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IMPLEMENTATION OF NON LINEAR COMPANDING TECHNIQUE FOR REDUCING PAPR OF OFDM

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Abstract- **To implement Orthogonal frequency division multiplexing (OFDM) system, one of the major limitation is High Peak-to-Average power ratio (PAPR) of transmitted signal. The new Non-Linear Companding Transform (NCT) algorithm which transform the OFDM signal into intended statistics form which are specified by a linear piecewise function are used. To achieve efficiency in performance and flexibility in the Non-Linear Companding (NCL) form the variable slopes and inflexion points are introduced in the probability density function (PDF) while comparing the PAPR and Bit error rate (BER). The expected transform gain and signal attenuation factor and all the theoretical value study of this algorithm are provided. The main parameters are evaluated specifically based on the selection criteria of the transform parameters focusing on robustness and execution aspects. The Analysis is exactly verified in simulink.**

Keywords: **Orthogonal frequency division multiplexing (OFDM), Peak-to-average power ratio (PAPR), Non-Linear Companding technique (NCL), High power amplifier (HPA).**

1. INTRODUCTION

As an Emerging technique, orthogonal frequency division multiplexing (OFDM) has been widely applied in modern wireless communications. OFDM offers high spectral efficiency, immunity to frequency selective fading channels, low susceptibility to the multipath propagation and power efficiency. However, there are some obstacles in OFDM which degrades the system performance. A most important problem of OFDM transmission system is its high instantaneous peak-to-average power ratio (PAPR). At the OFDM Transmitter , Presence of large number of sub-carriers with varying amplitude results in high PAPR of the system with large dynamic range, which leads to undesired in band distortion and out of band radiation, occurs if the linear range of the high power amplifier (HPA) is not sufficient [3], [2]. The complex envelope of baseband transmitted OFDM signal with N carriers can be written as,

$$x(t) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_n \, . \, e^{\frac{j2\pi kt}{T}} \quad , \quad 0 \leq \mathrm{t} \leq \mathrm{T}, \quad (1)$$

Where $j = \sqrt{-1}$ and the vector $X = [X_0, X_1, \dots\dots, X_{N-1}]^T$ denotes the frequency-domain OFDM symbols and T is the symbol duration. Guard interval in OFDM system is used to remove ISI which is generally introduced between consecutive OFDM symbols. To remove ISI entirely a guard band interval with no signal transmission can be used but it can produce ICI because of higher spectral components which occurred due to quickly change of waveforms.

Based on the Probability theorems i.e. central limit theorem, when N is large, $x(t)$ can be approximated as a complex Gaussian process; thus, it is possible that the maximum amplitude of OFDM signal may be well exceed its average amplitude [8]. To overcome this issue, various methods have been developed, among which, non-linear companding transform (NCT) is an efficient solution in reducing the PAPR of OFDM signal. The concept of NCT was uses the µ-law companding technique, which significantly outperform the traditional clipping.

Instructively, by compressing the large signals and enlarging the small signals ones, both the PAPR reduction and immunity of small signal from noise can be achieved. However, it is worth noting that NCT is an extra pre-distortion operation applied to transmitted signal, which results in performance degradation and increased sensitivity to the HPA. It was pointed that, due to the disadvantages of nonlinear distortion, such transform should be designed cautiously so that the amount of clipped signal is as little as possible [6]. For this reason, how to reallocate the power as well as the statistics of OFDM signal more reasonable to reduce the impact of companding distortion is the key challenge for a well designed NCT method. Moreover, a flexible and effective trade-off among the overall performance of OFDM system with respect to the reduction in PAPR (power efficiency), bit error rate (BER), spectral regret (bandwidth efficiency), and the implementation complexity also should be considered.

