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Ramanujan Fourier transform for Even signal

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Abstract: **In today's technology the power and time are the major parameters to be saved. Ramanujan transform will uses only integer values to compute transform for the given input sequence. The objective of this paper is to reduce the complexity of the algorithm and execution time. Ramanujan number of order of order 2 is more efficient than Ramanujan number of order 1.**

index terms : *Ramanujan numbers, Ramanujan sum and Even signals*

I INTRODUCTION

In 1918 the mathematician Srinivas Ramanujan was introduced the concept of Ramanujan sums for signal processing. The Ramanujan sums are orthogonal in nature and therefore they will give excellent energy conservation. RS operated only on integers hence can obtain reduced quantization error in implementation.

Discrete transform is well suited for periodic signal and not well suited for non-periodic signal such as low frequency signals.DFT is fails to obtain constructive features of a aperiodic sequences.RFT uses the prime numbers theory which reduces the number of samples.DFT uses the N^2Complex multiplications and N(N-1) complex additions and in FFT algorithm still it is reduced where as in RFT no multiplications only additions. The RFT can be computed with single adder in $O(N^2)$ addition times where as in parallel implementation it will be executed in O(N) addition times with O(N) number of adders.

II.RAMANUJAN NUMBERS

Ramanujan numbers were introduced to find DFT of the sequence without using any multiplication operations. Here we have discussed Ramanujan numbers of order-1 and order-2.The use of Ramanujan numbers of order-2 is more accurate than by using Ramanujan numbers of order-1.

The complex input sequence $\{y_k\}$ of length N is transformed to $\{Y_k\}$ by DFT.DFT is defined as

$$Yk = \sum_{n=0}^{(N-1)} y_n\, w_{nk}, \quad k = 0,1,2 \ldots (N-1) --(1)$$

where $y_n = p_{n1} + p_{n2}$, where p_{n1} and p_{n2} are real numbers. The Ramanujan number of order-1 is defined as

$$R_1(a) = \left[\frac{2\pi}{v_1(a)}\right], \qquad v_1(a) = 2^{-a} --(2)$$

The function [ ] which rounds off its argument to its nearest integer value.

Ramanujan numbers of order-2 are defined as bellow

$$R_{2j}(l,m) = \left[\frac{2\pi}{v_{2j}(l,m)}\right] \quad j = 1,2 --(3a)$$

$$v_{21}(l,m) = 2^{-l} + 2^{-m} \qquad m > l \geq 0 \quad --- (3b)$$

$$v_{22}(l,m) = 2^{-l} - 2^{-m} \quad (m-1) > l \geq 0 --(3c)$$

where l and m are integers.

$$\hat{\pi} = \frac{1}{2} R\, V --(4a)$$

where ε_π is relative error in π. For higher values of N , the absolute value of ε_π is smaller[1],[2].

Table 1 $R_{21}(l,m)$ type Ramanujan numbers and their characteristics

l , m	*$R_{21}(l , m)$*	$\hat{\pi}$	ε_π
0 , 2	5	3.125	-5.2816×10^{-3}
0 , 4	6	3.1875	1.4613×10^{-2}
1 , 2	8	3.0	-4.5070×10^{-2}

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2 ,3	17	3.1875	1.4613x10^{-2}
2 , 4	20	3.125	-5.2816x10^{-3}
3 , 4	34	3.1875	1.4613x10^{-2}
3 , 5	40	3.125	-5.2816x10^{-3}
4 , 5	67	3.140625	-3.0801x10^{-4}

Table 2 R_{22}(l,m) type Ramanujan numbers and their characteristics

l , m	*R_{22}(l , m)*	$\hat{\pi}$	ε_π
0 , 3	7	3.0625	-2.5176x10^{-2}
0 , 4	7	3.28125	4.4454x10^{-2}
1 , 6	13	3.148438	2.1788x10^{-3}
2 ,5	29	3.171875	9.6392x10^{-3}
2 , 6	27	3.164063	7.1524x10^{-3}
3 , 5	67	3.140625	-3.0801x10^{-4}
3 , 6	57	3.117188	-7.7684x10^{-3}
4 , 6	134	3.140625	-3.0801x10^{-4}

