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Fourier Transform-Based LMMSE Analysis of MIMO Channel Estimation

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ABSTRACT*:The paper we present an improved DFT-based channel estimation method. The conventional discrete Fourier transforms (DFT)-based approach will cause energy leakage in multipath channel with non-sample-spaced time delays. The improved method uses symmetric property to extend the LMMSE in frequency domain, and calculates the changing rate of the leakage energy, and selects useful paths by the changing rate. The computer simulation results show the improved method can reduce the leakage energy efficiently, and the performance of the LMMSE channel estimation method is better than the MMSE and ZF algorithm.*

Keywords: DFT, LMMSE, MMSE,ZF, channel estimation,Fourier transforms

1. INTRODUCTION:

OFDM is modulation method known for its capability to mitigate multipath. In OFDM the high speed data stream is divided into Nc narrowband data streams, Nc corresponding to the subcarriers or subchannels i.e. one OFDM symbol consists of N symbols modulated for example by QAM or PSK. As a result the symbol duration is N times longer than in a single carrier system with the same symbol rate. The symbol duration is made even longer by adding a cyclic prefix to each symbol. As long as the cyclic prefix is longer than the channel delay spread OFDM offers inter-symbol interference (ISI) free transmission.

Another key advantage of OFDM is that it dramatically reduces equalization complexity by enabling equalization in the frequency domain. OFDM, implemented with IFFT at the transmitter and FFT at the receiver, converts the wideband signal, affected by frequency selective fading, into N narrowband flat fading signals thus the equalization can be performed in the frequency domain by a scalar division carrier-wise with the subcarrier related channel coefficients. The channel should be known or learned at the receiver. The combination MIMO-OFDM is very natural and beneficial since OFDM enables support of more antennas and larger bandwidths since it simplifies equalization dramatically in MIMO systems.

MIMO-OFDM is under intensive investigation by researchers. This paper provides a general overview of this promising transmission technique.

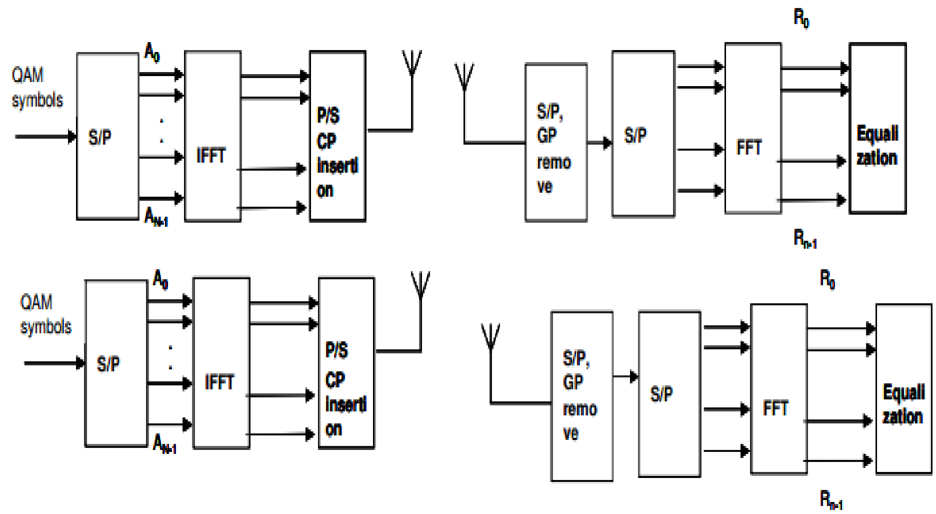
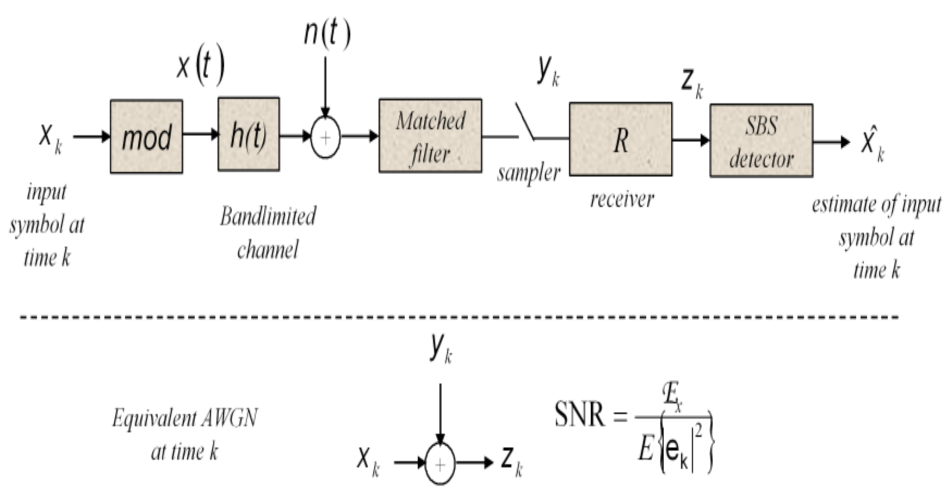


Figure 1.1: MIMO-OFDM transmission technique of data processing

2. ZERO FORCING AND MMSE CHANNEL ESTIMATION ON MIMO:

2.1 INTRODUCTION:

Equalization methods are used by communication engineers to mitigate the effects of the inter symbol interference. An equalizer is essentially the content of Figure receiver box. This chapter studies both inter symbol interference and several equalization methods, which amount to different structures for the receiver box. The methods presented in this chapter are not optimal for detection, but rather are widely used sub-optimal cost-effective methods that reduce the ISI. These equalization methods try to convert a band limited channel with ISI into one that appears memory less, hopefully synthesizing a new AWGN-like channel at the receiver output.



