

Performance Evaluation of Digital M-Ary Quadrature Amplitude Signalling Scheme

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Abstract— In this paper the modular work is presented to appraise the performance of digital Multilevel Quadrature Amplitude Modulation system. The performance is analyzed in terms of probability of error considering Additive White Gaussian Noise channel. The simulation is performed for M=4, 16, and 64 in MATLAB Simulink environment. The simulation and theoretical results are compared and found almost similar. It is observed that signal to noise ratio (SNR) requirement for a given bit error rate (BER) increases as M increases while bandwidth efficiency increases with M. Further M-ary QAM systems are bandwidth efficient and have large data carrying capacity.

Index Terms— Additive White Gaussian Noise (AWGN), Bit Error Rate (BER), Multilevel Quadrature Amplitude Modulation (M-QAM), Signal to Noise Ratio (SNR).

I. INTRODUCTION

The digital communication systems have now replaced old generation's analog communication systems because of many advantages, like, more data carrying capacity, more band width efficiency, better performance at low power, flexibility in design etc. In digital communication systems, initially, basic signaling systems like, Amplitude shift keying (ASK), Frequency Shift Keying (FSK), Phase shift Keying (PSK), Binary Phase Shift Keying (BPSK) were employed [1], [2]. To improve transmission rate and band width efficiency, now-a-day the M-ary signaling systems, like, M-ary ASK, M-ary FSK, M-ary PSK and M-ary QAM are being used normally. The M-ary QAM signaling scheme having some excellent properties like requirement of less signal to noise ratio, high capability, more band width efficient and better performance. M-ary QAM system is widely used in digital video broadcasting in existing satellite communication, cable TV, dish TV, microwave digital radio, modems, broad band set top box and digital video. It is also used in broad band wireless communication systems [3].

II. M-ARY QAM SIGNALLING SCHEMES

The quadrature amplitude modulation scheme is the combination of phase shift keying and amplitude keying in which phase and amplitude varies with respect to the input symbol. The digital quadrature amplitude modulated signal is normally specified as given below.

$$s_m(t) = A_m \cos(2\pi f_c t + \phi_m), \quad (1)$$

$$0 \leq t \leq T_s$$

where $m = 1, 2, \dots, M$ and T_s is symbol duration.

On expanding equation (1), it is obtained that

$$s_m(t) = A_m \cos\phi_m \cos 2\pi f_c t - A_m \sin\phi_m \sin 2\pi f_c t$$

$$s_m(t) = A_{cm} \cos 2\pi f_c t + A_{sm} \sin 2\pi f_c t \quad (2)$$

where $A_{cm} = A_m \cos\phi_m$ and

$$A_{sm} = -A_m \sin\phi_m$$

From equation (2) it is clear that QAM can be viewed as digital pulse amplitude modulation of two phase quadrature sinusoidal carriers. A_{cm} and A_{sm} are independent integers chosen in order of message point location. The energy of transmitted signals are different depending on values of A_{cm} and A_{sm} [4].

III. M-ARY QAM MODULATOR

The block diagram of M-ary QAM transmitter is given in Figure 1. In which a binary stream of bit rate $R_b = 1/T_b$ is fed to the amplitude and phase selector and evaluate different values of A_{cm} and A_{sm} . The quadrature carriers $\cos(2\pi f_c t)$ and $\sin(2\pi f_c t)$ are then modulated by A_{cm} and A_{sm} respectively. Both modulated signals are then added together to obtain a QAM modulated signal.

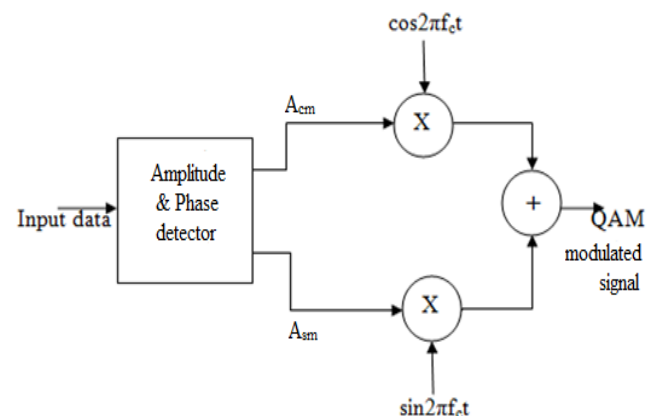


Figure 1: Block Diagram of M-ary QAM Transmitter

Two dimensional constellation diagram for values of M = 4, 16, 64 is given in Figures 2, 3, 4. To represent code words Gray coding is used.

