

# Kalman filter beamformer for speech processing and its applications

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## Abstract:

**The Kalman filter has long been regarded as the optimal solution to many tracking and data prediction tasks. Its use in the analysis of visual motion has been documented frequently. Statistically optimal spatial processors (also referred to as data-dependent beam formers) are widely-used spatial focusing techniques for desired source extraction. The Kalman filter-based beam former (KFB) is a recursive Bayesian method for implementing the beam former. We adopt the KFB framework to the task of speech extraction. We formalize the KFB with a set of linear constraints and present its equivalence to the linearly constrained minimum power (LCMP) beam former. We further show that the optimal output power, required for implementing the KFB, is merely controlling the white noise gain (WNG) of the beam former. We also show, that in static scenarios, the adaptation rule of the KFB reduces to the simpler affine projection algorithm (APA). The analytically derived results are verified and exemplified by a simulation study.**

***Index terms* – Kalman filter-based beamformer (KFB), linearly constrained minimum power beamformer (LCMP).**

## I. INTRODUCTION

Beam forming is one of the most common techniques in microphone array processing. Typically, a beam former is used to obtain a spatial focusing on the desired speech sources, while reducing the interfering sources and the background noise. Statistically optimal beam formers e.g., minimum variance distortion less response (MVDR) linearly constrained minimum variance (LCMV) and speech distortion weighted multichannel Wiener filter (SDW-MWF) are very useful and widely-used for speech extraction in a reverberant environment. Construction of the beam formers necessitates the

sources' statistics and the acoustic transfer functions (ATFs) relating the sources and the microphones (or merely the respective relative transfer functions), which have to be either known or estimated from the received signals.

The applicability of the constrained Kalman filter as a recursive estimator of a time-varying beam former has been addressed. A recursive solution of the MVDR criterion by a constrained Kalman filter in the Kalman filter-based beam former (KFB) was utilized to design a robust version of the MVDR beamformer. Considering the speech extraction task, the availability of the output power cannot be assumed, as the output power, corresponding to the desired speech power, is unknown and time-varying. In the current contribution, we adopt the KFB framework and apply it to non-stationary signals. The KFB with a set of linear constraints is formalized, and its equivalence with the LCMP beam former is presented. We show that the output power value is only required for controlling the KFB white noise gain (WNG), known to be closely related to the beam former robustness. Additionally, we also demonstrate that, in a static scenario, the KFB adaptation rule is reduced to affine projection algorithm (APA). It was shown that the adaptation rule of a single-channel, time-domain Kalman filter is reduced to APA in a static scenario. Here, we show that this simplification is also applicable to the multi-channel, frequency-domain case.

## II. LCMP BEAMFORMER

The adaptive linearly constrained minimum power (LCMP) beamformer can improve the robustness of the Capon beamformer. And quadratic constraints on the weighting vector of the LCMP beamformer can improve the robustness to pointing errors and to random perturbations in sensor parameters. But how to solve it and how to select the constraint parameters are its key problems. The Lagrange multiplier method is proposed to solve the LCMP

beamformer under quadratic inequality constraint (QIC). The problem of finding the optimal weight vector is solved, and the choice of the quadratic constraint parameter is analyzed and the selected bound is also given. Since the quadratic equality constraint (QEC) is stronger than the quadratic inequality constraint (QIC), the performance of the QECLCMP beamformer is more robust than that of the QICLCMP beamformer. Therefore, the QECLCMP beamformer is proposed and is solved effectively. Results show that the QECLCMP beamformer has the advantage of overcoming the steering vector mismatch, namely the optimal negative loading has the preferable robustness.

For the QICLCMP beamformer, the key problems are how to solve the optimal weighting vector and how to select the quadratic constraint parameter. Hence, the Lagrange multiplier method is proposed to solve the QICLCMP beamformer, and the problem of finding the optimal weight vector is solved. Furthermore, the choice of the quadratic constraint parameter is analyzed and the selected bound is given. Since the quadratic equality constraint (QEC) is stronger than the quadratic inequality constraint (QIC), the LCMP beamformer under quadratic equality constraint (QEC) has more ascendant robust performance than the QICLCMP beamformer. Therefore, the QECLCMP beamformer is proposed and is solved effectively. Numerical examples attest the correctness and the validity of the proposed algorithm, which show that the QECLCMP beamformer has the best performance to overcome the signal direction mismatch, namely the optimal negative loading has the preferable robustness. This paper is organized as follows. First, the signal model and the LCMP beamformer are introduced. Second, the Lagrange multiplier method is proposed to solve the QICLCMP beamformer, particularly the choice of the quadratic constraint parameter and the selecting bound is discussed. Third, the QECLCMP beamformer is proposed and is solved effectively. Finally, the simulation analyses and the conclusion are given.

