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A Novel Explicit FDTD Algorithm for Conformal Antenna Array

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***Abstract*— Conventional Finite Difference Time Domain (FDTD) technique fails to analyze conformal objects as they limit by stair case errors. This work presents an integrated frame work of conformal FDTD and Group Explicit scheme to provide computationally efficient analysis of conformal objects. In this work, magnetic field components of two adjacent points in the computational domain are updated at a time in each iteration. In order to reduce the reflections at the boundaries, an appropriate boundary conditions have been applied. A hexagonal antenna array is assumed as computational object to evaluate the performance of our proposed method. Numerical simulations shown that our method exhibits better performance.**

***Index Terms*—FDTD, CFDTD, GEFDTD, PML, Antenna analysis.**

I. INTRODUCTION

Finite difference Time Domain (FDTD) technique is a very efficient and widely used method for obtaining numerical solutions for electromagnetic problems. Maxwell's equations are first ordered partial differential equations (PDEs), relating the electric field, magnetic field and current excitation. The FDTD technique was developed by Yee based on Maxwell's equations in 1960's. The FDTD equations provide the Electric and Magnetic Fields generated at any point of time to estimate the wave propagation.

The partial difference equations are derived by using central finite difference equation as shown in (1).

$$f'(x) \approx \frac{f\left(x+\frac{\Delta x}{2}\right)-f\left(x-\frac{\Delta x}{2}\right)}{\Delta x} \qquad (1)$$

At any point '*x*', the function values at half a delta step are calculated both in forward and backward direction. The difference between these two is divided by delta to get the derivative of the function at the point '*x*'. The equation (1) is also applicable for multi variable function. This can be done by fixing all variables except one and taking the partial derivative with respective to that variable.

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To illustrate this concept, consider a discrete grid with square or rectangular cells in all the three directions *x*, *y*, and *z*. Each cell has a size of Δx, Δy and Δz. Then, according to Yee's cell, the electric and magnetic field components are in the middle of the edges and the surfaces. This made every electric filed to be surrounded by four magnetic field components. Similarly every magnetic field is surrounded by four electric components. The Yee's unit cell is presented in the figure below.

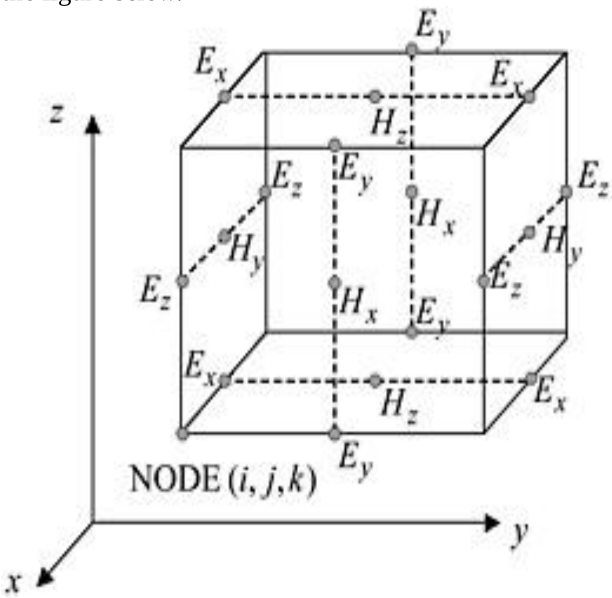

Fig. 1. Yee's unit cell

With help of Yee's unit cell architecture, the equations are never defined at the same point, rather they are staggered in space and time. The electric fields are calculated at regular time intervals along the edges. In the same manner the magnetic fields are calculated at the center of the faces.

In this paper, the first section describes the basics of FDTD and the Maxwell's equations. Section two describes the latest state of the art techniques and research works in the concerned area. Section three provides the details of the proposed algorithms. Section four contains the antenna modelling and experimental results. This section is followed by conclusions.

II. LITERATURE SURVEY

Over the years, there have been many efforts in developing mathematical models for analyzing the wave pattern in regular objects such as square and rectangular antennas. When these methods were applied on irregular objects, lower

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accuracies were produced. The researchers aimed at developing methods which were simple and effective for analyzing such irregular objects. Thus the Conformal Finite Difference Time Domain (CFDTD) was introduced for precisely calculating the wave parameters on irregular objects.

Seyed-Mojtaba Sadrpour and Vahid Nayyeri in [1] presented an FDTD method where only one of the two, electric or magnetic, components is to be updated over the time iterations. The author claimed that this method reduced the computational complexity. Robert A. Marshall, Tom Wallace and Michael Turbe in [2] used FDTD to simulate very low frequency signals. Authors reported that the numerical dispersion problem was solved through Richardson extrapolation.

The irregular surfaces in the boundaries create staircase errors as the FDTD update equations are not designed to handle non square cells. A traditional and simple method was to consider only the square part and neglect the boundaries. The resultant of this approach led to inefficient analysis of irregular antennas. Thus to overcome this problem, CFDTD method was introduced. In any CFDTD method, the computational cost increases as a grid spacing decreases.