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2.2 INTER SYMBOL INTERFERENCE AND RECEIVERS FOR SUCCESSIVE MESSAGE TRANSMISSION:

Inter symbol interference is a common practical impairment found in many transmission and storage systems, including voice band modems, digital subscriber loop data transmission.

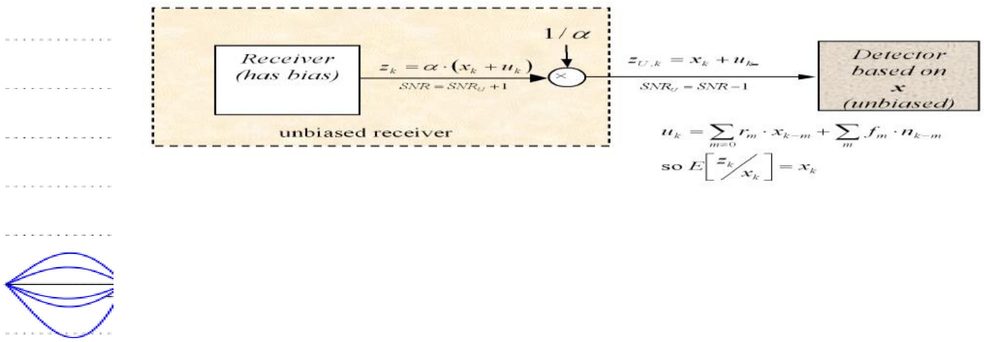

Figure 2.1: Inter symbol Interference and Receivers for successive Message Transmission

The concept of a receiver SNR facilitates evaluation of the performance of data transmission systems with various compensation methods (i.e. equalizers) for ISI. Use of SNR as a performance measure builds upon the simplifications of considering mean-square distortion, that is both noise and ISI are jointly considered in a single measure. The two right-most terms in have normalized mean-square value _2 + ¯D ms. The SNR for the matched filter output *y*k in Figure 2.1.a. is the ratio of channel output sample energy ¯ E*x*k*p*k2 to the mean-square distortion _2 + ¯D ms. This SNR is often directly related to probability of error and is a function of both the receiver and the decision regions for the SBS detector.

This text uses SNR consistently, replacing probability of error as a measure of comparative performance. SNR is easier to compute than *P*e, independent of *M* at constant Ex, and a generally good measure of performance: higher SNR means lower probability of error. The probability of error is difficult to compute exactly because the distribution of the ISI-plus-noise is not known or is difficult to compute.

2.3 PERFORMANCE ANALYSIS OF ZF AND MMSE EQUALIZER FOR MIMO SYSTEM:

Zero forcing (ZF) and minimum mean squared error (MMSE) equalizers applied to wireless multi-input multi-output (MIMO) systems with no fewer receive than transmit antennas. In spite of much prior work on this subject, we reveal several new and surprising analytical results in terms of output signal-to-noise ratio (SNR), by comparing the Bit Error Rate (BER) and the average detection time consuming. Simulation based on the platform of MATLAB. We discuss the case where there a multiple transmit antennas and multiple receive antennas resulting in the formation of a Multiple Input Multiple Output (MIMO) channel with Zero Forcing equalizer, MIMO with MMSE equalizer, MIMO with ZF Successive Interference Cancellation equalizer, MIMO with ML equalization, MIMO with MMSE SIC and optimal ordering

Let us now discuss the case where there a multiple transmit antennas and multiple receive antennas resulting in the formation of a Multiple Input Multiple Output (MIMO) channel. In this post, we will restrict our discussion to a 2 transmit 2 receive antenna case (resulting in a **2×2** MIMO channel). We will assume that the channel is a flat fading Rayleigh multipath channel and the modulation is BPSK.

2×2 MIMO channel:

In a 2×2 MIMO channel, probable usage of the available 2 transmit antennas can be as follows:
1. Consider that we have a transmission sequence, for example $\{x_1, x_2, x_3, \ldots, x_n\}$
2. In normal transmission, we will be sending x_1 in the first time slot, x_2 in the second time slot, x_3 and so on.
3. However, as we now have 2 transmit antennas, we may group the symbols into groups of two. In the first time slot, send x_1 and x_2 from the first and second antenna. In second time slot, send x_3 and x_4 from the first and second antenna, send x_5 and x_6 in the third time slot and so on.
4. Notice that as we are grouping two symbols and sending them in one time slot, we need only $\frac{n}{2}$ time slots to complete the transmission – **data rate is doubled.**
5. This forms the simple explanation of a probable MIMO transmission scheme with 2 transmit antennas and 2 receive antennas.

