

Enhancing Comb filter by using optimization technique

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ABSTRACT

The Comb filters are a class of optimized FIR filters also used for the purpose of decimation. The comb filters have a high droop in pass-band which is responsible for falsification of the incoming signal and also introduces stop-band attenuation in impulse response of filters. Various techniques have been designed for reducing pass-band droop as well as increasing attenuation in stop-band. The filter requires only additions and subtractions for their implementation and requires simple implementations. The design structure of cascaded integrator comb filters requires two simple basic building blocks (integrators and differentiator). The same filter can be used for higher decimation factor with simple change in circuit design. work is done to reduce droop in pass-band and to increase attenuation in stop-band by designing compensators or by designing an external filter for stop-band improvement. Designing of compensators or external filters improves the response of comb filters but at the expense of increased circuit complexity. In this paper we propose to use the work for comb filter by using an optimization technique of simplex algorithm which provided an improvement in stop-band attenuation.

General Terms

Interpolation, FIR filters, IIR filters, decimation, phase response, sharpening techniques.

Keywords

Comb filters, recursive structure of comb filter, frequency response of comb filters, techniques.

1. INTRODUCTION

1.1 Comb filters

Comb filters were introduced by Eugene Hogenauer, more than two decades ago. The Comb filters are a class of optimized FIR filters also used for the purpose of decimation. The coefficients of FIR filters are optimized or set to unity. The comb filters have a high droop in pass-band which is responsible for distortion of the incoming signal and also introduces stop-band attenuation in impulse response of filters. Various techniques have been designed for reducing pass-band droop as well as increasing attenuation in stop-band. Decimation of oversampled signal is done in a number of stages by cascading no. of decimation filters. First stage includes generally a comb filter because of their simplicity in design while decimation filters used in successive stages can be FIR filters [1]. The frequency-magnitude response of Comb filters approximates to that of sinc filters ($\sin(X)/X$).

The transfer function of non- recursive filter of length N is given by:

$$H(z) = \frac{\sum_{i=0}^N z^{-i}}{M} \quad (1)$$

The transfer function of recursive comb filters (Cascaded integrator Comb filter) is given by:

$$H(z) = \left[\frac{1}{M} \left(\frac{1-z^{-M}}{1-z^{-1}} \right) \right]^K \quad (2)$$

The transfer function of non-recursive structure of comb filter is given by:

$$H(z) = [1/M]^K [1 + z^{-1} + z^{-2} + z^{-3} + \dots + z^{-(M-1)}]^K \quad (3)$$

Where, K is the order of filter

M is the factor for decimation

Fig. 1.1 below shows a first order CIC filter with cascaded integrator and differentiator separated by down sampler or down sampler at the output stage. Both figures are equivalent because of noble identities

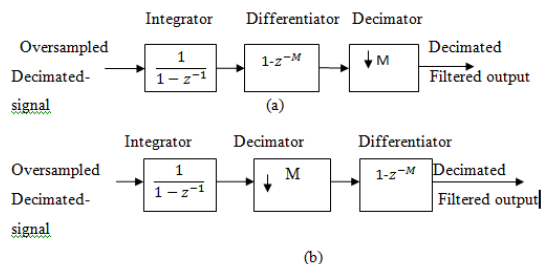


Fig.1.1 Noble identities for CIC Systems

1.2 Structure of Comb Filters

There are basically two structures of Comb filters used in hardware implementation.

1.2.1 Recursive structure of comb filters

The recursive structure of comb filter is a cascade of integrator operating at a higher sampling rate, separated by a down sampler with that of a differentiator. The advantage of recursive algorithm is that the differentiator operates at a lower rate of decimation factor as compared to that of integrator. The differentiator is known as a comb filter. The disadvantage of recursive architecture is that it requires higher power consumption and is unstable.

1.2.2 Non-recursive structure of comb filters

The Non-recursive architecture of comb filter is used to reduce the power consumption and to reduce circuit complexity. The sampling rate is reduced easily as compare to that of recursive architecture and hence helps to reduce power consumption and circuit complexity.

