Joint Spectrum Sensing, Power Allocation and Relay Gain Control in Cognitive Relay Networks

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Abstract—In recent years, cognitive radio (CR) technology has been proposed as a solution to overcome inefficient spectrum allocation strategies by enabling unlicensed users to opportunistically use allocated bands to the licensed users. In classical disjoint spectrum sensing and power allocation methods, CR system decides about the presence and absence of the PU, in each frequency band, by comparing the measured energy to a predefined threshold. After detecting vacant bands, CR transmitter allocates the power and subchannel to other CR users. Due to noise, fading and spectrum sensing errors, this approach may lose some opportunities in the CR system. In this paper, the problem of joint spectrum sensing and power allocation in a cognitive relay network is studied. To this end, an optimal power allocation scheme is proposed to maximize the CR achievable data rate under constraints of spectrum sensing performance and total power budget of CR user. The performance of the proposed method has been examined through numerical simulations in terms of the sensing parameters, available radio resources and relay parameters.

Index Terms—Cognitive Radio, Joint sensing and power allocation, Cognitive Relay Network.

I. INTRODUCTION

Recently, cognitive radio [1] has been proposed to solve the spectrum resource scarcity problem. In a cognitive radio (CR) network, unlicensed users called secondary users (SUs), are permitted to use allocated bands to the licensed primary users (PUs), as long as interference introduced to the PU remains tolerable [1]. So, cognitive network should sense the radio environment and after detecting vacant bands, shares its resources in a way, maximum exploitation is achieved and minimum interference is introduced to the PU. By utilizing spectrum sensing, secondary users can detect and reuse the vacant channels which are not used by primary users (PUs) in a certain time and location. CR resource allocation methods aiming at maximizing transmission capacity, operate based on spectrum sensing results and calculated interferences introduced to the CR and PU [14], [15].

Sensing a PU is a difficult task due to the spectrum sensing errors, fading and noise [9], [10] and [12]. Therefore, sensing is never completely reliable and will always produce some errors depending upon the channel conditions between the primary transmitter and CR sensors. Energy detection-based sensing is used in spectrum sensing phase of CR systems because of its simplicity and lower information needed about the PU [11]. The decision on the occupancy of a band can be obtained by comparing the measured energy against a fixed threshold which is obtained from Neman-Pearson criteria. If the decision metric is greater than threshold, CR network decides the PU is present. Otherwise, if the decision metric is lower than threshold, CR network decides that PU is absent and tries to use the spectrum.

However, due to noise and errors in spectrum sensing measurements, this method may result in missing lots of opportunities in CR network. Also, in the low signal-to-noise ratio (SNR), one energy detection cannot achieve reliable sensing. Amplify-and-forward (AF) relaying, could be used to improve sensing performance as well as to increase achievable CR data rate [6], [9].

On the other hand, joint spectrum sensing and radio resource allocation scheme for CR systems has been shown to achieve higher opportunities in accessing vacant bands and achieving higher information rates for CR users. CR radio resource allocation methods aiming at maximizing transmission data rate of CR users operate based on spectrum sensing results and calculated interferences introduced to the CR and PU.

In this work, a CR system consisting of a PU and a CR network composing of three nodes named cognitive source (CS), cognitive relay (CR), and cognitive destination (CD) is considered. The total transmission power in the CR network is limited. On the other hand, the sensing performance must be satisfied in order to restrict the interference to the PU. Therefore, it is worthy to design an optimal power allocation strategy by maximizing the CR achievable data rate under constraints on false alarm and miss detection probabilities and maximum available power budget in CR network. The constrained optimization problem is transformed into an unconstrained optimization problem which only requires finding an optimal CR amplifying gain. Simulation results show that optimal power allocation strategy always achieves higher data rates in CR network compared with classical approach named equal power allocation. Furthermore, the maximum achievable data rate in CR network using proposed power allocation scheme is maximized.
strategy increases by increasing the total power budget of CR network.

The rest of this paper is organized as follows. The system model and main assumptions in considered CR system model as well as the sensing parameters related to the energy detection methodology in our system model are presented in Section II. In Sections III, we formulate the problem of optimal power allocation. Section IV provides Simulation results and finally, in Section V, we conclude the paper.

II. SYSTEM MODEL AND MAIN ASSUMPTIONS

The CR system, consists of a PU and a CR network with three CR users, i.e., CS, CR and CD. The considered scenario operates on a frame-by-frame basis in which each frame is equally divided into two phases $T_1$ and $T_2$ ($T_1 + T_2$) [6]. In each frame duration of $(T = T_1 + T_2)$, the CR network senses the presence or absence of the PU. If the PU is not detected, the CR network will transmit data and senses the presence or absence of the PU simultaneously in the current frame, referred to as Scenario 1. Otherwise, if the PU is detected in the previous frame, the CR network will not transmit data but only senses the presence or absence of the PU in the current frame, referred to as Scenario 2. It should be noted that the CR network only monitors the presence of the PU but not transmit data in the first frame.

