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Determination of Troposphric Delay for GPS Satellite Signals

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***Abstract* - The GPS satellite signals propagate through atmospheric layers as they travel from the satellite to the receiver. Two layers are generally considered when dealing with GPS: the Ionosphere, which extends from a height of 70 to 1000 km above the Earth, and the Troposphere which stretches to about 16 kms. above the equator and 9 kms. above the poles from the surface of the earth. The troposphere causes a delay in both the code and carrier observations. Since it is not frequency dependent (within the GPS L band range) it cannot be canceled out by using dual frequency measurements but it can, however, be successfully modeled. The troposphere is split into two parts: the dry component, which constitutes about 90% of the total refraction, and the wet part, which constitutes the remaining 10%. Values for temperature, pressure and relative humidity are required to model the vertical delay due to the wet and dry part, along with the satellite elevation angle, which is used with an obliquity/mapping function. Models put forward by are all successful in predicting the dry part delay to approximately 1-cm and the wet part to 5 cm. This paper discusses and compares various methods of estimating the tropospheric delay for GPS users namely Hopfield, Saastamoninen and Marini models for various elevation angles of a GPS satellite.**

***Index Terms*: Global Positioning System (GPS), GPS Time (GPST)**

I. INTRODUCTION

The principal error source in the GPS technology is a delay experienced by the GPS signal in propagating through the electrically neutral atmosphere, usually referred to as a tropospheric delay. This delay is normally calculated in the zenith direction, and is referred to as a zenith tropospheric delay. The delay consists of a zenith hydrostatic delay, which can be modeled accurately using surface barometric measurements, and a zenith wet delay, which cannot be modeled from surface barometric measurements and depends on atmospheric water vapor.

The propagation delay of the troposphere reaches about 1.9 - 2.5m in the zenith direction and increases approximately with the cosecant of the elevation angle, yielding about a 20-28m delay at a 5 degree elevation angle. The delay depends on the temperature, humidity, and pressure of the atmosphere. For the measurement of tropospheric delay the atmosphere can be thought of as a mixture of two ideal gases, dry air and water vapour. The ‘dry’ part contributes about 90% of tropospheric refraction. It can be accurately modeled using surface measurements such as pressure and temperature. The dry air component is assumed to obey the ideal gas law. The problem with the “wet” contribution is that the distribution of water vapour cannot be accurately predicted. Even under normal conditions there are localized sources of water vapour, often in the form of liquid water. These water vapour sources along with turbulence in the low atmosphere cause variations in the concentration of water vapour that cannot be correlated over the time or space. These variations cannot be accurately predicted from surface measurements. Fortunately the wet contribution is only about 10 % of the total tropospheric refraction.

Delay due to troposphere can defined as

$$\Delta^{Trop} = \int (n-1)\, ds \qquad (1)$$

In general, instead of the refractive index ‘n’ the refractivity ‘N’ is used.

Where the refractivity $N^{Trop} = 10^{6}(n-1)$

Equation (1) becomes $\Delta^{Trop} = 10^{-6}\int N^{Trop} ds$ (2)

Hopfield (1969) showed the possibility of separating N^{Trop} into a dry and wet components as

$$N^{Trop} = N_d^{Trop} + N_w^{Trop} \qquad (3)$$

Where the dry part results from the dry atmosphere and the wet part from the water vapor. Correspondingly the relations become

$$\Delta_d^{Trop} = 10^{-6}\int N_d^{Trop} ds \quad \text{and} \quad \Delta_w^{Trop} = 10^{-6}\int N_w^{Trop} ds \qquad (4)$$

$$\Delta^{Trop} = \Delta_d^{Trop} + \Delta_w^{Trop} = 10^{-6}\int N_d^{Trop} ds + 10^{-6}\int N_w^{Trop} ds \qquad (5)$$

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There are a number of models which give us the dry and wet refractivity at the surface of the earth [3]

Dry component of refractivity is given by

$$N_{d,o}^{Trop} = \bar{c}_1 \frac{p}{T}, \quad \bar{c}_1 = 77.64 \ \text{K mb}^{-1} \qquad (6)$$