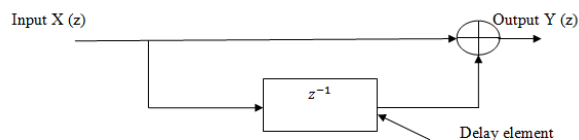


Fig 1.2 Basic comb filter

1.2.1 Frequency response of Comb filters

The frequency response of any LTI system is given by its transfer function. The frequency response of any LTI system is defined as the ratio of output signal divided by input signal. Comb filter's frequency response shows various characteristics as below [2]

- Comb filters shows the existence of periodically spaced nulls at a frequency interval of $\omega_k = 2\pi k/M$ ($k=1, 2, \dots, M$) which makes an appearance of comb and hence filters are known as comb filters.

2. The pass-band is defined by the relation as: $\omega_p = \frac{\pi}{M}$, where M is decimation factor. If a second stage of decimation follows a Comb filter the pass-band reduces to $\omega_p = \frac{\pi}{vM}$, where v is second stage of decimation filter.

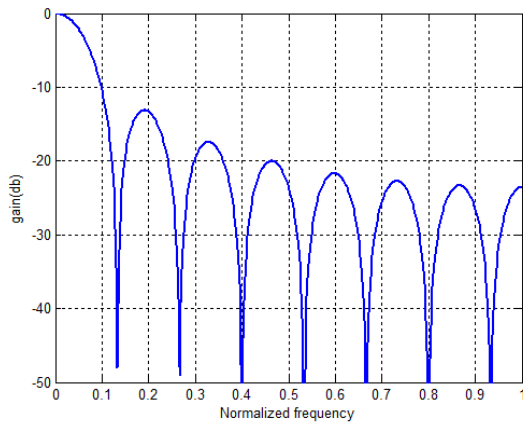


Fig. 1.3 Frequency response of single stage Comb filter for Decimation factor=15

1.2.2 Applications of Comb filter

1. CIC filters are used for removing aliasing due to decimation and interpolation during the process of sample rate conversion.
2. When two loudspeakers operating at similar frequency are placed at different distance from listener, to reduce the effect of different voice form different location comb filters are used.

For PAL and NTSC decoders, 2D and 3D comb filters are used to reduce the dot crawl effect in television systems.

1.2 Why Comb Filter

Comb filters because of low complexity filters are used for the operations of filtering the signal where sample rate conversion is required. The process where sample-rate reduction occurs comb filters are known as decimator while the processes requiring increase in sample-rate, comb filters are known as interpolator [3].

1.3.1 Interpolation

Interpolation does the reverse operation of decimation. The process used to convert a low sample rate signal to higher sample rate is known as interpolation. The device used for the operation of increasing sample rate is named as interpolator, expander, and up-sampler.

1.3.2 Decimation

The process used to convert an oversampled signal to lower sample rate is known as decimation. The decimator is also known as down-sampler, sampling rate compressor or also simply a compressor. The process of lowering the sampling rate means selecting only selected samples and neglecting the samples in between.

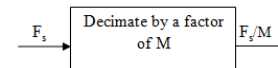


Fig. 1.4 Decimator

where, F_s is the input sample rate

M is the decimation factor (ratio of input sample rate to the output sample rate)

F_s/M is the output sample rate

1.3.1.1 Types of Decimation Filter

There are various types of filters used for the operation of decimation. The advantages and disadvantages of various decimation filters are explained below. Generally there are three types of filters used for the operation of decimation as explained below:

a) FIR filters

FIR filters can be used for the process of decimation and are also known as non-recursive filters. The transfer function of FIR filter can be expressed by equation (4). The output of FIR filter depends upon past and present values of input. The disadvantage of using FIR filter for the purpose of decimation is because of its complexity on circuit design. The FIR filters occupy a larger area on chip because of need of multipliers to store the coefficients for the filter design.

b) IIR filters:

The IIR filters can also be used for the purpose of decimation and known as recursive filters. The equation (1) represented the transfer function of IIR filters. The output of IIR filter depends upon past and present values of both input and output values. IIR filter require less memory and calculations as compare to the similar FIR filter. The FIR filters are more costly as compare to that of FIR filter. The IIR filters are more complex to design, unstable and are non-linear phase response and hence not suitable for the operation of decimation.

c) Comb filters

Comb filters were invented by Eugene Hogenauer in 1980. The Comb filters can be stated as an optimized FIR filters used for the purpose of decimation. The coefficients of FIR filters are optimized or set to unity. The comb filters introduces distortion of the incoming signal due to high pass-band droop and low stop-band attenuation in its frequency response.