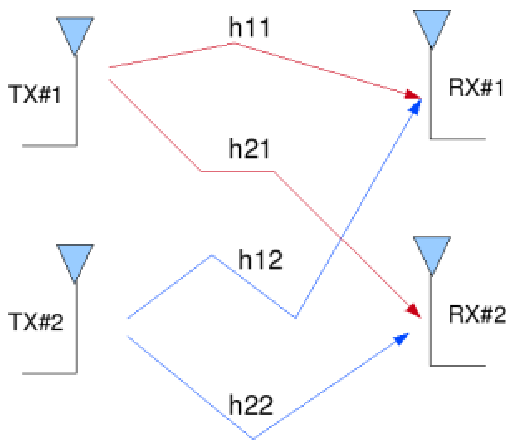

Figure 2.2 : 2 Transmit 2 Receive (2×2) MIMO channel

Other Assumptions:
1. The channel is flat fading – In simple terms, it means that the multipath channel has only one tap. So, the convolution operation reduces to a simple multiplication.
2. The channel experience by each transmit antenna is independent from the channel experienced by other transmit antennas.
3. For the i^{th} transmit antenna to j^{th} receive antenna, each transmitted symbol gets multiplied by a randomly varying complex number $h_{j,i}$. As the channel under consideration is a Rayleigh channel, the real and imaginary parts of $h_{j,i}$ are Gaussian distributed having mean $\mu_{h_{j,i}} = 0$ and variance $\sigma^2_{h_{j,i}} = \frac{1}{2}$.
4. The channel experienced between each transmit to the receive antenna is independent and randomly varying in time.
5. On the receive antenna, the noise n has the Gaussian probability density function with

$p(n) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-(n-\mu)^2}{2\sigma^2}}$ With $\mu = 0$ and $\sigma^2 = \frac{N_0}{2}$.

6. The channel $h_{j,i}$ is known at the receiver.

2.3.1 Zero forcing (ZF) equalizer for 2×2 MIMO channel:

Let us now try to understand the math for extracting the two symbols which interfered with each other. In the first time slot, the received signal on the first receive antenna is,

$$y_1 = h_{1,1}x_1 + h_{1,2}x_2 + n_1 = [h_{1,1} \ h_{1,2}] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + n_1 \quad \text{-------- 2.0}$$

The received signal on the second receive antenna is,

$$y_2 = h_{2,1}x_1 + h_{2,2}x_2 + n_2 = [h_{2,1} \ h_{2,2}] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + n_2 . \quad \text{-------- 2.1}$$

Where

y_1, y_2 are the received symbol on the first and second antenna respectively,
$h_{1,1}$ is the channel from 1^{st} transmit antenna to 1^{st} receive antenna,
$h_{1,2}$ is the channel from 2^{nd} transmit antenna to 1^{st} receive antenna,
$h_{2,1}$ is the channel from 1^{st} transmit antenna to 2^{nd} receive antenna,
$h_{2,2}$ is the channel from 2^{nd} transmit antenna to 2^{nd} receive antenna,
x_1, x_2 are the transmitted symbols and
n_1, n_2 is the noise on $1^{st}, 2^{nd}$ receive antennas.

We assume that the receiver knows $h_{1,1}$, $h_{1,2}$, $h_{2,1}$ and $h_{2,2}$. The receiver also knows y_1 and y_2. The unknown s are x_1 and x_2.

For convenience, the above equation can be represented in matrix notation as follows:

Equivalently,

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n} \quad \text{-------- 2.3}$$

To solve for $\mathbf{x}$, we know that we need to find a matrix $\mathbf{W}$ which satisfies $\mathbf{W}\mathbf{H} = \mathbf{I}$. The **Zero Forcing (ZF) linear detector** for meeting this constraint is given by,

$$\mathbf{W} = (\mathbf{H}^H \mathbf{H})^{-1} \mathbf{H}^H . \quad \text{-------- 2.4}$$

This matrix is also known as the pseudo inverse for a general m x n matrix.

The term,

BER with ZF equalizer with 2×2 MIMO:

Note that the off diagonal terms in the matrix $\mathbf{H}^H\mathbf{H}$ are not zero (Recall: The off diagonal terms where zero in Altamonte 2×1 STBC case). Because the off diagonal terms are not zero, the zero forcing equalizer tries to null out the interfering terms when performing the equalization, i.e. when solving for x_1 the interference from x_2 is tried to be mulled and vice versa. While doing so, there can be amplification of noise. Hence Zero Forcing equalizer is not the best possible equalizer to do the job. However, it is simple and reasonably easy to implement. Hence the BER for 2×2 MIMO channel in Rayleigh fading with Zero Forcing equalization **is** same as the BER derived for a 1×1 channel in Rayleigh **f**ading.

2.3.2 MMSE Analysis in MIMO OFDM System:

2×2 MIMO channel

In a 2×2 MIMO channel, probable usage of the available 2 transmit antennas can be as follows:

1. Consider that we have a transmission sequence, for example $\{x_1, x_2, x_3, \ldots, x_n\}$
2. In normal transmission, we will be sending x_1 in the first time slot, x_2 in the second time slot, x_3 and so on.

3. However, as we now have 2 transmit antennas, we may group the symbols into groups of two. In the first time slot, send x_1 and x_2 from the first and second antenna. In second time slot, send x_3 and x_4 from the first and second alternatively. Notice that as we are grouping two symbols and sending them in one time slot, we need only $\frac{n}{2}$ time slots to complete the transmission – **data rate is doubled**
4. This forms the simple explanation of a probable MIMO transmission scheme with 2 transmit antennas and 2 receive antennas.