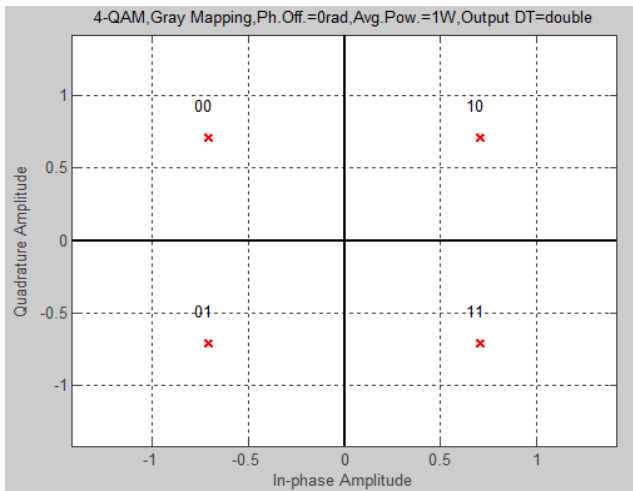


Figure 2: Constellation diagram of 4QAM

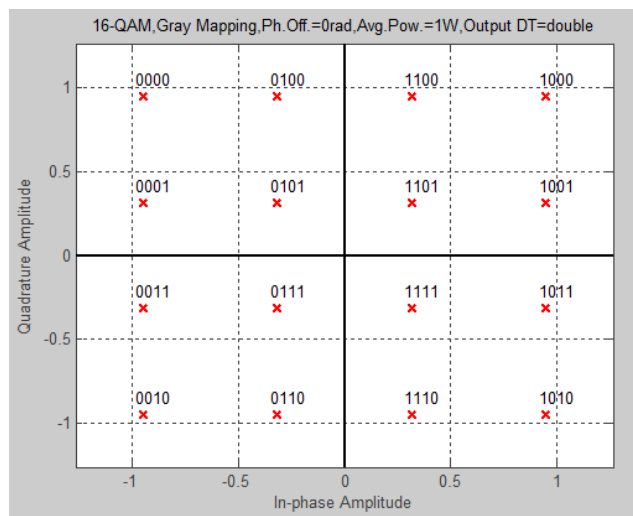


Figure 3: Constellation diagram of 16QAM

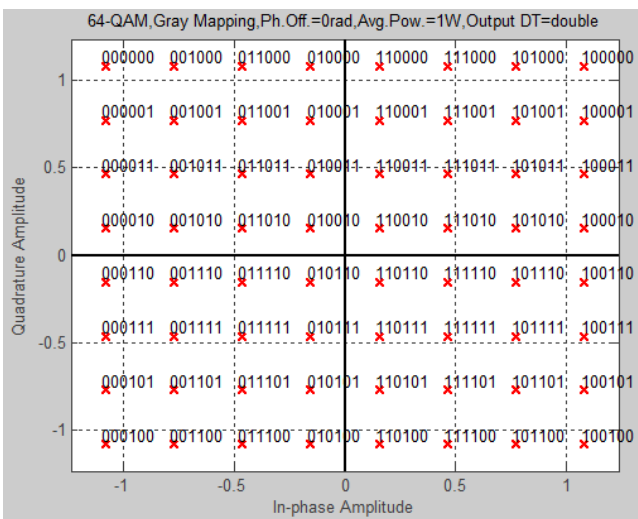


Figure 4: Constellation diagram of 64QAM.

IV. M-ARY QAM DEMODULATOR

The M-ary QAM demodulator is given in Figure 5, the received signals corrupted with a noise n(t) is processed in cross correlator receivers. The noise is taken white Gaussian

zero mean with power spectral density of $\eta/2$. The a_{cm} and a_{sm} are corrupted values of A_{cm} and A_{sm} due to noise. The receiver computes the distance between observed values of a_{cm} and a_{sm} with different symbol points and then decides in favor of minimum distance.