Further, the motive of PAPR reduction is to get better linear system performance as well as BER than that of the original OFDM system. Keeping the above points in view, we propose a new NCT algorithm that transforms Gaussian-distributed signal into a needed distribution form defined by a linear piecewise function with a choose of inflexion point and cut-off points. This algorithm can significantly reduce the impact of the companding distortion on the BER performance by choosing proper transform parameters. In addition, it also allows more flexibility and freedom in the companding form to satisfy various design requirements. The analytical expressions regarding the achievable reduction in PAPR, signal attenuation factor, and the selection criteria of transform parameter are derived and verified through computer simulations.

Notation: The expectation and maximal element operator are denoted by $E\{.\}$ and $\max\{.\}$. We use $[.]^T$, $(.)^{-1}$ and $|.|$ to denote the transpose, inverse and modulus operation, respectively. Sgn(.) stands for the sign function. $IFFT_N\{.\}$ represents the N-point inverse fast Fourier transform (IFFT) operation. Prob $\{A\}$ is the probability of the event A. Bold letters denotes the vectors.

II. LITERATURE REVIEW

In modern technology, a worldwide connection has occurred for use of orthogonal frequency division multiplexing (OFDM) as an emerging technology for high data rates. In the late 1950's, the birth of OFDM development started with the introduction of Frequency Division Multiplexing (FDM) for data communications. The concept of parallel data transmission by use of frequency division multiplexing (FDM) was published in middle of 1960's. In 1966, Chang presented the overview structure of OFDM and published the concept of using orthogonal overlapping multi-tone signals for data communication. With the attractive OFDM towards wireless technology, many wireless standards have adopted OFDM technique to make a move to future wireless communications i.e., WIMAX, IEEE 802.11a, LTE. Later in 1971, Weinstein and Ebert introduced the idea of using a Discrete Fourier Transform (DFT) for implementation of the generation and reception of OFDM signal, eliminating the requirement for banks of analog subcarriers oscillators. The reliance on DSP prevented the wide spread use of OFDM during the early development of OFDM. In spite of several advantageous features, the OFDM system have two major concerns i.e. high PAPR of the transmitted signal and synchronization (timing and frequency) at the receiver. Several algorithms have been proposed to handle this PAPR problem. However, none of these algorithms have produced significant reduction of PAPR in OFDM system. In 2008, Muller, S.H, Huber proposed Partial Transmit Sequence (PTS) [4]. In PTS, the information bearing subcarrier block is subdivided into disjointed carriers unblocks and introduced rotation blocks for each sub-block and modified the subcarrier amplitude vector. Thereby, PAPR was reduced with different rotation factors for different sub blocks. This needs number of iterations to find the optimum combination of factors for different sub-blocks. Later on Adaptive PTS was proposed to reduce the number of iterations by setting up a desired threshold and probably by trial for different weighting factors until PAPR dropped below the threshold. By using QPSK modulation with 256 subcarriers, PAPR can be reduced by 4.1 dB and 4.0dB without adaptive PTS and with adaptive PTS respectively. However, these two approaches need to send side information to the receiver which implies a reduction in the bandwidth efficiency. PTS with connected side information is used to effectively reduce this problem. Several algorithms such as Signal Distortion techniques, Scrambling Techniques, Coding techniques have been proposed in the literature. It wasn't until the late 1980's that work began on the development of OFDM for commercial use, with the introduction of the Digital Audio Broadcasting (DAB) system. OFDM involves in several application i.e, HIPERLAN/2, Wireless LAN Networks, HDTV and UMTS Terrestrial Radio Access. Cyclic extension was first introduced by Peled and Ruiz in 1980 for OFDM system. With the advent of Companding Transform, compressing large signal while enhancing small signal that can achieve a desired PAPR but with a increased in the Bit error rate (BER).