III.RAMANUJAN SUM

The Trigonometrical sums is defined as

$$C_s(n) = \sum_{\lambda} \cos\frac{2\pi\lambda n}{s} ----------- (1)$$

where λ is prime to s and it is not greater than s. it is plain that

$$C_s(n) = \sum \alpha^n ----------------(2)$$

is the primitive root of the equation

$$x^s - 1 = 0 -----------(3)$$

The objective is to obtain expression for a variety of well known arithmetical functions of n in the form of series

$$\sum_s a_s\, C_s(n) ---------- (4)$$

The typical formula is

$$\sigma(n) = \frac{\pi^2 n}{6}\left\{\frac{C_1(n)}{1^2} + \frac{C_2(n)}{2^2} + \frac{C_3(n)}{3^2} + \ldots.\right\} \quad -(5)$$

where σ(n) is the sum of the divisors of n.we have two distinct methods for proof of this . The formulae is involving only finite algebraic and simple general theorems concerning infinite series. These are however some which are of a "deeper" character and can only be analytic functions.

A typical formula of this class is

$$C_1(n) + \frac{1}{2}C_2(n) + \frac{1}{3}\ C_3(n) + \ldots\ldots = 0 \ --(6)$$

The above formulae which depends upon prime number theorem of Hadamard and de la Vallee Poussin

Let F(u,v) be any function of u and v

$$D(n) .= \sum_{\delta} F(\delta, \delta') --------(7)$$

where is a divisor of n and For instance

D(1) = F(1,1) D(2) = F(1,2) +F(2,1)
D(3) = F(1,3) + F(3,1) D(4) = F(1,4) + F(2,2) + F(4,1)
D(5) = F(1,5) = F(5,1) D(6) = F(1,6) + F(2,3) + F(3,2) + F(6,1) . . . .

D(n) can be also expressed as

$$D(n) = \sum_{\delta} F(\delta', \delta) --------(8)$$

suppose now that

$$\eta_s = \sum_{0}^{s-1} \cos\left(\frac{2\pi v n}{s}\right) -------- (9)$$

if $\eta s = s$ it means that s is divisor of n otherwise 0. Then

$$D(n) = \sum_{1}^{t} \frac{1}{v}\ \eta_v(n) F\left(v, \frac{n}{v}\right) ------(10)$$

where t is any number not less than n. Now let

$$Cs(n) = \sum_{\lambda} \cos\left(\frac{2\pi\lambda n}{s}\right) ------(11)$$

where is prime to s and it will not exceed s.

For example

$$C_1(n) = 1; C_2(n) = \cos n\pi \ ;$$

$$C_3(n) = 2\cos\frac{2}{3}n\pi$$

$$C_4(n) = 2\cos\left(\frac{1}{2}n\pi\right) \ ;$$

$$C_5(n) = 2\cos\left(\frac{2}{5}n\pi\right) + 2\cos\left(\frac{4}{5}n\pi\right)$$

$$C_6(n) = 2\cos\left(\frac{1}{3}n\pi\right);$$

$$C_7(n) = 2\cos\left(\frac{2}{7}n\pi\right) + 2\cos\left(\frac{4}{7}n\pi\right) + 2\cos\left(\frac{6}{7}n\pi\right)$$

$$C_8(n) = 2\cos\left(\frac{1}{4}n\pi\right) + 2\cos\left(\frac{3}{4}n\pi\right);$$
$$C_9(n) = 2\cos\left(\frac{2}{9}n\pi\right) + 2\cos\left(\frac{4}{5}n\pi\right) + 2\cos\left(\frac{8}{9}n\pi\right)$$
$$C_{10}(n) = 2\cos\left(\frac{1}{5}n\pi\right) + 2\cos\left(\frac{3}{5}n\pi\right)$$

A. Euler's totient function ϕ(n)

The Euler's totient function is defined as the number of positive integers which are less than (or) equal to integers and are prime to the integers. some of the values are as bellow

N	ϕ(n)
1	1
2	1
3	2
4	2
5	4
6	2
7	6
8	4
9	6
10	4
11	10
12	4
13	12
14	6
15	8
16	17
17	16
18	6
19	18
20	8