1.3.2.2 Comb filters as Decimation Filters

Generally a low-pass filter is followed by the process of decimation and hence it is known as decimation filter. The relation between input-output for decimation filters in time-domain can be expressed as:

$$Y(n) = \sum_{m=-\infty}^{\infty} x(m)h(nM - m) \quad \text{M decimation factor} \quad (5)$$

Noble identities: Decimation includes two processes first lowering the sample rate and filters the decimated signal to avoid aliasing created due to decimation. Using noble Identities the operations of linear filtering and decimation can be interchanged to accomplish the desire results.

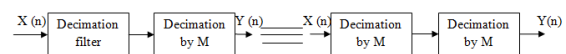


Fig 1.5 Equivalent structures using Noble Identities

Advantages of using Comb filters as decimation filter

1. The Comb filters a class of optimized FIR filters are used for the purpose of decimation filters because of multiplier less design in hardware.
2. The coefficients of comb filters are set to unity to eliminate the need of multipliers from the design.
3. The filter requires only additions and subtractions for their implementation and requires simple implementations.

1.4 Issues in design of comb filters

1.4.1 Issues

1. The Comb filter requires simple implementation, besides of this advantage Comb filter introduces droop in pass-band and low attenuation in stop-band.
2. The comb filter causes distortion in the signal in pass-band because of droop introduced by the filters in pass-band [5].

1.4.2 Techniques

There are various techniques used to improve droop in pass-band and to increase attenuation in stop-band of Comb filters. Generally four techniques are used for improving attenuation and to reduce droop in pass-band as explained below [3]

Multistage decimation filters

A number of stages can be used for decimation. The first stage is of Comb filters or CIC filters while different techniques are used to design further stages of decimation. Filters for decimation in further stages can be design using FIR filters or IIR filters or by somehow modifying the Comb filters to improve frequency response of Comb filters.

1. Design of droop compensator

The droop of Comb filter introduced in pass-band can be reduced by designing another filter known as compensator. The compensator can be design using FIR filter, using various other techniques mentioned in literature review.

2. Sharpening Technique

The sharpening technique was introduced by Kaiser and Hamming in 1977. The technique is suitable for improvement in non-recursive filters. The Sharpening technique improves attenuation in stop-band as well as improves droop in pass-band without increasing complexity of the circuit design. The sharpening technique improves both stop-band and pass-band by making multiple use of identical filter.

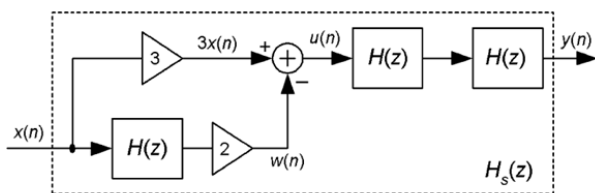


Fig.1.6 Sharpening technique introduced by Kaiser and Hamming [7]

2. RELATED WORK

The research work performed in this field by different researchers is presented as follows:

Kalpna et al. (17) dealt with the design of sharpened maximally flat Cascaded integrator Comb filters. The technique is used to improve the response of Cascaded integrator Comb filter. The technique improves droop in pass-band as well as attenuation in stop-band. To improve droop in pass-band in case of wideband region fourth order linear phase filters are used. To calculate the sharpening polynomial random search algorithm is used. The results of second order,

third order and fourth order sharpened compensated Cascaded integrator comb filters are shown. The results illustrated that designed second order, third order and fourth order filter provided better attenuation as well as better gain. The results for parameters of decimation factor equal to 32 for 5 stages of comb filter are shown.