Since the quadratic constraints on the weight vector of the LCMP beamformer can improve the robustness to pointing errors and random perturbations in sensor parameter, the Lagrange multiplier method is developed to solve the LCMP beamformer under quadratic constraint which includes the inequality and equality constraint. Therein, the exact Lagrange multiplier or loading

level is obtained, and the optimal weight vector can be computed exactly. The choice of the quadratic constraint parameter is analyzed and the selecting bound is given. Above all, this method gives the efficient solution to find the optimal loading level for the QCLCMP beamformer. From the theory analysis and the simulations, the QECLCMP beamformer has the best performance to overcome the steering vector mismatch.

### III. KALMAN FILTERING

The Kalman Filter is an estimator for what is called the “linear-quadratic problem”, which focuses on estimating the instantaneous “state” of a linear dynamic system perturbed by white noise by using measurements linearly related to the state but corrupted by white noise. The resulting estimator is stastically optimal with respect to any quadratic function of estimation error. The block diagram of proposed system is as shown in the figure 1.

The Kalman filter addresses the general problem of trying to estimate the state  $x \in \mathcal{R}^n$  of a discrete-time controlled process that is governed by the linear stochastic difference equation.

$$x_k = Ax_{k-1} + Bu_{k-1} + w_{k-1} \quad (1)$$

with a measurement  $Z \in \mathcal{R}^m$  that is

$$Z_k = Hx_k + V_k \quad (2)$$

$W_k$  and  $V_k$  represent the process and measurement noise respectively. They are assumed to be independent of each other

$$P(w) \sim N(0, Q) \quad (3)$$

$$P(v) \sim N(0, R) \quad (4)$$

The process noise covariance  $Q$  and measurement noise covariance  $R$  matrices might change with each time step or measurement.

In equation (1)  $A$  is  $n \times n$  matrix that relates to the previous time step  $k-1$  to the current step  $k$ . we assume the matrix to be a constant.

$B$  is a  $n \times l$  matrix that relates the optimal control input  $u \in \mathcal{R}^l$  to the state  $x$ .

$H$  is a  $m \times n$  matrix that relates the state to the measurement  $Z_k$ . it is also assumed to be a constant.

We can define a priori and a posteriori estimate errors as,

$$e_k^- = x_k - \hat{x}_k^- \quad (5)$$

$$e_k = x_k - \hat{x}_k \quad (6)$$

where  $\hat{x}_k^-$  to be priori state estimate at step  $k$ . and  $\hat{x}_k$  to be posteriori state estimate at step  $k$  given measurement  $Z_k$ .

then a priori estimate error covariance is then

$$P_k^- = E[e_k^- e_k^{-T}] \tag{7}$$

The posteriori estimate error covariance

$$P_k = E [e_k e_k^T] \tag{8}$$

$$X_k = \hat{x}_k^- + K (Z_k - H \hat{x}_k^-) \tag{9}$$

The difference  $(Z_k - H \hat{x}_k^-)$  in equation (9) is called the measurement innovation or the residual. This indicates the the similarity between the actual measurement and predicted measurement.

The  $n \times m$  matrix  $K$  in equation (9) is chosen to be the gain or blending factor that minimizes thea posteriori error covariance.

$$K_k = P_k^- H^T (H P_k^- H^T + R)^{-1} \tag{10}$$

The measurement error covariance  $R$  approaches zero, the gain  $K$  weights the residual more heavily.

$$\lim_{R \rightarrow 0} K_k = H^{-1} \tag{11}$$

As the a priori error covariance  $P_k^-$  approaches zero, the gain  $K$  weights the residual less heavily

$$\lim_{P_k^- \rightarrow 0} K_k = 0 \tag{12}$$

As the a priori estimate error covariance  $P_k^-$  approaches zero the actual measurement  $Z_k$  is trusted less, while the predicted measurement  $H \hat{x}_k^-$  is trusted more.