B. Mobius function($\mu(n)$)

The $\mu(n)$ is 0 if n contains a square factor, 1 if n = 1 and $(-1)^k$ if n is a product of k distinct prime numbers.

$$\mu(n) = \begin{cases} 0 & \text{if } n \text{ contains a square factor} \\ 1 & \text{if } n = 1 \\ (-1)^k & \text{if } n \text{ is the product of } k \text{ distinct prim} \end{cases}$$

N	μ(n)
1	1
2	-1
3	-1
4	0
5	-1
6	1
7	-1
8	0
9	0
10	1
11	-1
12	0
13	-1
14	1
15	1
16	0
17	-1
18	0
19	-1
20	0

The q^{th} Ramanujan sum($q \geq 1$) is a sequence defined as

$$C_q(n) = \sum_{\substack{k=1 \\ (k,q)=1}}^{q} e^{j2\pi kn/q} = \sum_{\substack{k=1 \\ (k,q)=1}}^{q} W_q^{-kn} \quad --(12)$$

the summation runs over only the values of k that are co prime to the q.At n=0, sum has precisely ϕ(q).

$$C_q(n) = \emptyset(q) \quad ----(13)$$

from (12) we can also show that the DFT of i Cq(n) is

$$C_q[k] = \begin{cases} q & if(k,q) = 1 \\ 0 & Otherwise \end{cases} \quad --- (14)$$

if (k,q) = 1, then it follows that (q - n) = 1 as well. since $W_q^k = (W_q^{-k})^*$ this shows the summation is real valued thus it can got the form as

$$Cq(n) = \sum_{\substack{k=1 \\ (k,q)=1}}^{q} \cos\left(\frac{2\pi kn}{q}\right) ------(15)$$

Ramanujan sum can be written in following forms

$$Cq(n) = \sum_{\substack{k=1 \\ (k,q)=1}}^{q} W_q^{kn} = \sum_{\substack{k=1 \\ (k,q)=1}}^{q} W_q^{-kn} = \sum_{\substack{k=1 \\ (k,q)=1}}^{q} \cos\left(\frac{2\pi kn}{q}\right) -(16)$$

Equation (5) it is said that Cq(n) is real sequence and furthermore it is symmetric, i.e Cq(n) = Cq(-n) and periodic

$$Cq(n) = Cq(q - n) \quad ---(17)$$

Thus any Ramanujan sum is Cq(n) is real, symmetric and periodic sequence in n. Here few Ramanujan sequences are shown in one period $0 \le n \le q - 1$

C1(n) = {1 }

C2(n) = {1 , -1}

C3(n) = {2 , -1, -1}

C4(n) = {2, 0, -2, 0}

C5(n) = {4, -1, -1, -1, -1}

C6(n) = {2, 1, -1, -2, -1, 1}

C7(n) = {6, -1, -1, -1, -1, -1, -1}

C8(n) = {4, 0, 0, 0, -4, 0, 0, 0}

C9(n) = {6, 0, 0, -3, 0, 0, -3, 0, 0}

C10(n) = {4, 1, -1, 1, -1, -4, -1, 1, -1, 1}

C11(n) = { 10, -1, -1, -1, -1, -1, -1, -1, -1, -1}

C12(n) = {4, 0, 2, 0, -2, 0, -4, 0, -2, 0, 2, 0}

C13(n) = {12, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1}

C14(n) = {1, -1, 1, -1, 1, -1, -6, -1, 1,-1, -1, 1, 1, 6}

C15(n) = {1, 1, -2, 1, -4, -2, 1, 1, -2, -4, 1, -2, 1, 1, 8}

C16(n) = {0, 0, 0, 0, 0, 0, 0, -8, 0, 0, 0, 0, 0, 0, 0, 8}

C17(n) = {-1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, 16}

C18(n) = {0, 0, 3, 0, 0, -3, 0, 0, -6, 0, 0, -3, 0, 0, 3, 0, 0, 6}

C19(n) = {-1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, 18}

C20(n) = {0, 2, 0, -2, 0, 2, 0, -2, 0, -8, 0, -2, 0, 2, 0, -2, 0, 2, 0, 8}

we notice that Ramanujan sum $C_q(n)$ is always integer valued.