Dolecek et al., (13) presented a wideband compensator of second order by using simple trigonometric identities. The parameter B for the design of compensation filter depends upon the no. of cascaded stages K and can be implemented by using only addition and subtraction. The designed filter uses only a maximum of four addition and subtraction elements and can be applied to any value of decimation factor. The droop value of δ is chosen to be 0.4dB so that better compensation is obtained. The designed parameter depends upon B and values can be evaluated using MATLAB. To obtain multiplier less compensator B is expressed as sum of powers of two. The value is rounded off by using value $r=2^{-2}$. The rounded value depends upon three parameters the number of cascaded filter, droop in pass-band and order of filter. The parameters used for the compensator design are $K=3$ & $K=4$ for any value of decimation factor. The number of adders for compensators is three for $K=1, 2, 4$ and for $K=3$ & 5 requires a maximum of 4 adders. The paper provided a better compensation as compared to that of compensators of similar complexity. The more complex droop compensators provided better compensation by increasing number of multipliers and increased number of adders.

Jovanovic et al., (2) presented a multiplier lacking decimation filters and improve frequency characteristics by adding more attenuation around so called folding bands as well as minimize droop in pass-band by using an extra block of droop compensator. The designed decimation filter is applicable for down sampling an oversampled signal at the output of sigma delta modulator. The proposed filter is reconfigurable, less complex and can be implemented by using only addition and subtraction. Non-recursive architecture is used to design proposed filters. The proposed decimation filters are able to better the response of classical comb filters while other parameters are kept retained. The proposed filter decreases droop in pass-band by introducing droop compensator block and attenuation around the folding bands is increased. The introduced filter is also applicable for multistage applications. The characteristics of designed filter make it suitable for downsampling the output of $\Sigma\Delta$ modulators. A comparison of proposed decimation filter introducing a droop of maximum of 0.9dB is made with that of 5-th order comb filter which introduces a maximum droop of 5 dB depending upon different parameters. Non-recursive architecture is implemented using poly phase decomposition. Proposed filter comparison with classical comb filters with design parameters $M=16$, $K=5$ and $\nu=2$. The proposed filter introduces a droop of 0.9dB while comb filter introduces a droop of 4.5 dB. The designed filter is very less in complexity because of no requirement in its implementation. The target of proposed decimation filter is to improve response while keeping in mind the most important features of low complexity, multiplier less design. The droop compensator design is independent of decimation factor M and depends upon cascaded no of filters. To improve stop-band attenuation designed filter is multiplier less and can be implemented by using only two additions at a lower rate.

Dolecek et al., (7) explained the design of two stage comb decimation filter for improvement in magnitude response of comb filter. The designed filter is multiplier less and designed using frequency sampling and IFIR methods. The coefficients

of filter designed using frequency sampling method are then rounded off to nearest integer. To increase further improvement in droop pass-band is expanded by changing some design parameters. The designed filter is having power to reduce droop in the pass-band and to increase attenuation in folding bands. The filters can be sampled down to low sample rate by using simple multirate identity. The designed filter doesn't depend upon decimation factor M and depends only on the number of cascaded comb filters. The existing techniques for the design of multiplier less comb filters needs to be designed again for different decimation factor while the designed technique used in the paper doesn't require the redesign of comb filter as it doesn't depend upon decimation factor. The maximum number of adders used in the design technique is 16. The proposed decimation comb filter introduces a 0.25dB of droop in pass-band and increase in attenuation in stop-band. The attenuation around folding bands can be increased further by cascading the comb filter with decimation factor $M=M/2$ with that of designed decimation comb filter. The proposed method is limited to only even decimation factor. The designed decimation droop compensated comb filter is applicable only for even values of decimation.

3. PROPOSED WORK

3.1 Problem Formulation

This dissertation uses the Simplex Algorithm optimization technique to improve attenuation in stop-band. The technique is based on designing linear equations on the basis of transfer function of comb filter. The Simplex algorithm is applied to solve linear equations to get a feasible point. A multiplier less filter based on the decimation factor is designed to be connected in cascade with the comb filter. The coefficient of cascaded designed filter with comb filter is multiplied with the feasible point obtained using simplex algorithm of optimization in order to improve attenuation around the folding bands. The simulation results are obtained in the dissertation using 4 different parameters such as decimation factor, number of stages of comb filter, number of stages of multiplier less filter and parameter for wideband or narrowband design. The simulation is done on decimation factor 15, 11 and 27 for stages 6, 5 and 3 respectively. The various different stages of designed multiplier less filters are used for different decimation factors. The dissertation uses 3, 4 and 3 stages of designed multiplier less filter according to decimation factor of 15, 11 and 27 respectively. The parameter used for narrowband and wideband design depends upon the value of second stage decimation factor. The values of second stage decimation factor are chosen to be 4, 2 and 4 respectively. The results obtained by using simplex algorithm optimization technique on comb filter are compared with response of different stages and decimation factor of comb filter are shown in the dissertation. The results obtained are approximately 10 dB or more higher than that of comb filter around the folding bands while an improvement of approximately 30dB higher for side-lobes.