Where 'p' is the atmospheric pressure in millibar (mb) and T is the temperature in Kelvin (K). Wet component is given by

$$N_{w,o}^{Trop} = \bar{c}_2 \frac{e}{T} + \bar{c}_3 \frac{e}{T^2}, \qquad (7)$$

$$\bar{c}_2 = -12.96 \ \text{K mb}^{-1} \text{ and } \bar{c}_3 = 3.718*10^5 \ \text{K}^2 \ \text{mb}^{-1}$$

Where 'e' is the partial pressure of water vapour in mb.

In this paper the tropospheric delay is estimated using three different models namely the Hopfield model, Saastomoinen model and the mapping function model of Marini which are described below.

II. HOPFIELD MODEL

According to Hopfield model [2] the dry refractivity as a function of the height 'h' above the surface is given by

$$N_d^{Trop}(h) = N_{d,o}^{Trop} \left[\frac{h_d - h}{h_d} \right]^4 \qquad (8)$$

Where h_d = 40136 +148.72 (T – 273.16) meters. Substituting the above equation results in

$$\Delta_d^{Trop} = 10^{-6} \ N_{d,o}^{Trop} \int \left[\frac{h_d - h}{h_d} \right]^4 ds \qquad (9)$$

The integral can be solved if the delay is calculated along the vertical direction and if the curvature of the signal path is neglected. Thus, for an observation site on the surface of the earth (i..e., h = 0), the above equation becomes

$$\Delta_d^{Trop} = 10^{-6} \ N_{d,o}^{Trop} \frac{1}{h_d^4} \int_{h=0}^{h=h_d} (h_d - h)^4 dh \qquad (10)$$

Where the constant denominator has been extracted.

$$\Delta_d^{Trop} = 10^{-6} \ N_{d,o}^{Trop} \frac{1}{h_d^4} \left[-\frac{1}{5}(h_d - h)^5 \Big|_{h=0}^{h=h_d} \right] (11)$$

The evaluation of the expression between the brackets gives $h_d^5/5$ so that

$$\Delta_d^{Trop} = \frac{10^{-6}}{5} N_{d,o}^{Trop} h_d \qquad (12)$$

is the dry portion of the tropospheric zenith delay.

The wet portion is much more difficult to model because of the strong variations of the water vapour with respect to time and space. Nevertheless, due to the lack of an appropriate alternative, the Hopfield model assumes the same functional model for both the wet and dry components. Thus,

$$N_w^{Trop}(h) = N_{w,0}^{Trop} \left[\frac{h_w - h}{h_w} \right]^4$$; Where the mean value $h_w = 11{,}000m$ is taken. (13)

The integration of above equation results in

$$\Delta_w^{Trop} = \frac{10^{-6}}{5} N_{w,o}^{Trop} h_w \qquad (14)$$

Therefore, the total tropospheric zenith delay is given by

$$\Delta^{Trop} = \frac{10^{-6}}{5} \left[N_{d,o}^{Trop} h_d + N_{w,o}^{Trop} h_w \right] \text{ meters} \quad (15)$$

The model in its present form does not account for an arbitrary zenith angle of the signal. Considering the line of sight, an obliquity factor must be applied which is the projection from the zenith onto the line of sight. Frequently, the transition of the zenith delay to a delay with arbitrary zenith angle is denoted as the application of a mapping function.