III.CHARACTERIZATION OF OFDM SIGNAL

Generally, an OFDM signal is the sum of N independent data symbols modulated by phase-shift keying (PSK) or quadrature amplitude modulation (QAM). In discrete-time domain, since the Nyquist rate samples might not represent the peaks of the continuous-time signal, it is preferable to approximate the true PAPR on an oversampled signal. The oversampled time-domain OFDM symbols x=$[x_0, x_1, \dots, x_{PN-1}]^T$ can be calculated as

$$x_n = \frac{1}{\sqrt{PN}} \sum_{k=0}^{N-1} X_k \, . e^{\frac{j2\pi kt}{T}} \text{ , } 0 \le n \le PN-1, \quad (2)$$

Where n=0, 1, 2…, PN-1 is the time index and P is the oversampling ratio. Usually, P≥4 is used to accurately describe the PAPR of the continuous-time signal. This oversampling process can be achieved by performing a PN-point IFFT with extending X to a PN-length vector by inserting (P-1) N zeros in its middle, i.e.

$$X_e = \left[X_0, \dots, X_{\frac{N}{2}-1}, 0, \dots, 0, X_{\frac{N}{2}}, \dots\dots, \dots\dots X_{N-1}\right]^T$$

(3)

It is clear that x= $IFFT_{PN}\{X_e\}$. For a large N (e.g. N≥64), the real and imaginary part of x_n may be approximated as Gaussian random variables with zero mean and a variance. Based on this assumption, the signal amplitude $|x_n|$ follows a Rayleigh distribution with the probability density function (PDF) as

$$f_{|x_n|}(x) = \frac{2x}{\sigma^2} e^{-\frac{x^2}{\sigma^2}} \text{ , } x \ge 0, \quad (4)$$

The cumulative density function (CDF) of $|x_n|$ is defined as

$$F_{|x_n|}(x) = \text{Prob } \{|x_n| \le x\} = \int_0^x e^{-\frac{y^2}{\sigma^2}} dy = 1 - e^{-\frac{x^2}{\sigma^2}} \text{ , } x \ge 0. \quad (5)$$

The PAPR of OFDM signal in a given frame is defined as

$$PAPR_x = \frac{max_{\, n \, \varepsilon \, [0, PN-1]}\{|x_n|^2\}}{E\{|x_n|^2\}}. \quad (6)$$

It is more helpful to consider the PAPR as a random variable and utilize a statistical description given by the complementary cumulative density function (CCDF), defined as the probability that the PAPR of x exceeds an assigned level $\gamma_0 > 0, \le$

$CCDF_x(\gamma_0)$=Prob{$PAPR_x$>γ_0}=1-$(1-e^{-\gamma_0})^N$. (7)

The Principle of NCT is described as follows. The original signal x_n is companded before converted into analog waveform and amplified by HPA. The companded signal is denoted as y_n = h (x_n), where h (.) is the companding function that only changes the amplitudes of x_n. In the case of addition Gaussian white noise (AWGN) channel, the received signal $r_n = y_n + v_n$ can be recovered by the de-companding function h^{-1}(.), namely $x_n' = h^{-1}(y_n + v_n) = x_n + h^{-1}(v_n)$, where v_n is channel noise.

IV. COMPANDING TECHNIQUE

The Companding is nothing but compressing the signal before transmitting and expanding the signal after receiving it. The Companding reduces the Dynamic range of the signal. The µ-law companding algorithm, which makes possible to improve the signal-to-noise ratio without the addition of more data and also reduces the dynamic of the signal i.e. clipping is predominantly used. For a given input x, the µ-law encoding is given as,

$$F(y) = \text{sgn}(x)\,\frac{In(1+\mu|x|)}{In(1+\mu)} \quad -1 \le x \le 1 \qquad (8)$$

Where µ=225 (8 bits) in the North America and Japanese standards. The range of this function is -1 to 1.