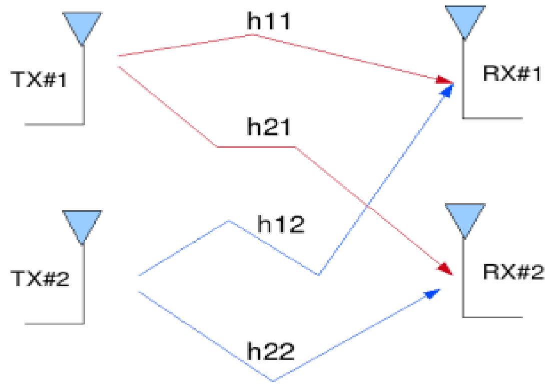

Figure 2.2 : 2 Transmit 2 Receive (2×2) MIMO channel

Other Assumptions:

1. The channel is flat fading – In simple terms, it means that the multipath channel has only one tap. So, the convolution operation reduces to a simple multiplication
2. The channel experience by each transmit antenna is independent from the channel experienced by other transmit antennas.3. For the i^{th} transmit antenna to j^{th} receive antenna, each transmitted symbol gets multiplied by a randomly varying complex number $h_{j,i}$. As the channel under consideration is a Rayleigh channel, the real and imaginary parts of $h_{j,i}$ are Gaussian distributed having mean $\mu_{h_{j,i}}=0$ and variance $\sigma^2_{h_{j,i}}=\frac{1}{2}$.
4. The channel experienced between each transmit to the receive antenna is independent and randomly varying in time.
5. On the receive antenna, the noise n has the Gaussian probability density function with

$$p(n)=\frac{1}{\sqrt{2\pi\sigma^2}}e^{\frac{-(n-\mu)^2}{2\sigma^2}}$$ With $\mu=0$ and $\sigma^2=\frac{N_0}{2}$.

6. The channel $h_{j,i}$ is known at the receiver.

2.3.3 Minimum Mean Square Error (MMSE) equalizer for 2×2 MIMO channel:

Let us now try to understand the math for extracting the two symbols which interfered with each other. In the first time slot, the received signal on the first receive antenna is,

$$y_1=h_{1,1}x_1+h_{1,2}x_2+n_1=[h_{1,1}\ h_{1,2}]\begin{bmatrix}x_1\\x_2\end{bmatrix}+n_1. \qquad \text{-------- 2.5}$$

The received signal on the second receive antenna is,

$$y_2 = h_{2,1}x_1 + h_{2,2}x_2 + n_2 = [h_{2,1} \ h_{2,2}]\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + n_2. \quad \text{-------- 2.6}$$

Where

y_1, y_2 are the received symbol on the first and second antenna respectively,

$h_{1,1}$ is the channel from 1^{st} transmit antenna to 1^{st} receive antenna,

$h_{1,2}$ is the channel from 2^{nd} transmit antenna to 1^{st} receive antenna,

$h_{2,1}$ is the channel from 1^{st} transmit antenna to 2^{nd} receive antenna,

$h_{2,2}$ is the channel from 2^{nd} transmit antenna to 2^{nd} receive antenna,

x_1, x_2 are the transmitted symbols and

n_1, n_2 is the noise on 1^{st},2^{nd} receive antennas.

We assume that the receiver knows $h_{1,1}$, $h_{1,2}$, $h_{2,1}$ and $h_{2,2}$. The receiver also knows y_1 and y_2. For convenience, the above equation can be represented in matrix notation as follows:

Equivalently,

$$\boldsymbol{y} = \boldsymbol{H}\boldsymbol{x} + \boldsymbol{n} \quad \text{-------- 2.7}$$

The **Minimum Mean Square Error (MMSE)** approach tries to find a coefficient $\boldsymbol{W}$ which minimizes the criterion,

$$E\left\{[\boldsymbol{W}\boldsymbol{y} - \boldsymbol{x}][\boldsymbol{W}\boldsymbol{y} - \boldsymbol{x}]^H\right\}. \quad \text{-------- 2.8}$$

Solving,

$$\boldsymbol{W} = \left[\boldsymbol{H}^H\boldsymbol{H} + \boldsymbol{N}_0\boldsymbol{I}\right]^{-1}\boldsymbol{H}^H. \quad \text{-------- 2.9}$$

When comparing to the equation in Zero Forcing equalizer, apart from the $\boldsymbol{N}_0\boldsymbol{I}$ term both the equations are comparable. In fact, when the noise term is zero, the **MMSE** equalizer reduces to Zero Forcing equalizer.

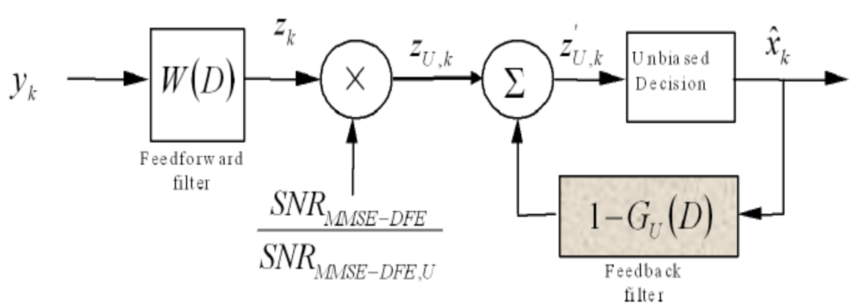

Figure 2.3: Minimum Mean Square Error (MMSE) equalizer for 2×2 MIMO channel

(a) Generate random binary sequence of +1′s and -1′s.
(b) Group them into pair of two symbols and send two symbols in one time slot
(c) Multiply the symbols with the channel and then add white Gaussian noise.
(d) Equalize the received symbols
(e) Perform hard decision decoding and count the bit errors
(f) Repeat for multiple values of $\frac{E_b}{N_0}$ and plot the simulation and theoretical results.