3.2 Proposed Work

This dissertation uses the work for comb filter by using an optimization technique of simplex algorithm which provided an improvement in stop-band attenuation. The equations are designed in order to obtain a feasible solution. The technique provides better attenuation at the points where worst case aliasing occurs or simply named as aliasing or imaging bands also provides better stop-band attenuation at the side lobes. The technique provided better stop-band attenuation in

folding bands and around don't care bands as compared to comb filter.

This dissertation performs following mentioned tasks to improve stop-band attenuation of comb filters:

1. Study of Comb Filters
2. Design of multiplier less filter depending upon decimation factor.
3. Implement Comb filter by using optimization technique of simplex algorithm.
4. Multiply coefficients obtained in step 2 and step 3.
5. Compare the results with original values of Comb filters

4. RESULTS AND ANALYSIS

Result based on the specifications performed in MATLAB (R2012b). MATLAB is a tool used for research, analysis and development. There are five windows defined in MATLAB which performs different functions.

To compare the performance of Comb filter and Optimized Comb filter (proposed method), we are considering the performance specifications such as: $M=27$, $K=3$, $L=3$ and $v=4$ in third design example. This design example shows an improvement in attenuation around the folding bands for narrowband design of comb filter. Various plots of results such as overall response, folding band zoom, phase response and pole-zero plots are demonstrated in the chapter. Overall response, folding band zoom and phase plots are used to show a comparison of designed approach with that of comb filter while pole zero plot of designed approach illustrates stability according to the location of poles and zeros.

Steps of Design Process:

1. Connect three stages ($K=3$) of Comb filter using Convolution in Cascade with decimation factor $M=27$.
2. Convolution of resulting coefficients of three stage Comb filter connected in Cascade with designed multiplierless filter depending upon decimation factor.
3. Solve linear equations mentioned in Implementation using Simplex Algorithm.
4. Multiply Coefficients obtained in step 2 with feasible points obtained in step 3.
5. Plot response of Optimized coefficients obtained in step 4 inset with coefficients obtained in step 1 of Comb filter.

The normalized frequency range of pass-band is defined as $\omega_p = \frac{\pi}{vM} = 0.0092\pi$. The folding band is defined as $\Omega = [2\pi k/M - \omega_p; 2\pi k/M + \omega_p]$ where $k=1$ for first folding band. Thus first folding band is defined in the range of $[0.06, 0.1]$. Since $\omega = 2\pi f$, where x-axis represented the normalized frequency $\frac{\omega}{\pi}$ and y-axis represent the attenuation values.

Frequency Response of Comb filter and Optimized Comb filter:

Fig 5.1 shows a comparison of frequency response between Comb filter and Optimized Comb filter.

The response of Comb filter is demonstrated with blue line while Optimized Comb filters response is demonstrated with red line. A key observation is that the response of Optimized Comb filter lies below the response of Comb filter in case of side-lobes thus yields a higher signal rejection around the side-lobes. The fig 5.1 illustrates an improvement in attenuation at all side-lobes of comb filter. The first side-lobe occurs at frequency approximately equal to 0.1π . Thus the fig.5.1 demonstrates that attenuation at first side-lobe of Comb filter is 39.71dB while for Optimized Comb filter is

65.66 dB and hence has been increased by approximately 26dB with small ripples in pass-band 0.7028dB.