Introducing the mapping function the above equation becomes

$$\Delta^{Trop} = \frac{10^{-6}}{5} \left[N_{d,o}^{Trop} h_d m_d(E) + N_{w,o}^{Trop} h_w m_w(E) \right] \quad (16)$$

Where $m_d(E)$ and $m_w(E)$ are the mapping functions for the dry and the wet part and E (expressed in degrees) indicates the elevation at the observing site (where the line of sight is simplified as straight line). Explicitly,

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$$m_d(E) = \frac{1}{\sin\sqrt{E^2 + 6.25}} \quad (17)$$

$$m_w(E) = \frac{1}{\sin\sqrt{E^2 + 2.25}} \quad (18)$$

are the mapping functions for the Hopfield model. The above equation can be represented as

$$\Delta^{Trop}(E) = \Delta_d^{Trop}(E) + \Delta_w^{Trop}(E) \quad (19)$$

$$\Delta_d^{Trop}(E) = \frac{10^{-6}}{5}\,\frac{77.64}{\sin\sqrt{E^2+6.25}}[40.136+148.72(T-273.16)]$$

(20)

$$\Delta_w^{Trop}(E) = \frac{10^{-6}}{5}\,\frac{-12.96+3.718(10^5)}{\sin\sqrt{E^2+2.25}}[\frac{e}{T^2}](11000) \quad (21)$$

Measuring the atmospheric pressure (p), temperature in Kelvin (T) and the partial pressure of water vapor (e) at the observation location and calculating the elevation angle E, the total pospheric path delay is obtained in meters by using the above ıations.

III. SAASTOMOINEN MODEL

This model is deduced from gas laws. The tropospheric delay is expressed in meters as [4]

$$\Delta^{Trop} = \frac{0.002277}{\cos z}\left[p + \left(\frac{1255}{T} + 0.05\right)e - \tan^2 z\right] \quad (22)$$

Where 'Z' denotes the zenith angle of the satellite, 'p' is the atmospheric pressure in millibar, 'T' is the temperature in Kelvin and 'e' is the partial pressure of water vapor in millibar.

Subsequently he refined this model by adding two correction terms, one being dependent on the height of the observing site and the other on the height and zenith angle. The refined model is given below:

$$\Delta^{Trop} = \frac{0.002277}{\cos z}\left[p + \left(\frac{1255}{T} + 0.05\right)e - B*\tan^2 z\right] + \delta R \quad (23)$$

The correction terms B, δR can be interpolated from Table 1 and Table 2.

Table 1 Correction term B for the refined Saastomoinen model

Height in Km.	0.0	0.5	1.0	1.5	2.0	2.5	3.0	4.0	5.0
B (mb)	1.156	1.079	1.006	0.938	0.874	0.813	0.757	0.654	0.563

Table 2 Correction term δR in meters for refined Saastamoinen model

Zenith angle	Station height above sea level [km]							
	0	0.5	1.0	1.5	2.0	3.0	4.0	5.0
60^0 00'	0.003	0.003	0.002	0.002	0.002	0.002	0.001	0.001
66^0 00'	0.006	0.006	0.005	0.005	0.004	0.003	0.003	0.002
70^0 00'	0.012	0.011	0.010	0.009	0.008	0.006	0.005	0.004
73^0 00'	0.020	0.018	0.017	0.015	0.013	0.011	0.009	0.007
75^0 00'	0.031	0.028	0.025	0.023	0.021	0.017	0.014	0.011
76^0 00'	0.039	0.035	0.032	0.029	0.026	0.021	0.017	0.014
77^0 00'	0.050	0.045	0.041	0.037	0.033	0.027	0.022	0.018
78^0 00'	0.065	0.059	0.054	0.049	0.044	0.036	0.030	0.024
78^0 30'	0.075	0.068	0.062	0.056	0.051	0.042	0.034	0.028
79^0 00'	0.087	0.079	0.072	0.065	0.059	0.049	0.040	0.033
79^0 30'	0.102	0.093	0.085	0.077	0.070	0.058	0.047	0.039
79^0 45'	0.111	0.101	0.092	0.083	0.076	0.063	0.052	0.043
80^0 00'	0.121	0.110	0.100	0.091	0.083	0.068	0.056	0.047

IV. THE MAPPING FUNCTION OF MARINI

In 1972, Marini developed a continued fraction of the mapping function [5]. Herring (1992) specified this function with three constants and normalized to unity at the zenith. For the dry component, the following mapping function is used.

$$m_d(E) = \frac{1 + \cfrac{a_d}{1 + \cfrac{b_d}{1 + c_d}}}{\sin E + \cfrac{a_d}{\sin E + \cfrac{b_d}{\sin E + c_d}}} \quad (24)$$

