \end{bmatrix} x|^2 \\ &= \text{argmin } x \in x^M |\hat{y} - Rx|^2 \end{aligned} \quad (12)$$

Q is $N \times N$ and orthogonal, R is $M \times M$, upper invertible and triangular, and 0 is an $(N-M) \times M$ matrix of zeros. As the objective function is invariant under orthogonal transformation, minimization problem can be written as. Where $\hat{y} = [Q^T]y$, the lower limit 1 and the upper limit M extract the first M elements of the orthogonally transformed target.

Based on [5] analysis, SD achieves quasi-ML performance with average computational complexity for a large range of signal-to-noise ratios. Hence, instead of testing all the transmitted vector, SD restricts the search in to the lattice points that inside of sphere radius d .

$$\hat{X} = \text{argmin } x \in x^M |\hat{y} - Rx|^2 \quad (13)$$

The estimated value is must inside the sphere radius

$$\hat{X} = \text{argmin } x \in x^M |\hat{y} - Rx|^2 \leq d^2 \quad (14)$$

The original sphere decoder, after the computation of the first point in the lattice, reduces the radius of the sphere to the value of the distance of this new point to the received point. H represents a channel matrix for a communication channel, it can be assumed that the norm of Hx is not too large because of the power constraint of the channel. This means that when the channel matrix H is applied to the source signal vector x , it does not significantly magnify the length of the source signal vector. In other words, $\|Hx\|^2$ and $\|x\|^2$ is the same magnitude order. Based on this assumption, an initial radius can be determined by the following deterministic method. [16] Suppose that the QR decomposition is already applied to and it is reduced. First, we find the real solution for the triangular system $Rs = \hat{y}$, which is the real least squares solution for the problem $\min \|Hx - \hat{y}\|^2$. Then we round the entries of s to their nearest integers to obtain the lattice point:

$$\hat{x} = [x] \in Z^M \quad (15)$$

This \hat{x} is known as the Babai estimate discussed by Grottschel. M., L. Lov'asz, A. Schriver. A radius \hat{d} is then set to the distance, in the Euclidean sense, between $R\hat{x}$ and \hat{y} :

$$\hat{d} = \|R\hat{x} - \hat{y}\|^2 \quad (16)$$

It is clear that the hyper sphere of radius \hat{d} and centered at \hat{y} , contains at least one lattice point, namely \hat{x} . Thus the sphere

decoding using this radius will find the integer least squares solution in this sphere, possibly on its surface. If the real least squares solution s happens to be an integer vector, that is, $\hat{x} = x$, then the radius \hat{d} is zero, meaning that s is the integer least squares solution [2]. In communication application, this situation means that both channel and signal are perfect, no channel distortion on the transmitted signal and no additive noise to the transmitted signal, which is only possible in theory. The radius computed by this Algorithm ensures that the search sphere contains at least one lattice point namely \hat{x} , therefore the integer least squares solution lies in the search sphere if the radius is computed exactly. In practice, however, in the presence of inexact arithmetic due to rounding errors introduced by floating-point computation, the computed radius contains error. If the computed radius is smaller than the exact radius and \hat{x} happens to be the integer least squares solution, the computed search sphere will contain no lattice point and sphere decoding fails.

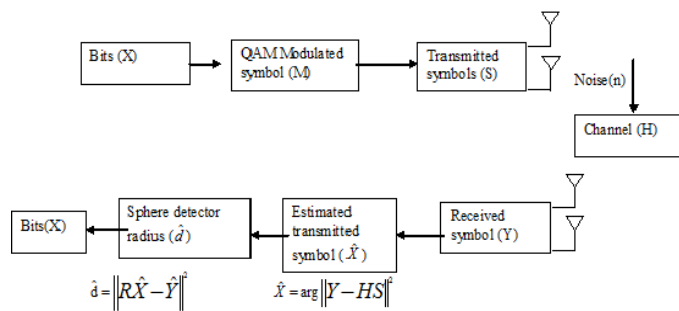


Fig.2: Flow diagram

IV RESULT AND DISCUSSION

In this section we showed sphere decoder is good when compared with other decoder such as zero forcing, maximum mean square error(biased and unbiased) detector. From the result we clearly understand if the zero forcing SNR is above 40 we get BER is 10^{-3} Using MMSE the bit error rate is decreases. Using sphere decoder, the SNR is 25 we get the minimized BER of 10^{-4} . In low symbol error rate we minimizing bit error rate using sphere decoder. So the sphere decoder only gives the good performance when compared with others.

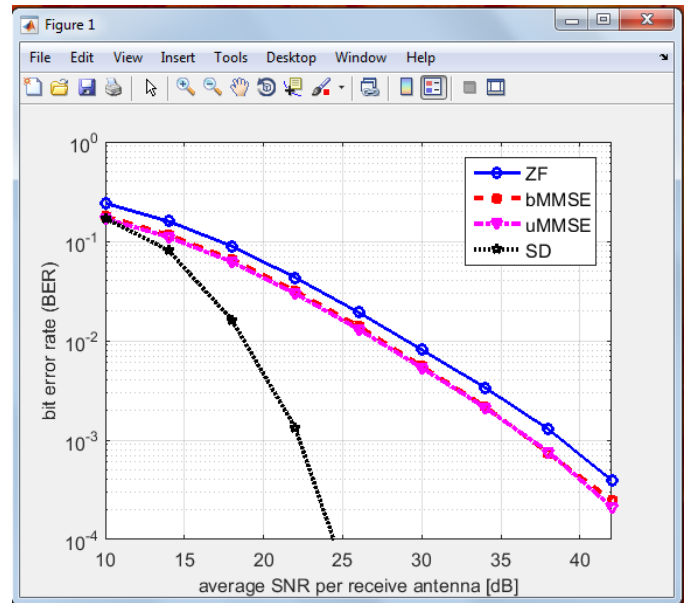


Fig 3: Performance Comparison Result

CONCLUSION

We considered receiver design of MIMO systems with Gaussian channel estimation errors. In this paper we analyzed performance and comparison of the MIMO detectors with sphere decoder. Our sphere decoder is robust to CSI errors and is near optimal in the sense of minimizing the probability of symbol error. We demonstrated, via simulations, that our sphere decoder incurs very small performance loss (when compared to the exact ML solution) with significantly lower computational complexity. Thus, our sphere decoder is implementable in practical MIMO systems. We also study about the complexity and performance of the MIMO decoders. Future research directions will study about how we reduce the complexity again decreasing the radius of the sphere decoder technique using tree search algorithm in MIMO receiver system

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