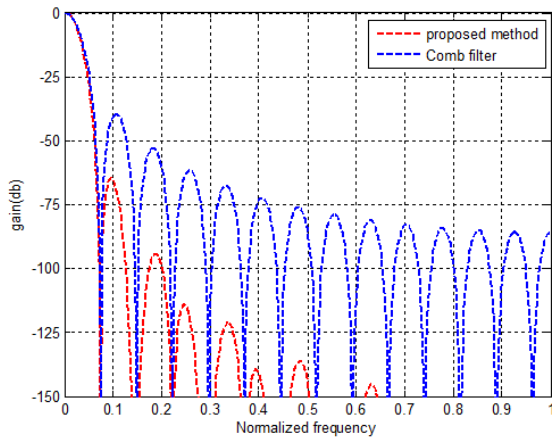


Fig 5.1 Frequency responses of comb filter and optimized comb filter

Attenuation details of first folding Band:

First folding band is the region where worst case of attenuation occurs during the process of decimation. The behavior of attenuation at first folding band of Comb filter at frequency from equation: $[2\pi k/M - \omega_p; 2\pi k/M + \omega_p] = [0.0648\pi; 0.0834\pi]$ and around Optimized Comb filter is equal to 51dB and 61dB respectively is shown in fig.5.10.

Table 5.1 compares the results obtained for comb filter and optimized comb filter at first side-lobe as well as at first folding band. The attenuation of Comb filter is demonstrated with blue line while attenuation of Optimized Comb filters is demonstrated with red line. A key observation is that the response of Optimized Comb filter lies below the response of Comb filter in case of folding band thus yields a higher signal rejection around the folding band. Thus the fig.5.2 demonstrates that attenuation at first folding band has been increased by approximately 10dB.

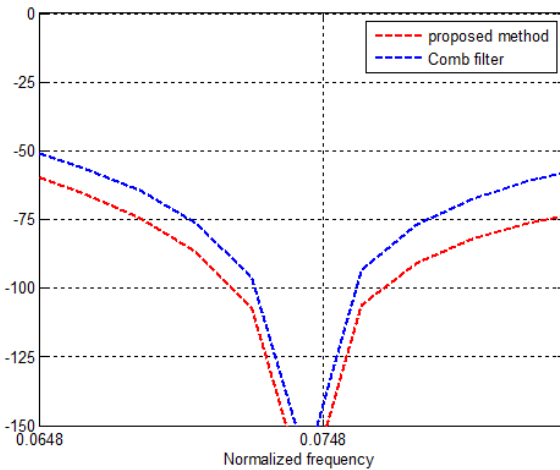


Fig 5.2 Attenuation details of first folding band

Table 5.1 illustrates the design parameters used to obtain the desired attenuation around the first folding band and attenuation obtained at first side-lobe. The attenuation around the first side-lobe of Comb filter and Optimized Comb filter is 51dB and 61dB respectively. The attenuation around the first folding band of Comb filter and Optimized Comb filter is 39.71dB and 65.66dB respectively.

Table 5.1 Attenuation Values for M=27, K=3, L=3 and v=4

Method Used	Attenuation around first folding band	Attenuation of first side lobe
Optimization Method	61dB	65.66dB
Comb filters	51dB	39.71dB

Phase Response:

Phase of output signal when input is taken as a reference. There are basically three types of phase response a filter can have such as zero phase, linear phase and non-linear phase. The phase response of Comb filter and Optimized Comb filter are demonstrated in fig. 5.3 The phase response of Comb filter is demonstrated with blue line while Optimized Comb filters phase response is demonstrated with red line. The phase response of filter designed using proposed approach i.e. Simplex algorithm is linear. The fig 5.3 illustrates that the wave shape is preserved from being distorted as it is passed through designed filter. In the designed approach filter doesn't have any phase delay and hence doesn't result in any phase distortion.