IV. RAMANUJAN FOURIER TRANSFORM FOR EVEN SIGNALS

In this we have analyze the class of discrete time periodic signals, which will be called as even signals. The signals are defined with fixed positive integer r for all integer values of index n.Let gcd(a, b) denotes the greatest common divisor of integers a and b.The signal $x_r(n)$ is even signal if

$$x_r(n) = x_r(\gcd(n,r)) \qquad \forall n - - - (18)$$

From definition it shows that even signal(mod r) is periodic with period r. To show even(mod r) and periodicity we have used the algorithm called as Euclidian algorithm .The Euclidian algorithm will help in computation of greatest common divisor.

The algorithm can shown bellow

$$\gcd(0,b) = b$$

$$\gcd(a,b) = \gcd(a \bmod b, b), \quad b > 0 \quad -(19)$$

For integer k , we can use Eqn(1) and Eqn(2) , it has got the form

$$x_r(n + kr) = x_r(\gcd(n + kr, r))$$
$$= x_r(\gcd((n + kr) \bmod r , r)$$
$$= x_r(\gcd(n \bmod r , r)) = x_r(n) \text{-}(20)$$

$x_r(n)$ possesses an even symmetry i.e $x_r(n) = x_r(r - n)$. Proof for this is shown bellow

$$x_r(n) = x_r(\gcd(n , r))$$
$$= x_r(\gcd(-n , r)) = x_r(\gcd((-n) \bmod r , r))$$
$$= x_r(\gcd((r - n) \bmod r , r)) = x_r(\gcd(r-n , r))$$
$$= x_r(r - n) \text{ -------------}(21)$$

Ramanujan sum is an integer sum of the form

$$Cd(n) = \frac{\phi(d)\mu(m)}{\phi(m)}$$

$$where \quad m = \frac{d}{\gcd(n,d)} \text{----}(22)$$

Ramanujan sums are even functions(mod d) i.e.

$$Cd(n) = Cd(\gcd(n,d)) - - - (23)$$

The RFT Equation is given by

$$X_r(n) = x_r(0) + \sum_{\substack{d|r \\ d>1}}^{r} x_r\left(\frac{r}{d}\right) Cd(n) - - - (24)$$

where $x_r(n)$ is the even signal(mod r) and $X_r(n)$ is RFT sequence.

IRFT Equation is written as

$$x_r(n) = \frac{1}{r}\left(X_r(o) + \sum_{\substack{D|r \\ D>1}} X_r\left(\frac{r}{D}\right) C_D(n) \right) - - - -(25)$$

The Equation (1) gives the Inverse Ramanujan Fourier transform.

RESULTS

The table 1 and 2 shows RFT of the even sequence as shown in table 1 and table 2.

Table 1 RFT of N=4 sequence

Sequence 1 of length 4				
Input Sequence x(n)	1	1	1	1
DFT	4	0	0	0
RFT	4	0	0	0
Sequence 2 of length 4				
Input Sequence x(n)	1	2	1	2
DFT	6	0	-2	0
RFT	6	0	-2	0

The table 1 shows the RFT of sequence of length N =4 and Table 2 gives RFT of sequence of length N=8

Table 2 RFT of N=8 sequence

Sequence 3 of length 8								
Input Sequence x(n)	1	1	1	1	0	1	1	1
DFT	7	1	-1	1	-1	1	-1	1
RFT	7	1	-1	1	-1	1	-1	1

Table 3 Comparison of DFT and RFT

No. of points $N=2^M$	No. of stages $M=\log_2 N$	Number of complex multiplications			Number of complex additions			Speed improvement factor from DFT to FFT $N^2/((N/2)\log_2 N)$	Speed improvement factor from FFT to RFT
		DFT N^2	FFT $(N/2)\log_2 N$	RFT	DFT $N^2 - N$	FFT	RFT		
2	1	4	1		2	2		4	
4	2	16	4		12	8		4	
8	3	64	12		56	24		5.33	
16	4	256	32		240	64		8	
32	5	1024	80		992	160		12.8	
64	6	4096	192		4032	384		21.33	
128	7	16,384	448		16256	896		36.57	

CONCLUSION

By using RFT , Number of complex multipliers, complex additions and execution time can be reduced. This will inturn increase the speed of the DSP processor.

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