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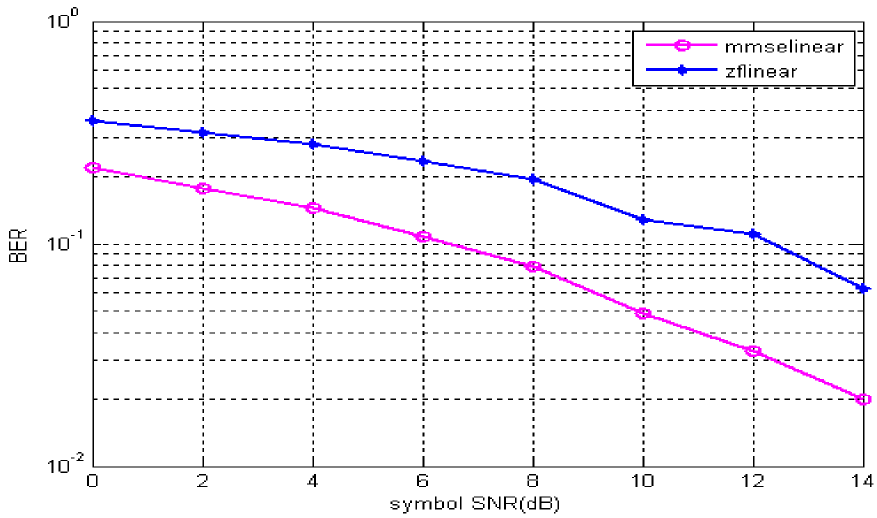

Figure 2.4 : Comparision between MMSE vs.ZF Using MIMO Process

3.ANALYSIS OF LMMSE METHOD:

We present an improved DFT-based channel estimation method. The conventional discrete Fourier transforms (DFT)-based approach will cause energy leakage in multipath channel with non-sample-spaced time delays.The improved method uses symmetric property to extend the LMMSE in frequency domain, and calculates the changing rate of the leakage energy, and selects useful paths by the changing rate. The computer simulation results show the improved method can reduce the leakage energy efficiently, and the performance of the LMMSE channel estimation method is better than the MMSE and ZF algorithm.