µ-law expansion is then given by the inverse equation,

$$F^{-1}(y) = \text{sgn}(y)\,\left(\frac{1}{\mu}\right)\left((1+\mu)^{|y|}-1\right), \quad -1 \le y \le 1 \qquad (9)$$

V. PROPOSED ALGORITHM

The main aim of the proposed algorithm is to transform the statistics of the amplitude $|y_n|$ into desirable PDF defined by a piecewise function, which consists of two linear functions with an inflexion point and cut-off point. Assume the inflexion and cut-off point of the PDF of $|x_n|$ are cA(0<c<1) and A (A>0), respectively [10]. Thus, the desirable target PDF can be expressed as

$$f_{|y_n|}(x)= \begin{cases} k_1 x, & 0 \le x \le cA \\ k_2 x + (k_1 - k_2), & cA < x \le A \end{cases} \qquad (10)$$

Where two slopes $k_1 > 0$ and $k_2 < 0$ are variable parameters that determine the desired companding form i.e. the ultimate PAPR, while controlling the average output power in this transform. Based on the definition of PDF $\int_{-\infty}^{+\infty} f_{|y_n|}(x)\,dy = 1$

We have

$$k_1 = \frac{2 - A^2 k_2 (c-1)^2}{A^2 c (2-c)}$$

From (10), the CDF of $|y_n|$ can be represented as

$$F_{|y_n|}(x)= \begin{cases} \frac{k_1}{2} x^2 & 0 \le x \le cA \\ \frac{k_2}{2} x^2 + (k_1 - k_2) - \frac{k_1 - k_2}{2}(cA)^2 & cA < x \le A \\ 1 & x \ge A \end{cases} \qquad (11)$$

Here, CDF is a strictly monotonic increasing function and it has corresponding inverse function given as,

$$F^{-1}_{|y_n|}(x) = \begin{cases} \sqrt{\frac{2x}{k_1}} & x \le \frac{k_1}{2}(cA)^2 \\ \frac{1}{k_2}(k_1 - k_2)cA + \sqrt{(k_1 - k_2)k_1 c^2 A^2 + 2k_2 x} & x > \frac{k_1}{2}(cA)^2 \end{cases} \qquad (12)$$

Given that h(x) is also a strictly monotonic increasing function, we can obtain the following relationship,

$$F_{|x_n|}(x) = Prob\{|x_n| \leq x\}$$

$$= Prob\{h(|x_n| \leq h(x)\}$$

$$= F_{|y_n|}(\text{h(x)}). \qquad (13)$$

Therefore,

$$\text{h(x)} = \text{sgn(x)}.\ F^{-1}_{|y_n|}(F_{|x_n|}(x)). \qquad (14)$$

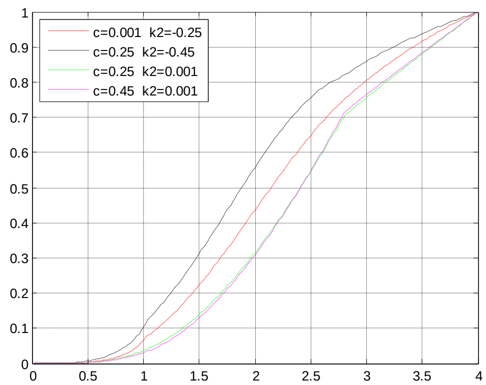


Fig 1. Transfer curves of the proposed companding function

Therefore, proposed companding function is given by (14). Where

$\chi_0 = \sigma(-In\left(1 - \left(\frac{k_2}{2}\right)c^2A^2\right))^{\frac{1}{2}}$. Additionally, in order to keep the input and output signal with a constant average power level, namely

E $\{|y_n|^2\}$ = E$\{|x_n|^2\}$ = σ^2, we can obtain

$$\text{A} = \left(\frac{1}{2\zeta_2}\left((\zeta_1^2 - 4\zeta_1\zeta_2)^{\frac{1}{2}} - \zeta_1\right)\right)^{\frac{1}{2}} \qquad (15)$$

Where, $\zeta_0 = 12\sigma^2(c-2), \zeta_1 = -2(c^3 -$ *4and ζ2=k2c3−3c+2.* From Fig 1, transform can be achieved more reduction in the PAPR with k_2 or c increasing.