The channels over the links PU→CS, PU→CR, PU→CD, CS→CR, CR→CD, and CS→CD are modeled to be Rayleigh flat fading channels with channel coefficients $h_{pt}$, $h_{pr}$, $h_{pd}$, $h_{sr}$, $h_{rd}$ and $h_{sd}$, respectively. We assume each channel coefficient follows a complex Gaussian distribution, i.e., $h_x \sim \mathcal{CN}(0,d_x^2)$, where $d_x$ is the normalized distance of the $X$-th path. It is assumed that these channel coefficients are independent at the CS and the CR, and are constant during one frame [10]. Let us denote the aforementioned channel gains by $G_{ps} = |h_{pt}|^2$, $G_{pc} = |h_{pr}|^2$, $G_{pd} = |h_{pd}|^2$, $G_{sr} = |h_{sr}|^2$, $G_{rd} = |h_{rd}|^2$, $G_{sd} = |h_{sd}|^2$. Also, $x_p[n]$ and $x_r[n]$ denote the primary signal and the secondary signal, respectively, which are zero mean and have unit variance, i.e.,

$$E\left\{ |x_p[n]|^2 \right\} = 1,$$

and

$$E\left\{ |x_r[n]|^2 \right\} = 1.$$  

Let $P_p$ and $P_d$ denote the transmission power of PU and CS, respectively. Since Scenario 2 is just a reduced version of Scenario 1, we only discuss Scenario 1 hereafter. In the first phase $T_1$, the CR and the CD listens to the PU while the CS transmits data to the CR and the CD. Then, the signals received by the CR and the CD in $T_1$ can be written as

$$y_{rt1}(n) = \sqrt{P_p} h_{sr} x_{ct1}(n) + \theta \sqrt{P_d} h_{pr} x_{pt1}(n) + u_{rt1}(n)$$

$$y_{dt1}(n) = \sqrt{P_p} h_{sd} x_{ct1}(n) + \theta \sqrt{P_d} h_{pd} x_{pt1}(n) + u_{dt1}(n)$$

respectively, where $n = 1, ..., N$, $N = T_1 f_s$, $f_s$ denotes the sampling frequency, $\theta$ is the primary signal indicator: $\theta = 1$ implies the presence of the primary signal and $\theta = 0$ implies the absence of the primary signal. $x_{ct1}[n]$ and $x_{pt1}[n]$ denote the signals sent from the CS and the PU in $T_1$, respectively, which are zero mean and have unit variance, i.e., $E\{ |x_{ct1}(n)|^2 \} = 1$, $E\{ |x_{pt1}(n)|^2 \} = 1$. The additive noises $u_{rt1}[n]$ and $u_{dt1}[n]$ are independent and identically distributed (i.i.d.), circularly symmetric complex Gaussian (CSCG) random sequences with zero mean and variances $E\{ |u_{rt1}(n)|^2 \} = P_r$ and $E\{ |u_{dt1}(n)|^2 \} = P_d$.

In the second phase $T_2$, the CR amplifies its received signal in $T_2$ with an amplifying factor $\beta$ and forwards this amplified signal to the CS and the CD. Moreover, the CS and the CD listen to the PU. Thus, the signals received by the CS and the CD in $T_2$ can be expressed as

$$y_{st2}(n) = \beta h_{sr} y_{rt1}(n) + \theta \sqrt{P_p} h_{pr} x_{pt2}(n) + u_{st2}(n)$$

$$= \beta \sqrt{P_p} h_{sr} x_{ct1}(n) + \beta \sqrt{P_p} h_{sr} x_{ct1}(n) + u_{st2}(n)$$

and

$$y_{dt2}(n) = \beta h_{rd} y_{rt1}(n) + \theta \sqrt{P_d} h_{pd} x_{pt2}(n) + u_{dt2}(n)$$

$$= \beta \sqrt{P_p} h_{rd} x_{ct1}(n) + \beta \sqrt{P_p} h_{rd} x_{ct1}(n) + u_{dt2}(n)$$

respectively, where $n = N + 1, ..., N$, $x_{pt2}[n]$ denotes the signal sent from the PU in $T_2$, with zero mean and unit variance

$$E\left\{ |x_{pt2}(n)|^2 \right\} = 1.$$  

The noises $u_{st2}[n]$ and $u_{dt2}[n]$ in $T_2$ are also i.i.d., CSCG random sequences with zero mean and variances $E\{ |u_{st2}(n)|^2 \} = P_r$ and $E\{ |u_{dt2}(n)|^2 \} = P_d$.