3.1 BLOCK DIAGRAM:

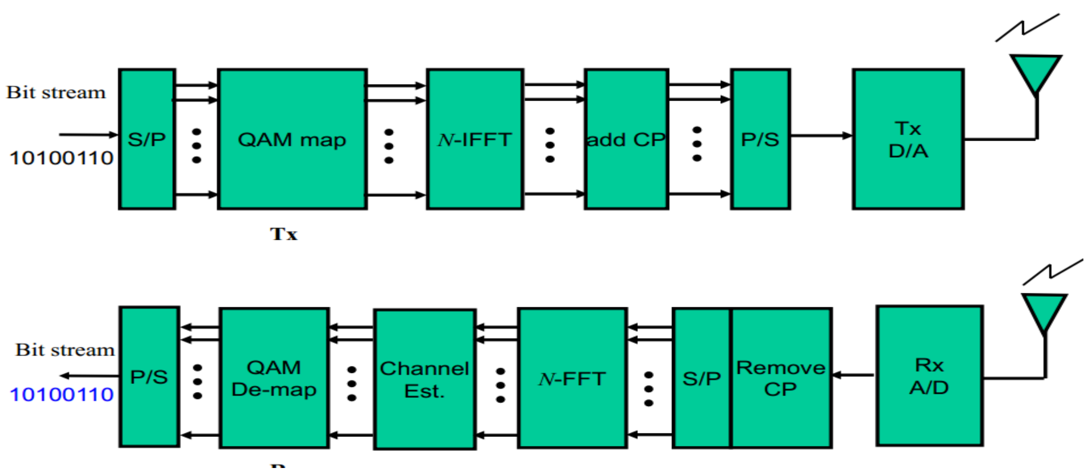

Figure 3.1:Block Diagram MIMO-OFDM

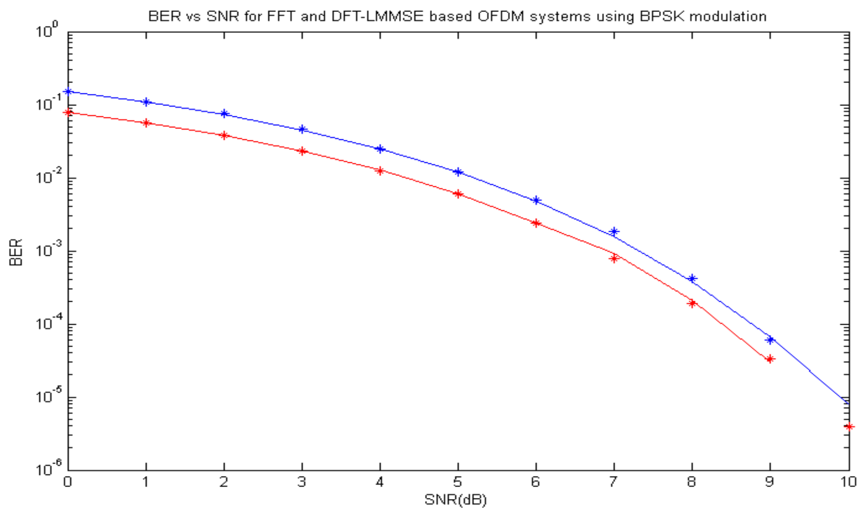

Figure 3.2: BER vs. SNR for FFT and LMMSE based OFDM using BPSK Modulation

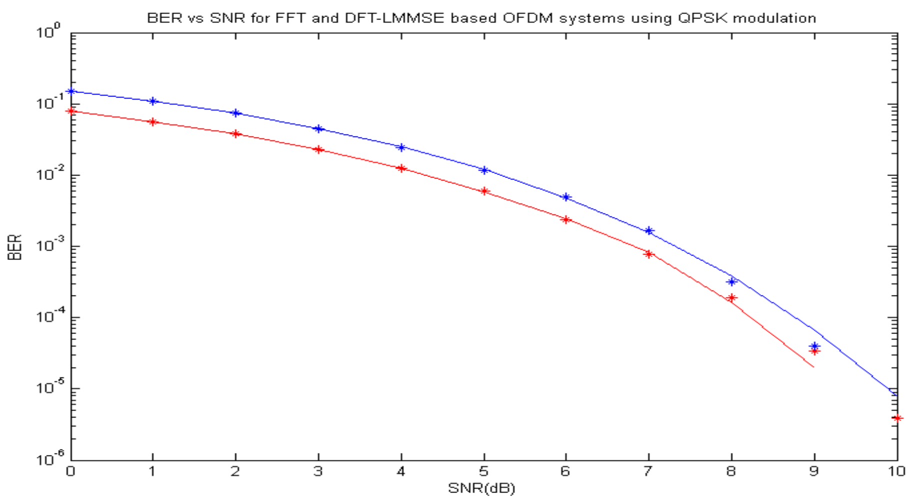

Figure 3.3 : BER vs. SNR for FFT and DFT-LMMSE based of OFDM systems using QPSK Modulation

Table 3.1: Gives a summary of the bit rates of different forms of QAM and PSK.

MODULATION	BITS PER SYMBOL	SYMBOL RATE
BPSK	1	1 x bit rate
QPSK	2	1/2 bit rate
8PSK	3	1/3 bit rate
16QAM	4	1/4 bit rate
32QAM	5	1/5 bit rate
64QAM	6	1/6 bit rate

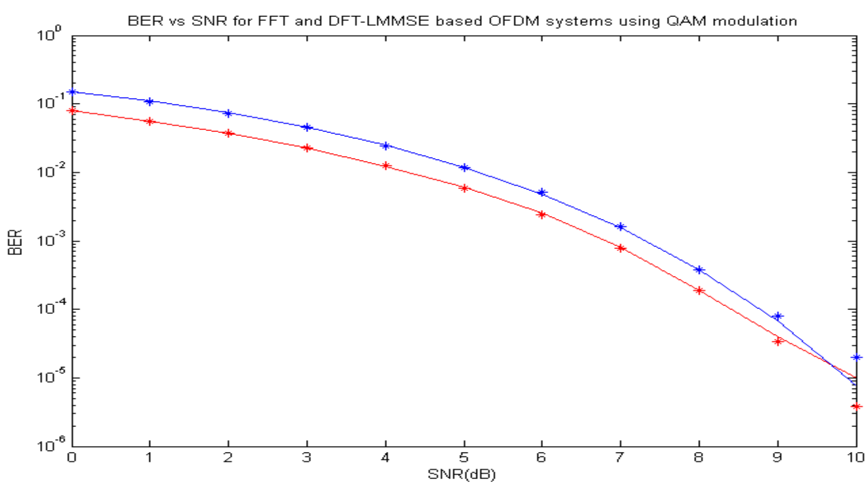

Figure 3.4: BER vs. SNR for FFT and DFT-LMMSE based OFDM systems using QAM modulation

4. SIMULATION RESULTS:

4.1 Bit error rate :

As the name implies, a bit error rate is defined as the rate at which errors occur in a transmission system. This can be directly translated into the number of errors that occur in a string of a stated number of bits. The definition of bit error rate can be translated into a simple formula:

$$\text{Bit Error Rate, BER} = \frac{\text{Number of errors}}{\text{Total number of bits sent}}$$

If the medium between the transmitter and receiver is good and the signal to noise ratio is high, then the bit error rate will be very small - possibly insignificant and having no noticeable effect on the overall system However if noise can be detected, then there is chance that the bit error rate will need to be considered.

The main reasons for the degradation of a data channel and the corresponding bit error 2rate, BER is noise and changes to the propagation path (where radio signal paths are used). Both effects have a random element to them, the noise following a Gaussian probability function while the propagation model follows a Rayleigh model. This means that analysis of the channel characteristics are normally undertaken using statistical analysis techniques.

For fibre optic systems, bit errors mainly result from imperfections in the components used to make the link. These include the optical driver, receiver, connectors and the fibre itself. Bit errors may also be introduced as a result of optical dispersion and attenuation that may be present. Also noise may be introduced in the optical receiver itself.

Typically these may be photodiodes and amplifiers which need to respond to very small changes and as a result there may be high noise levels present.

Another contributory factor for bit errors is any phase jitter that may be present in the system as this can alter the sampling of the data.

BER and Eb/No

Signal to noise ratios and Eb/No figures are parameters that are more associated with radio links and radio communications systems. In terms of this, the bit error rate, BER, can also be defined in terms of the probability of error or POE. The determine this, three other variables are used. They are the error function, erf, the energy in one bit, Eb, and the noise power spectral density (which is the noise power in a 1 Hz bandwidth), No.

It should be noted that each different type of modulation has its own value for the error function. This is because each type of modulation performs differently in the presence of noise. In particular, higher order modulation schemes (e.g. 64QAM, etc) that are able to carry higher data rates are not as robust in the presence of noise. Lower order modulation formats (e.g. BPSK, QPSK, etc.) offer lower data rates but are more robust.