At the receiver side, the companding signal can be recovered by the corresponding de-companding function.In practice, since actual signal processed at the transmitter and receiver are the quantized signal with finite set of values, the functions in [15] and inverse function of [14], can be numerically pre-computer and performed via look-up tables. Thus, its implementation complexity can be, significantly reduced.

VI. PERFORMANCE STUDY

The theoretical performance of the proposed algorithm is characterized in this section by using two main evaluation criteria: the achievable reduction in PAPR and the impact of companding distortion on the BER performance at the receiver.

A. Desirable Reduction in PAPR

By making appropriate substitution in (6), with the new algorithm the ultimate PAPR of the companding signal is given by

$$PAPR_y = \frac{max_{\ n} \in[0,PN-1]\{|y_n|^2\}}{E\{|y_n|^2\}}$$

$$= \frac{A^2}{\sigma^2} = \frac{(\zeta_1^2 - 4\zeta_2\zeta_0)^{\frac{1}{2}} - \zeta_1}{2\zeta_2\sigma^2} \qquad (16)$$

Furthermore, a transform gain G is defined as the ratio of the PAPR of original signal to that of the companded signal, i.e.

$$\text{G(dB)} = 10log_{10}\frac{PAPR_x}{PAPR_y}$$

$$= 10log_{10}\frac{2\zeta_2 A^2_{i\ max}}{(\zeta_1^2 - 4\zeta_2\zeta_0)^{\frac{1}{2}} - \zeta_1} \qquad (17)$$

Where $A_{i\ max} = max_{0 \leq n \leq PN-1}\{|y_n|\}$

The theoretical results of $PAPR_y$ and G are depicted below. As can be seen, by adjusting the values of k_2 and c this algorithm offers an adequate flexibility in the PAPR reduction. Consequently, the ultimate PAPR of the companded signal can be effectively confined in the interval [4.1 dB, 5.7 Db], or in other words, the achievable transform gain G in the PAPR is from 6 dB to 7.7 Db. Moreover, substituting (17) into (7), the CCDF of the PAPR with the proposed algorithm cab be written as follows,

$CCDF_y(\gamma_0) = \text{Prob}\{PAPR_y > \gamma_0\}$

$$=CCDF_x\left(\frac{2\zeta_2 A_{i\,max}^2}{(\zeta_1^2-4\zeta_2\zeta_0)^{\frac{1}{2}}-\zeta_1}\gamma_0\right) \qquad (18)$$

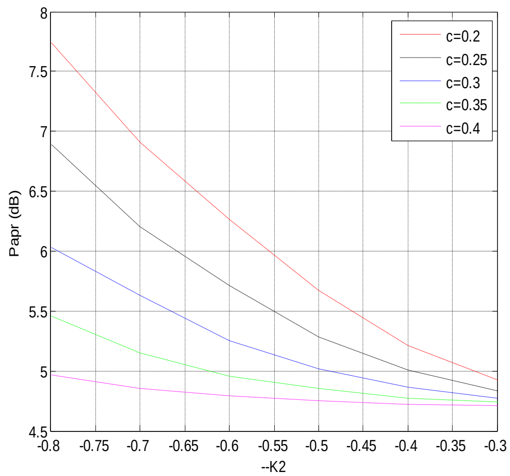


fig.2 (a) the ultimate PAPR of the companded signal

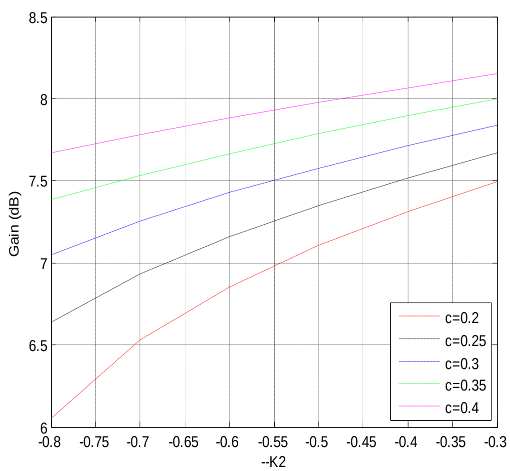


fig.2 (b) Transform Gain G

Fig. 3. Theoretical results of PAPR and G versus of the proposed algorithm.