Noted that the first term of (3) is self-interference at the CS. By applying self-interference cancelation (canceling $x_{ct1}[n]$ that originates from the CS) [3], the remaining signal can be represented by

$$y_{st2}(n) = \theta \sqrt{P_p} h_{sr} y_{rt1}(n) + x_{pt2}(n) + \theta \sqrt{P_p} h_{sr} x_{ct1}(n) + u_{st2}(n)$$

An energy detector is adopted at the CS for spectrum sensing by utilizing the signal $y_{st2}[n]$. Then, the test statistic is given by
\[ Y = \frac{1}{N} \sum_{n=1}^{2N} |\bar{Y}_{st2}(n)|^2. \]

By the central limit theorem, for a sufficiently large \( N \), the test statistic \( Y \) approximately follows a Gaussian distribution under the hypothesis \( H_1 \) when \( \theta = 0 \) or the hypothesis \( H_0 \) when \( \theta = 1 \). For analysis simplicity, \( x_{pr1}[n] \) and \( x_{pr2}[n] \) are assumed to be CSCG random sequences, and \( x_{st1}[n], x_{st2}[n], u_{st1}[n], \) and \( u_{st2}[n] \) are pairwise independent. Then, the mean and the variance of \( Y \) under \( H_1 \) and \( H_0 \) can be expressed respectively

\[
E(Y) = \begin{cases} 
\beta G_{sr} P_u + P_u, & H_0 \\
G_{ps} P_p + \beta G_{sr} (G_{pr} P_p + P_u) + P_u, & H_1
\end{cases}
\]

and

\[
\text{Var}(Y) = \begin{cases} 
\frac{2}{N} (\beta G_{sr} P_u + P_u)^2, & H_0 \\
\frac{2}{N} (G_{ps} P_p + \beta G_{sr} (G_{pr} P_p + P_u) + P_u)^2, & H_1
\end{cases}
\]

Let \( \lambda \) denote the decision threshold. Then, the detection probability \( p_d \) and the false alarm probability \( p_f \) are calculated by

\[
p_d = \text{Pr} \{ Y \geq \lambda | H_1 \} = Q \left( \frac{\lambda - E(Y_1)}{\sqrt{\text{Var}(Y_1)}} \right) = Q \left( \frac{\lambda - \mu_1}{\sqrt{\text{Var}(Y_1)}} \right),
\]

and

\[
p_f = \text{Pr} \{ Y \geq \lambda | H_0 \} = Q \left( \frac{\lambda - E(Y_0)}{\sqrt{\text{Var}(Y_0)}} \right) = Q \left( \frac{\lambda - \mu_0}{\sqrt{\text{Var}(Y_0)}} \right),
\]

where \( Q(\cdot) \) is the complementary cumulative distribution function of the standard Gaussian random variable. For a given detection probability \( \tilde{p}_d \), from (6) and (7), by canceling out the threshold \( \lambda \), the false alarm probability \( p_f \) can be written as

\[
p_f = Q \left( Q^{-1}(\tilde{p}_d) \mu_1 + \sqrt{\frac{N}{2}} (\mu_1 - \mu_0) \right)
\]

\[
= Q \left( Q^{-1}(\tilde{p}_d) + \sqrt{\frac{N}{2}} \gamma_p + Q^{-1}(\tilde{p}_d) \right)
\]

where

\[
\gamma_p = \frac{\mu_1 - \mu_0}{\mu_0} = \frac{G_{ps} P_p + \beta G_{sr} G_{pr} P_u}{\beta G_{sr} P_u + P_u} = \frac{\gamma_{ps} + \beta G_{sr} \gamma_{pr}}{1 + \beta G_{sr}}
\]

\[
\gamma_{ps} = \frac{G_{ps} P_p}{P_u}
\]

\[
\gamma_{pr} = \frac{G_{pr} P_p}{P_u}
\]

In fact, \( \gamma_{ps} \) is the SNR of the signal transmitted over the link PU\(\rightarrow\)CS and \( \gamma_{pr} \) is the SNR of the signal transmitted over the link PU\(\rightarrow\)CR.