The energy per bit, Eb, can be determined by dividing the carrier power by the bit rate and is a measure of energy with the dimensions of Joules. No is a power per Hertz and therefore this has the dimensions of power (joules per second) divided by seconds). Looking at the dimensions of the ratio Eb/No all the dimensions cancel out to give a dimensionless ratio. It is important to note that POE is proportional to Eb/No and is a form of signal to noise ratio.

Factors affecting bit error rate, BER

It can be seen from using Eb/No, that the bit error rate, BER can be affected by a number of factors. By manipulating the variables that can be controlled it is possible to optimise a system to provide the performance levels that are required. This is normally undertaken in the design stages of a data transmission system so that the performance parameters can be adjusted at the initial design concept stages.

- **Interference:** The interference levels present in a system are generally set by external factors and cannot be changed by the system design. However it is possible to set the bandwidth of the system. By reducing the bandwidth the level of interference can be reduced. However reducing the bandwidth limits the data throughput that can be achieved.

- **Increase transmitter power:** It is also possible to increase the power level of the system so that the power per bit is increased. This has to be balanced against factors including the interference levels to other users and the impact of increasing the power output on the size of the power amplifier and overall power consumption and battery life, etc.
- **Lower order modulation:** Lower order modulation schemes can be used, but this is at the expense of data throughput.
- **Reduce bandwidth:** Another approach that can be adopted to reduce the bit error rate is to reduce the bandwidth. Lower levels of noise will be received and therefore the signal to noise ratio will improve. Again this results in a reduction of the data throughput attainable.

It is necessary to balance all the available factors to achieve a satisfactory bit error rate. Normally it is not possible to achieve all the requirements and some trade-offs are required.