A. Companding Distortion Impact on OFDM

NCT is an extra nonlinear operation applied to the transmitted signal. For this reason, choosing the optimal companding form and parameters is the key reason to minimize the impact of companding distortion on the BER performance. Based on the analysis results for the Gaussian signal in (16) and (17), two performance criteria: signal attenuation and companding noise b_n can be used to characterize this impact, i.e.

$$y_n = \alpha x_n + b_n \qquad (19)$$

Where attenuation factor, α which is given by,

$$\alpha = \frac{1}{\sigma^2}\int_0^{\infty} xh(x)f_{|x_n|}dx \qquad (20)$$

Smaller value obviously gives larger companding distortion and the reduced BER performance. It is shown in [12] that noise power of b_n is also increasing as decreases. The attenuation factor of the new algorithm can be calculated as

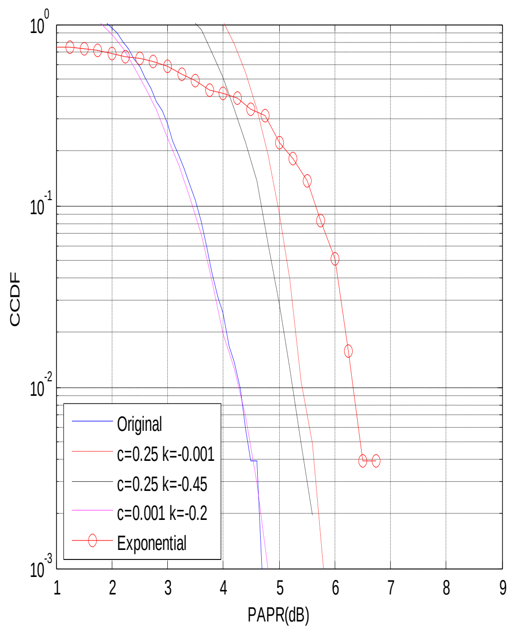
$$\alpha=\frac{2}{\sigma^4}\int_0^{\chi_0} x^2 e^{\frac{-x^2}{\sigma^2}}\left(\frac{2}{k_1}\left(1-e^{\frac{-x^2}{\sigma^2}}\right)\right)^{\frac{1}{2}}\text{dx}$$

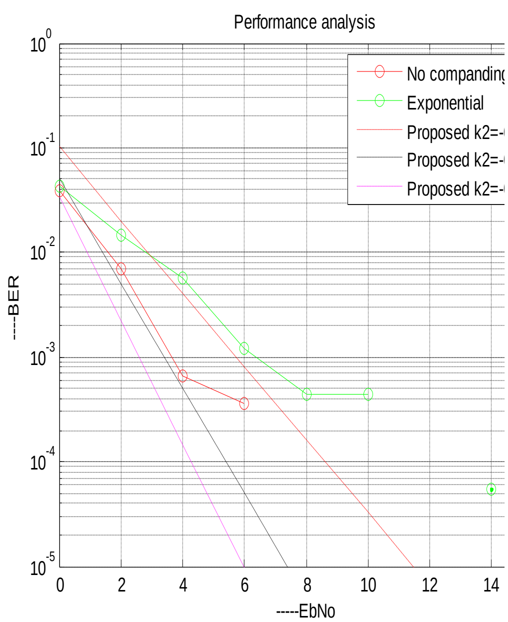
$$+\frac{2}{k_2\sigma^4}\int_{\chi_0}^{\infty} x^2 e^{\frac{-x^2}{\sigma^2}} \ \text{X} \left((k_2-k_1)cA + \right.$$