### III. Optimal Power Allocation

In this section, an optimum power allocation strategy is derived for the CR network that can maximize secondary throughput. A data packet transmitted by a CU is considered lost if the transmission of it conflicts with the PU (i.e., \( \theta = 1 \)). These secondary throughput in the CR network is calculated by

\[
R = \frac{1}{2} P(D_o, H_o) \log_2 (1 + \gamma_0) + \frac{1}{2} P(D_o, H_0) \log_2 (1 + \gamma_1), \quad (10)
\]

Equation (10) can be written as

\[
R = \frac{1}{2} P(D_o|H_0) P(H_0) \log_2 (1 + \gamma_0) + \frac{1}{2} P(D_o|H_1) P(H_1) \log_2 (1 + \gamma_1)
\]

\[
= \frac{1}{2} (1 - \alpha) (1 - p_f) \log_2 (1 + \gamma_0) + \frac{1}{2} \alpha (1 - p_d) \log_2 (1 + \gamma_1)
\]

where \( \gamma_0 \) denotes the received cognitive signal-to-noise ratio (CSNR) at the CD when the PU is inactive (\( \theta = 0 \)), \( \alpha \) denotes the probability that the PU is active, i.e., \( \alpha = P(\theta = 1) = P(H_1) \). The CD applies maximal-ratio combining for the received signals from the CS in \( T_1 \) and the CR in \( T_2 \). Using (2) and (4), the CSNR \( \gamma_0 \) can be expressed as

\[
\gamma_0 = \frac{G_{sd} P_s + \beta G_{rd} G_{sd} P_s P_u}{P_u} + \beta G_{sr} G_{sd} P_s + P_u
\]

and

\[
\gamma_1 = \frac{G_{pd} P_p + \beta G_{rd} G_{pd} P_p + P_p (\beta G_{rd} G_{pr} + G_{pd})}{P_u} + \beta G_{sr} P_p + P_u + \gamma_0
\]

Using (1), the transmission power of the CR can be expressed as

\[
P_r = \beta (G_{sr} P_s + \beta G_{pr} P_p + P_p)
\]

The average transmission power of the CR is given by

\[
P_r = \alpha \beta (G_{sr} P_s + G_{pr} P_p + P_p) + (1 - \alpha) \beta (G_{sr} P_s + P_p)
\]

Note that the CR network has a total transmission power budget \( P_{max} \) and the PU should be sufficiently protected (i.e., the detection probability \( p_d \) must be higher than a predefined threshold \( \tilde{p}_d \)). Then, the optimum power allocation problem can be formulated as

**Optimization Problem:**

\[
\max_{P_s, P_p, \alpha} R
\]

s.t. \( P_s + P_r \leq P_{max} \)

\[
p_d \geq \tilde{p}_d
\]
From (6), (7), (10) and (11), it is easy to verify that the throughput $R$ decreases with $p_d$, and increases with $P_s$ and $\beta$ (proportional to $P_r$). Therefore, the solution to the optimization problem (14) is obtained when the inequality constraints are satisfied at equality.

IV. SIMULATION RESULTS

In the simulation, the parameters are set as $N = 100$, $\alpha = 0.3$ (it means that the PU is present for 30% of the time), $p_d = 0.95$, $P_u = 0$ dBW, $P_p = 0$ dBW. To obtain the simulation value of false alarm probability 20000 Monte Carlo simulations are performed, which is further used to calculate the simulation value of achievable throughput by (10).

Fig. 1 shows the false alarm probability $p_f$ vs. CR amplifying gain $\beta$. It can be easily observed that the false alarm probability decreases as $\beta$ increases. Also, by increasing the target detection probability $p_d$, the false alarm probability increases, as expected.

Figures 2 and 3 depict the achievable CR data rates vs CR amplifying gain $\beta$, for different values of the CR network total power budget and CR transmitted power. As can be seen, by increasing amplifying gain $\beta$, the achievable data rate increases at first and then decreases. It means that an optimal amplifying gain exist in which the maximum data rate can be achieved for the CR network. Also, by increasing the power budget of the CR user, the achievable data rate increases.

Fig. 4 shows the achievable CR data rate $R$ vs total power budget of the CR network. As can be seen, by increasing $P_u$ and/or $P_{\text{max}}$, the CR achievable data rates increases, as expected.

CONCLUSION

The problem of joint spectrum sensing, power allocation and relay gain optimization in a cognitive relay network is studied in terms of the achieved data rate in CR network under sensing and radio resources constraints. To this end, an optimal power allocation scheme is proposed to maximize the CR achievable data rate under constraints of spectrum sensing performance and total power budget of CR user. The performance of the proposed method has been examined through numerical simulations in terms of the sensing parameters, available radio resources and relay parameters.
network, for different values of the CR network transmitted power, \( P_u \).

REFERENCES