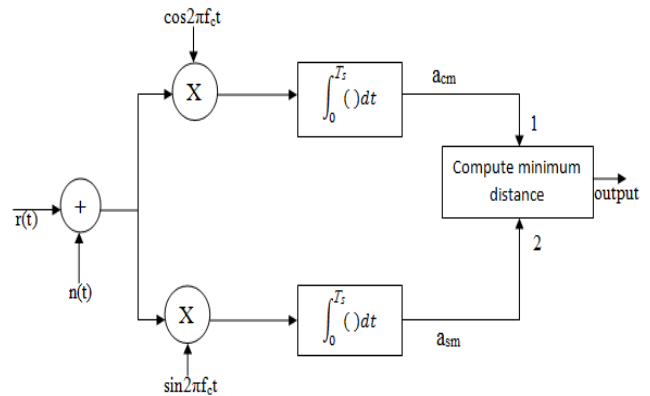


Figure 5: Block Diagram of M-ary QAM Receiver

V. PROBABILITY OF ERROR FOR M-ary QAM

To derive expression for probability of symbol error, it is assumed that channel is an ideal channel. Further the attenuation coefficient of the channel is taken unity and time delay between the transmitted and received signal is negligible. Under these conditions the received signal is same as transmitted signal. The received signal under above mentioned will be given by

$$r(t) = A_{cm} \cos 2\pi f_c t + A_{sm} \sin 2\pi f_c t \tag{3}$$

Both terms in equation (3) behaves like a digital PAM and A_{cm} and A_{sm} are recovered in the same manner as in PAM case. The probability of symbol error for digital PAM is evaluated by us and reported in equation (4). It is given by

$$P_M = \frac{M-1}{M} \operatorname{erfc} \left(\sqrt{\frac{3}{M^2-1} \gamma_{avg}} \right) \tag{4}$$

Here γ_{avg} is average SNR per bit. The above expression may be used in calculating the symbol error probability for $(\sqrt{M} \times \sqrt{M} = M)$ QAM case with simply replacing M by \sqrt{M} and γ_{avg} to $\gamma_{avg}/2$, so that total value of γ_{avg} remain same. After these replacements, the probability of symbol error at point 1 and 2 is given by

$$P_{\sqrt{M}} = \frac{\sqrt{M}-1}{\sqrt{M}} \operatorname{erfc} \left(\sqrt{\frac{3}{M-1} \gamma_{avg}/2} \right) \tag{5}$$

The probability of correct decision at point 1 and 2 is given by

$$P_c = 1 - P_{\sqrt{M}} \tag{6}$$

And at the output of minimum distance detector, it will be

$$P_c = (1 - P_{\sqrt{M}})^2 \tag{7}$$

Finally the probability of symbol error for M-QAM is written as

$$P_M = 1 - P_c = 1 - (1 - P_{\sqrt{M}})^2 \quad (8)$$

This expression can be simplified as

$$P_M \leq 2 \left(1 - \frac{1}{\sqrt{M}}\right) \operatorname{erfc} \left(\sqrt{\frac{3}{2(M-1)} \gamma_{avg}} \right) \quad (9)$$

In above expression, equal sign holds when $k = \log_2 M$ is even. For odd value of k , the P_M with equal sign will represent upper bound on probability [4].

VI. BANDWIDTH EFFICIENCY

The bandwidth efficiency is defined as the ratio of transmitted data rate, R , and required value of bandwidth.

$$\text{Bandwidth efficiency} = R/W$$

In case of QAM; R & W are

$$R = 1/T_b \text{ and } W = 1/2T_s = 1/2kT_b \text{ where } T_b \text{ is bit duration and } T_s \text{ is symbol duration.}$$

therefore bandwidth efficiency

$$R/W = 2k = 2\log_2 M \quad (10)$$

The bandwidth efficiency of QAM increases as M increases. The bandwidth efficiency for different values of M is given in table-1. It is also shown in figure 6.

Table -2: Value of M and bandwidth efficiency

Value of M	Bandwidth efficiency R/W
2	2
4	4
8	6
16	8
32	10
64	12

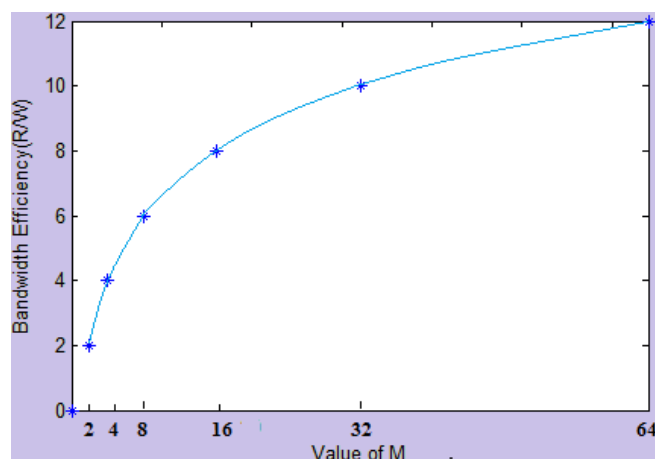


Figure 6: Value of M Vs Bandwidth efficiency.