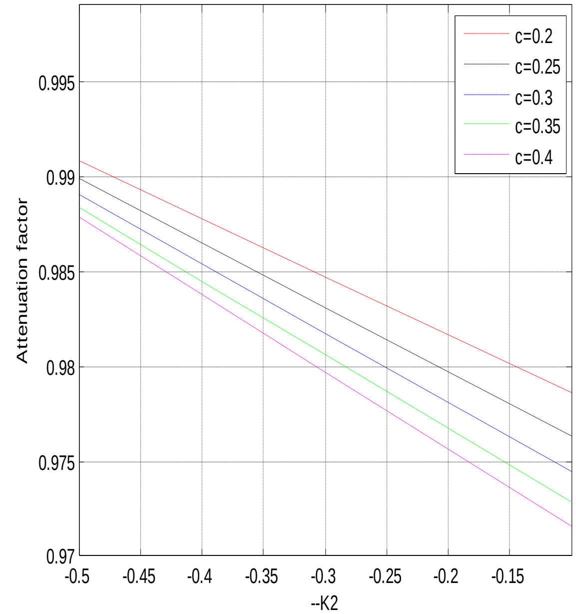
$k2{-}k1c2{+}2k21{-}e{-}x2\sigma2$dx

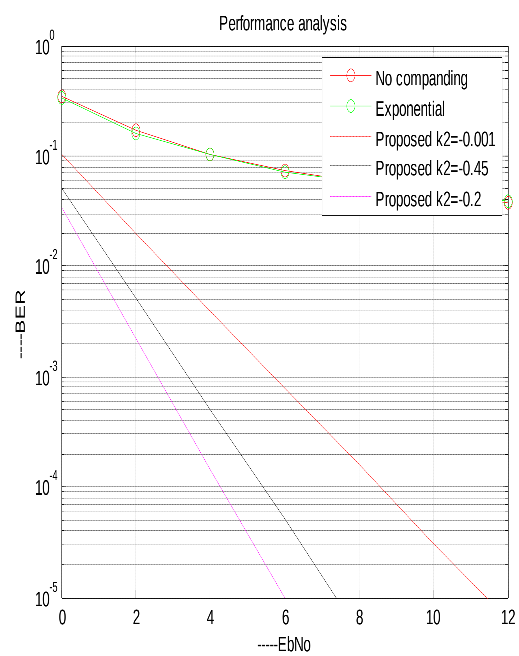
The theoretical results of α is depicted in fig.3 from that we can see α gradually tends to 1 as k_2 and c decrease. As a result, fig.2 and fig.3 demonstrate that, to obtain an expected PAPR reduction, it may be preferable for this algorithm to make the undesired signal distortion as small as possible by choosing proper parameters. This conclusion is quite helpful to design the optimal companding form to offer an effective trade-off between the PAPR reduction and BER performance in practice.

VII. SIMULATION RESULTS









VIII. CONCLUSIONS

Higher PAPR is the major disadvantage for implementation of OFDM. NCT is an attractive solution to reduce the PAPR of OFDM signal due to its simplicity and effectiveness. In this paper, we investigate a new NCT algorithm which changes the statistics of original signal from the complex Gaussian to a desirable PDF defined as a linear piecewise function. Thus, an effective and flexible trade-off between the PAPR and BER performance can be achieved to satisfy various system requirements. Theoretical performance of this algorithm is characterized by means of the achievable reduction in PAPR and the signal attenuation factor. Compared to the original signal the proposed algorithm can offer the transform gain in PAPR of 6 dB to 7.7 dB. The companding distortion can be significantly reduced by choosing proper transform parameters. Simulation results indicate that the new proposed algorithm substantially outperform the existing NCT methods in the overall performance of OFDM system regarding the reduction in PAPR, BER and out of band interference under the multipath fading channel or with the High Power Amplifier.

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