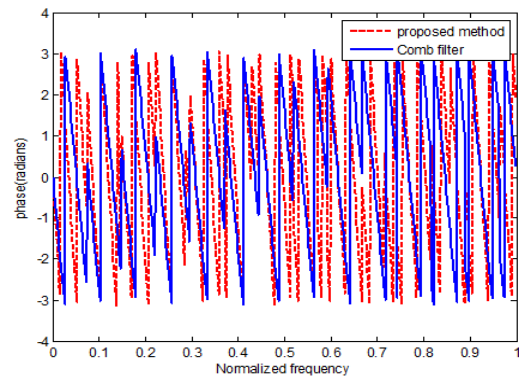


Fig 5.3 Phase response of comb filter and optimized comb filter

Pole-Zero Plot

Pole-zero plot in signal processing, control theory and mathematics is a graphical representation of rational transfer function. Stability is defined on the basis of location of poles and zeros on the unit circle in discrete time system. As fig 5.4 depicts that all poles are on the unit circle and zeros are located at the centre of unit circle which illustrates the stability of designed Optimized Comb filter. Fig.5.4 explains the stability of optimized Comb filter in terms of pole-zero plots. As shown in figure there are a total of 105 poles lying at the centre of unit circle and 106 zeros lying on the unit circle in designed approach. Table 5.2 illustrates the position and number of poles and zeros as obtained in the dissertation.

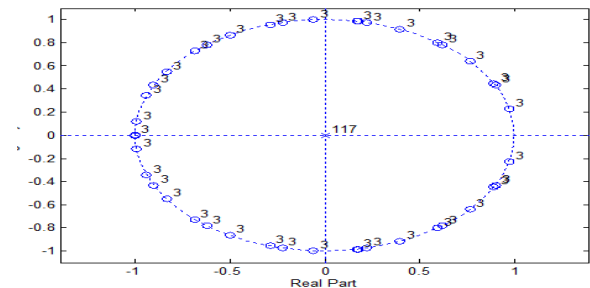


Fig 5.4 Pole-Zero Plot of optimized comb filter

Table 5.2 Location and number of poles and zeros for M=27, K=3, L=3 and v=4

S. No.	Subject	Poles	Zeros
1	Number	117	118
2	Location	At the centre of unit circle	On the unit circle

5. CONCLUSION AND FUTURE SCOPE

Comb filters are an important class of decimation filters and can be represented as an optimized class of FIR filters with coefficients set equal to unity. Thus comb filters can be designed without using multipliers in hardware. The comb filters have the issue in the design of having high droop in pass-band and low attenuation in stop-band. Comb filters have applications in the systems where sample rate reduction is to be done. Due to sample rate reduction process known as decimation aliasing happens in pass-band. To reduce aliasing the simplest decimation filters are known as Comb filters. In this dissertation the behavior of comb filter in pass-band and stop-band have been studied. Various techniques to refine droop in pass-band and attenuation in stop-band for comb filters have been studied. The technique to optimize the results using Simplex algorithm is studied and implemented in this dissertation. The goal of dissertation is to improve the attenuation around the folding-bands and attenuation of side-lobes of comb filter. The attenuation around the comb filter using proposed technique is improved as compared to original values of attenuation. The designed multiplier less filter is connected in cascade with Comb filter. The Simplex Algorithm improves stop-band attenuation of comb filters at both folding bands and side-lobes. The simulation is done using MATLAB R2007b for different decimation factor and different number of stages of comb filters. The dissertation designs the optimized comb filter for specifications such as decimation factor equal to 15 and stages of comb filter are 6. The design is also accomplished for decimation factor 11 and corresponding number of 5 stages of comb filters are used for improvement in attenuation around folding band. Similarly two other design examples for decimation factors 27 and 23 are demonstrated in the research work for 3 and 2 stages of comb filter respectively.

Research in the area of Comb filter design is still actively done. Due to the time restraint and code constraints the current work i.e. designs of comb filter was only focused on improvement in stop-band attenuation. The current research work focuses on improvement in stop-band using Simplex algorithm of optimization. A number of problems can be solved for proper improvement in the response of comb filter. These problems proposed a class of research directions that can be opted to make response of comb filter much better and feasible. One such possibility can be to improve droop in pass-band of comb filter using various techniques of improvement in pass-band. The pass-band improvement can be done by using most commonly used technique known as sharpening technique introduced by Kaiser and Hamming in 1977. Another possibility for future work of the research work can be stated in terms of decimation factor of even values. It would be valuable to perform design improvements for even decimation factors. Improvement in stop-band for even decimation factor can be possible by choosing linear equations in some modified ways. Finally, in terms of applications of the comb filter, there are a number of possible areas in which obtained results can be applicable. The most commonly used areas where improved results can be applicable include the decimated signal output of sigma delta modulator etc. The particular interest of the research work would be to use the sharpening technique in improvement of pass-band response of comb filters.

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