VII. SIMULATION MODEL OF M-QAM

The simulation model of M-ary QAM is shown in Figure 6. On transmitter side, the system model consists of a random integer generator and a rectangular QAM modulator. The modulated signal passes through the AWGN channel block to add white Gaussian noise with required signal to noise ratio. This signal is processed in rectangular QAM demodulator and error rate calculator. Probability of symbol error is evaluated for $M = 4, 16$ and 64 . The results are displayed in Figure 8 and table-2.

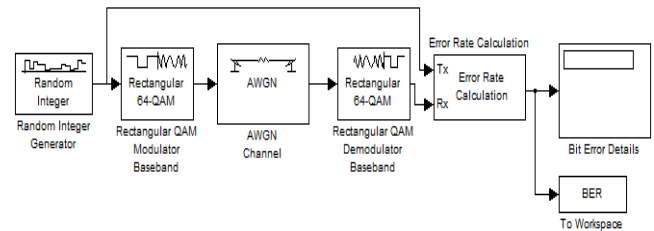


Figure 7: Simulation model of M-ary QAM

VIII. SIMULATION RESULTS

In Figure 8 both simulated and theoretical results are shown, it is observed that the simulated results are almost similar with the theoretical results for low BER. The simulated results are also shown in tabular form in table-2. Further it is observed that for a given BER, the requirement of signal to noise ratio (SNR) increases as M increases.

Table-2: E_b/N_0 and BER table for 4-QAM, 16-QAM, 32-QAM and 64-QAM

E_b/N_0 in dB	BER for 4-QAM	BER for 16-QAM	BER for 64-QAM
0	0.1727	0.4545	0.7575
1	0.1155	0.4184	0.685
2	0.0823	0.3518	0.63
3	0.0485	0.2801	0.585
4	0.0263	0.2222	0.545
5	0.0114	0.1618	0.5033
6	0.0050	0.1098	0.44
7	0.0015	0.0673	0.35
8	4.18E-4	0.0363	0.31
9	6.70E-5	0.0173	0.258
10	8.76E-6	0.0068	0.1733
11	4.97E-7	0.0021	0.1122
12	1.98E-8	5.9E-4	0.0594
13	2.00E-10	9.27E-5	0.0315
14		1.29E-5	0.0136
15		7.56E-7	0.0047
16		2.54E-8	0.0011
17		4.0E-10	3.33E-4
18			3.48E-5
19			3.80E-6
20			1.58E-7
21			3.8E-9
22			1.00-10

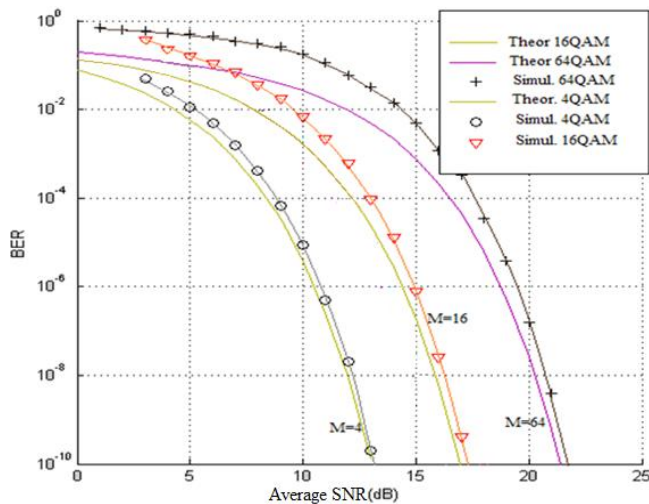


Figure 8: (γ_{avg}) average SNR Vs BER plots of M-ary QAM.



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IX. CONCLUSION

The bit error rate performance has been analyzed in this paper for digital M-QAM signaling scheme. It is found that simulated results are almost identical with theoretical results. For a fixed value of BER the bandwidth efficiency increases with increase of M which is a very significant advantage over other signaling schemes. However, SNR requirement increases with M but this can be tolerated. Higher order QAM systems are only solution for ever- increasing communication demand. Presently 64-QAM is widely employed in communication. When demand will increase in near future 256-QAM and higher order QAM will be certainly employed.

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