The justification for equation (9) is in the a priori estimate  $x_k^-$  conditioned on all priori measurements  $Z_k$  (Baye's rule). Kalman filter maintains the first two moments of the state distribution

$$E[x_k] = \hat{x}_k \tag{13}$$

$$E[(x_k - \hat{x}_k) (x_k - \hat{x}_k)^T] = P_k \tag{14}$$

The posteriori state estimate of (9) reflects the mean (first moment) of the state distribution. The posteriori estimate error covariance of (8) reflects the variance of the state distribution (the second moment)

$$P(x_k|Z_k) \approx N(E[x_k], E[(x_k - \hat{x}_k) (x_k - \hat{x}_k)^T]) = N(\hat{x}_k, P_k) \tag{15}$$

Let us consider the signal model as

$$Y_k = \frac{1}{1 - \sum_{i=1}^N a_i Z^{-i}} \tag{16}$$

Therefore,

$$Y_k = a_1 y_{k-1} + a_2 y_{k-2} + \dots + a_N y_{k-N} + w_k \tag{17}$$

Where,

$K$ =no. of iterations

$Y_k$ = current input signal

$Y_{k-N}$  =  $(N-1)^{th}$  sample of speech signal

$a_N$  =  $N^{th}$  kalman filter coefficient

$w_k$  = white noise

in order to apply kalman filtering to the speech signal, it must be expressed in state space form :

$$H_k = X H_{k-1} + w_k \tag{18}$$

$$Y_k = g H_k \tag{19}$$

Where,

$$X = \begin{bmatrix} a_1 & a_2 & \dots & a_{N-1} & a_N \\ 1 & 0 & \dots & 0 & 0 \\ 0 & 0 & \dots & 1 & 0 \end{bmatrix} \tag{20}$$

$X$  is the system matrix or transition matrix.

$H_k$  consists of series of speech samples

$W_k$  is the excitation vector

$g$  = output vector

$$H_k = \begin{bmatrix} y_k \\ y_{k-1} \\ y_{k-2} \\ \vdots \\ y_{k-N+1} \end{bmatrix} \quad w_k = \begin{bmatrix} w_k \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

$$g = [1 \quad 0 \quad \dots \quad 0]$$

kalman filter functions in a looping method.  $I$  denotes the following steps within the loop of the filter. Define matrix  $H_{k-1}^T$  as a row vector

$$H_{k-1}^T = [y_{k-1} \quad y_{k-2} \quad \dots \quad y_{k-N}] \tag{21}$$

$$Z_k = H_{k-1}^T x_k + w_k \tag{22}$$

$X_k$  will be updated according to no. of iterations  $K$ .  $K=0$  the matrix  $H_{k-1}$  cannot be determined. When the time  $z_k$  is detected, the value in matrix  $H_{k-1}$  is known. Thus the kalman filter is defined as

$$X_k = [1 - K_k H_{k-1}^T] X_{k-1} + K_k Z_k \tag{23}$$

$$I = \begin{bmatrix} 1 & 0 & \dots & 0 & 0 \\ 0 & 1 & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & 1 & 0 \\ 0 & 0 & \dots & 0 & 1 \end{bmatrix}$$

$$K_k = P_{k-1} H_{k-1} [H_{k-1}^T P_{k-1} H_{k-1} + R]^{-1} \quad (24)$$

Where  $K_k$  is the kalman gain matrix,  $P_{k-1}$  is the priori error covariance matrix,  $R$  is the measurement noise covariance,

$$P_k = P_{k-1} - P_{k-1} H_{k-1} [H_{k-1}^T P_{k-1} H_{k-1} + R]^{-1} H_{k-1}^T P_{k-1} + Q \quad (25)$$

Where,  $P_k$  is the posteriori error covariance matrix.

$$Q = \begin{bmatrix} 1 & 0 & \dots & 0 & 0 \\ 0 & 1 & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & 1 & 0 \\ 0 & 0 & \dots & 0 & 1 \end{bmatrix}$$

Thereafter the reconstructed speech signal,  $Y_k$  after Kalman filtering will be formed in a manner similar to equation (26).

$$Y_k = a_1 Y_{k-1} + a_2 Y_{k-2} + \dots + a_N Y_{k-N} + w_k \quad (26)$$

Since the value of  $y_k$  is the input at the beginning of the process, there will be no problem forming  $H_{k-1}^T$ . In that case a question rises,  $Y_k$  formed by the parameters  $w_k$  and  $\{a_i\}_{i=1}^N$  are determined from application of the Kalman filter to the input speech signal  $Y_k$ . That is in order to construct  $Y_k$ , we will need matrix  $X$  that contains the Kalman coefficients and the white noise,  $w_k$  which both are obtained from the estimation of the input signal. This information is enough to determine  $H_{k-1}$ .