Table 4.1: BER Rate Comparison results

	MMSE	ZF	LMMSE-BPSK	LMMSE-QPSK	LMMSE-QAM
BER	0.1039	0.2106	0.0769	0.1538	0.615

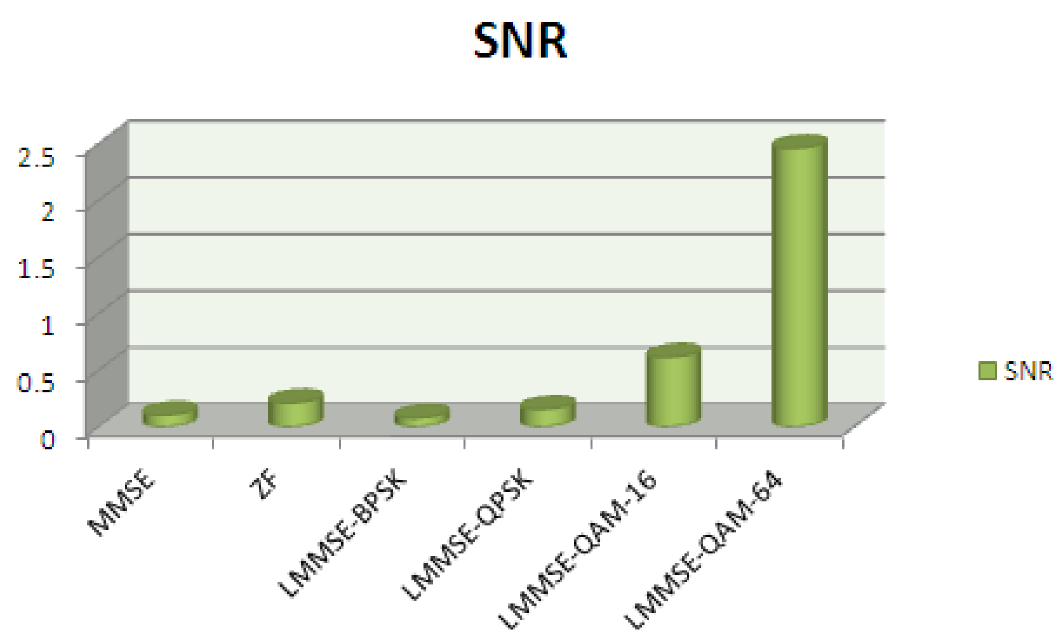

Figure 4.2: Comparison between MMSE; ZF; LMMSE for Different Modulation Process

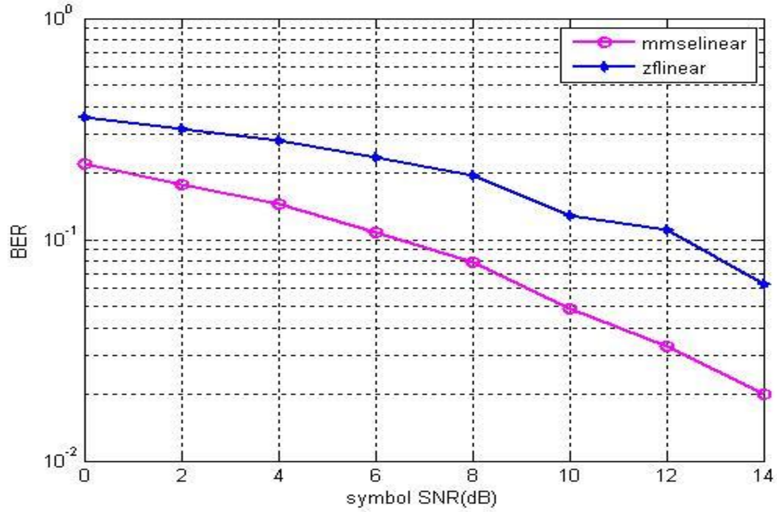

Figure 4.3: Comparison between MMSE and ZF

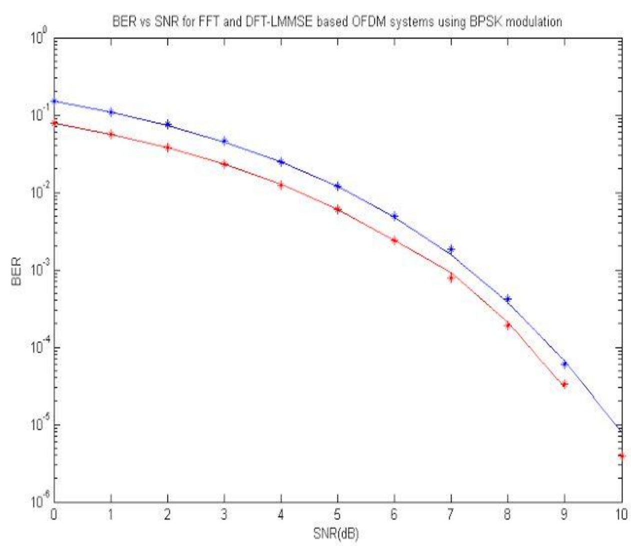

Figure 4.4: BER vs SNR for FFT and DFT-LMMSE based OFDM systems using BPSK modulation

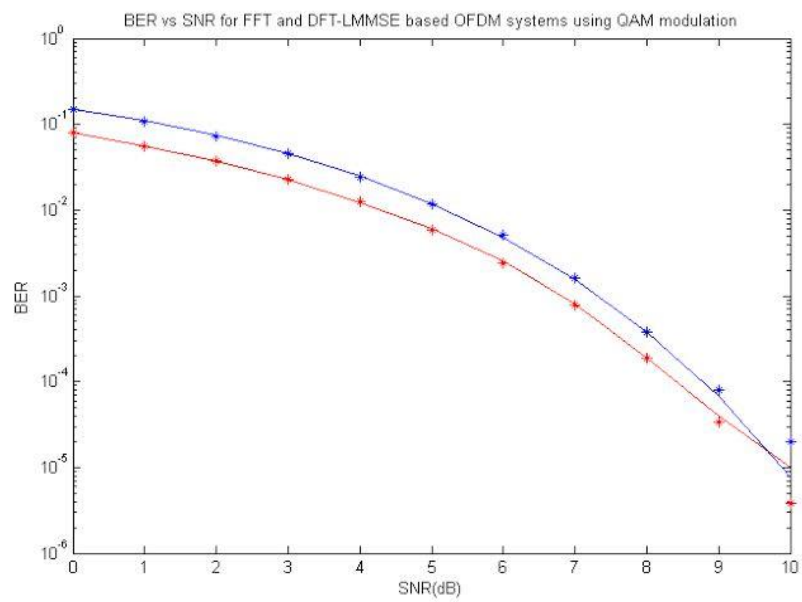

Figure 4.5: BER vs SNR for FFT and DFT-LMMSE based OFDM systems using QAM modulation

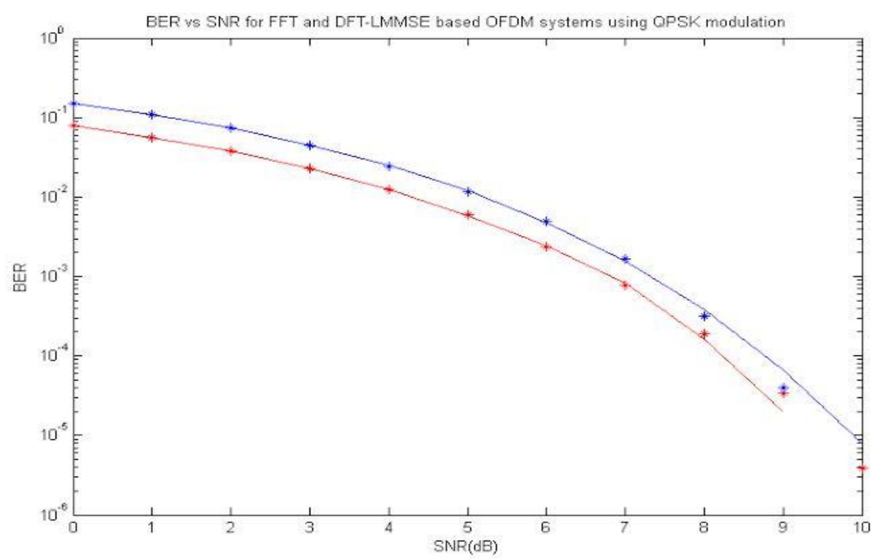

Figure 4.6: BER vs SNR for FFT and DFT-LMMSE based OFDM systems using QPSK modulation

5. CONCLUSION AND FUTURE SCOPE:

5.1 CONCLUSION:

BER performance of the FFT based OFDM systems can be found over AWGN and Rayleigh fading channel using different modulation schemes like BPSK, QPSK, and QAM. From the plots of the BER as a function of the Signal to Noise Ratio (SNR), it can be concluded that when the Signal to Noise Ratio (SNR) is very low and does not have any impact on the BER but if Signal to Noise Ratio (SNR) increased the BER is reduced.

5.2 FUTURE SCOPE:

Interference: The interference levels present in a system are generally set by external factors and cannot be changed by the system design.

Increase transmitter power*:* It is also possible to increase the power level of the system so that the power per bit is increased. This has to be balanced against factors including the interference levels to other users and the impact of increasing the power output on the size of the power amplifier and overall power consumption and battery life, etc.

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