Where the coefficients are defined as

$$a_d = [1.2320 + 0.0139\cos\Phi - 0.0209h + 0.00215(T - 283)]\ (10^{-3}) \quad (25)$$

$$b_d = [3.1612 - 0.1600\cos\Phi - 0.0331h + 0.00206(T - 283)]\ (10^{-3}) \quad (26)$$

$$c_d = [71.244 - 4.293\cos\Phi - 0.149h - 0.0021\ (T - 283\ )]\ (10^{-3}) \quad (27)$$

For the wet part, the mapping function is the same as in equation but the subscript‘d’ must be replaced by ‘w’. The corresponding coefficients are obtained as

$$a_w = [0.583 – 0.011\cos\Phi - 0.052h + 0.0014(T - 283)]\ (10^{-3}) \quad (28)$$

$$b_w = [1.402 - 0.102\cos\Phi - 0.1018h + 0.0020(T - 283)]\ (10^{-3}) \quad (29)$$

$$c_w = [45.85 - 1.91\cos\Phi - 1.29h + 0.015(T - 283)]\ (10^{-3}) \quad (30)$$

V. RESULTS AND CONCLUSION

Tropospheric delay was estimated using the above models for all the possible elevation angles of a GPS satellite. The results are summarized in Table 3 and plotted in Fig. 1. The atmospheric pressure (p) is taken as 101.325 millibar and the partial pressure of water vapor (e) is taken as 0.85 millibar for estimating the delay. The results clearly indicate that the delay is more at low elevation angles and less high elevation angles. All these models display the same amount of accuracy in estimating the tropospheric delay especially for high elevation angles.

Table 3 Estimation of tropospheric delay (in meters) using various models

Model	Elevation angle of GPS satellite in degrees								
	10^0	20^0	30^0	40^0	50^0	60^0	70^0	80^0	90^0
Hopfield	13.4	6.95	4.77	3.72	3.12	2.76	2.55	2.43	2.39
Saasta moinen	13.36	6.94	4.77	3.72	3.12	2.76	2.55	2.43	2.39
Marini	14.74	7.07	4.80	3.73	3.13	2.76	2.55	2.43	2.39

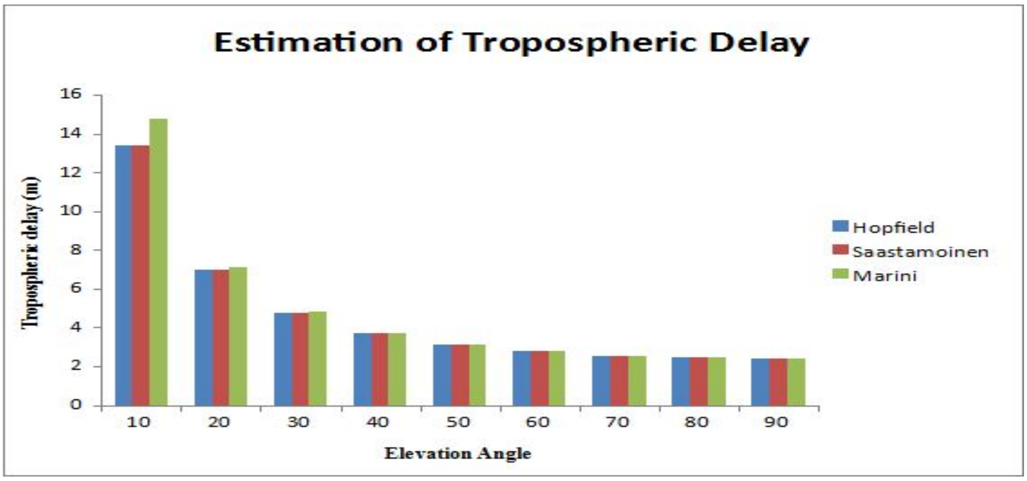

Fig. 1 Estimation of Tropospheric delay using various models

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