Where

$$H_{k-1} = \begin{bmatrix} y_{k-1} \\ y_{k-2} \\ y_{k-3} \\ \vdots \\ y_{k-N+1} \end{bmatrix}$$

#### IV. RESULTS

When the input signal is applied to the kalman filter the estimated output is shown in the figure 1.

Figure 2 shows the combined plot of the input signal and the noise suppressed output signal

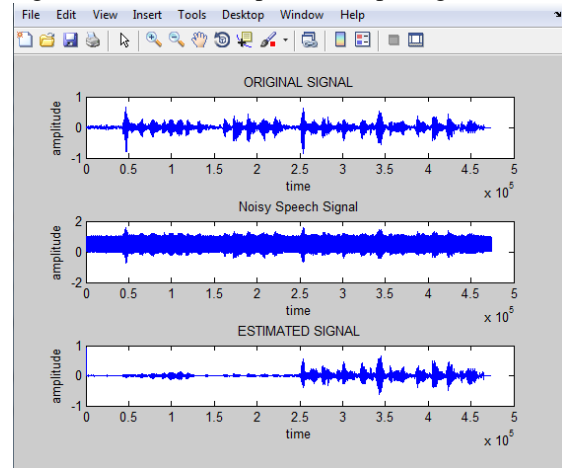


Fig1: estimated output of the kalman filter

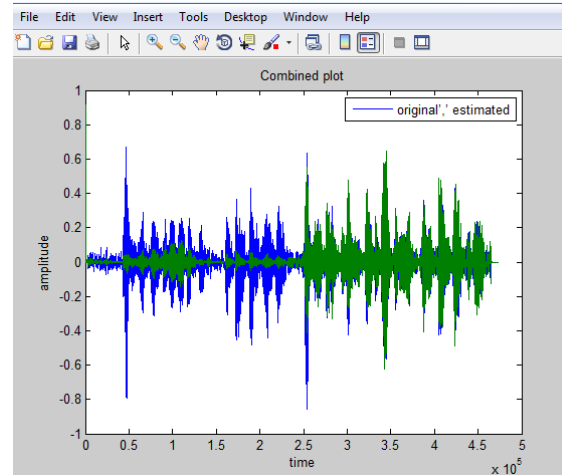


Fig2: combined plot of the input and output signal.

Figure 3 shows the mean square error of the signal

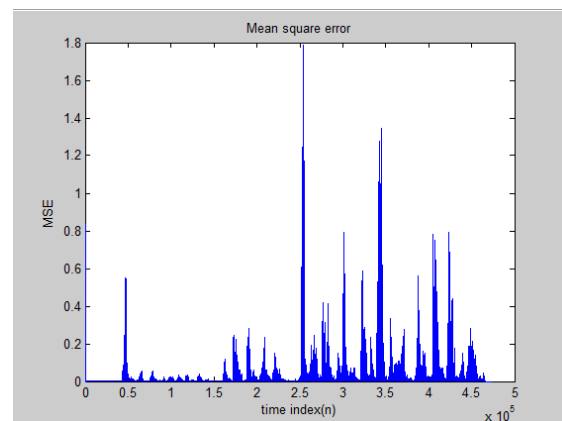


Fig 3: mean square error plot

Figure 4 shows the power spectral density of the signal.

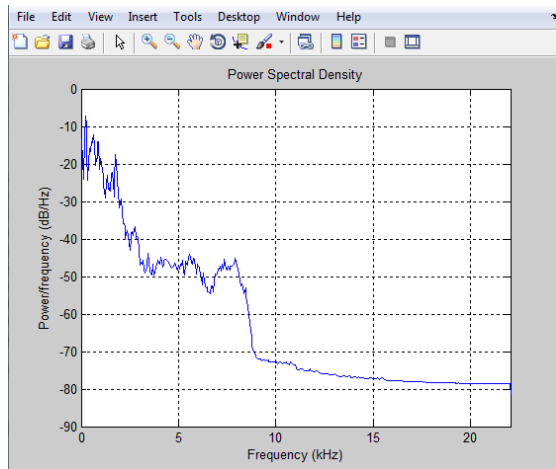


Fig 4: power spectral density plot

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