On the other hand some of the researchers are looking towards reducing the computational cost. Among them Hideaki Muraoka, Yuta Inoue [3] proposed an efficient electromagnetic simulation technique which is constructed by combining a Hybrid Implicit–Explicit (HIE)-FDTD method and CFDTD method. In this method the excitation pulse was applied on Perfectly Electric Conductor (PEC) in free space. Guanbo Chen, John Stang and Mahta Moghaddam [4] demonstrated a CFDTD technique with accurate wave-port excitation and S-parameter extraction. This was applied on an antenna which is a best example for objects with curved surfaces.
Though the conformal FDTD addressed the problem of staircase error, the computational complexity of the algorithm grew exponentially. In most physical structures, the grid has to be truncated by a certain boundary condition to avoid reflections at the edges and to eliminate the interference between adjacent grids. These boundary conditions play a major role in producing better accuracy of algorithm. These boundary conditions can be categorized into two types: Absorbing boundary conditions (ABC) and Perfectly Matched Layer (PML). Each boundary condition has its own advantages and disadvantages. Impedance mismatch occurs when ABC boundary conditions are applied to CFDTD technique. Such ordinary boundary conditions were not sufficient to accurately update the propagating wave. In order to overcome these addressed problems the alternate solution of PML was adopted for reducing the time complexity and to provide better accuracy. Noraini Md Nusi, Mohamed Othman and Mohamed Suleiman in [5] proposed a numerical method for two dimensional electromagnetic wave propagation. This method increased the maximum time step size to reduce its computational time.

Chandra Sekhar et all, [6] proposed novel Group explicit FDTD (GEFDTD) algorithm to reduce the computational complexity. The authors are used absorbing boundary conditions (ABC) to eliminate the reflections at the boundaries. In this work the parallel plate wave guide is used as computational domain to validate the GEFDTD algorithm. The authors are reported that, GEFDTD algorithm is computationally more efficient than conventional CFDTD algorithms.

III. Proposed Group Explicit Conformal FDTD

As discussed in the literature, the FDTD algorithm is applicable only for surfaces with linear geometry. To overcome this drawback, researchers adapted the conformal FDTD technique to eliminate staircase errors. To the best of authors knowledge limited work is done in that reduces the computational complexity of the CFDTD algorithm. Hence, in order to reduce the complexity, the proposed algorithm is developed. To achieve good accuracy, the Perfectly Matched Layer (PML) boundary condition has been applied in the proposed algorithm. By integrating PML boundary conditions with the proposed algorithm, mismatch impedance can be avoided.
The basis for calculating the magnetic field components is depicted in figure 2. In the figure, the contours are represented as C_1, C_2 and so on. The area of each contour is represented by A_1, A_2 and A_3 respectively. The conventional setup is modified in contour C_2 by including the tangential electric filed component rather than including the E_x which is usable. In the contour the length l_3 represents the usable Electric field component along the 'Y' direction E_y.

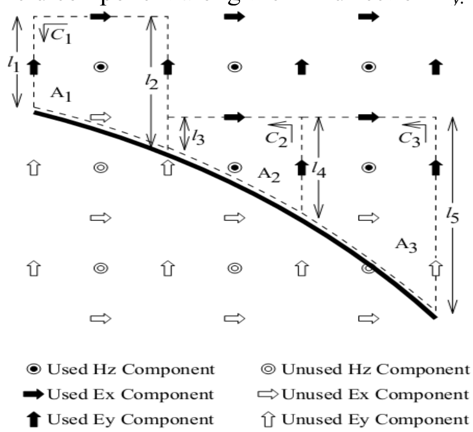

Fig. 2 Faraday contours used to calculate magnetic field components

The scalar equations can be expressed from the Maxwell's equations as represented below.

$$\varepsilon_x \frac{\partial E_x}{\partial t} = \frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}$$

$$\varepsilon \frac{\partial \bar{E}}{\partial t} = \nabla \times \bar{H} \quad \Rightarrow \quad \varepsilon_y \frac{\partial E_y}{\partial t} = \frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} \qquad (2)$$

$$\varepsilon_z \frac{\partial E_z}{\partial t} = \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}$$

$$\mu_x \frac{\partial H_x}{\partial t} = \frac{\partial E_y}{\partial z} - \frac{\partial E_z}{\partial y}$$

$$\mu\frac{\partial \bar{H}}{\partial t} = -\nabla \times \bar{E} \Rightarrow \mu_x\frac{\partial H_y}{\partial t} = \frac{\partial E_z}{\partial x} - \frac{\partial E_x}{\partial z} \quad (3)$$

$$\mu_x\frac{\partial H_z}{\partial t} = \frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x}$$

From the above scalar equations the update equations for GE-CFDTD technique are derived and expressed in equation 4.

$$\begin{bmatrix} H_z^{n+\frac{1}{2}}(i,j) \\ H_z^{n+\frac{1}{2}}(i,j+\frac{1}{2}) \end{bmatrix} = \alpha \begin{bmatrix} \frac{H_z^{n-\frac{1}{2}}(i,j)}{\alpha} & E_y^n(i+\frac{1}{2},j) & E_y^n(i-\frac{1}{2},j) & E_x^n(i,j+\frac{1}{2}) \\ \frac{H_z^{n+\frac{1}{2}}(i,j+\frac{1}{2})}{\alpha} & E_y^n(i+\frac{1}{2},j+\frac{1}{2}) & E_y^n(i-\frac{1}{2},j+\frac{1}{2}) & E_x^n(i,j+\frac{1}{2}) \end{bmatrix} \begin{bmatrix} 1 \\ l_2 \\ -l_1 \\ -\Delta \end{bmatrix} \quad (4)$$

Where $\alpha = \frac{\Delta t}{A_1\mu}$.

The PML boundary conditions were first proposed by Berenger in 1994. PML is more elegant and has extended capability with better accuracy. Berenger introduced additional freedom to the PML boundary conditions by splitting one of the field into sub-components. The CFDTD approximation is derived from Maxwell's curl equations within the uniaxial medium as defined as below:

$$\varepsilon = \mu = \begin{bmatrix} \frac{S_yS_z}{S_x} & 0 & 0 \\ 0 & \frac{S_xS_z}{S_y} & 0 \\ 0 & 0 & \frac{S_xS_y}{S_z} \end{bmatrix} = S \quad (5)$$

$$\nabla \times E = -j\omega\mu SH, \quad \nabla \times H = j\omega\varepsilon \quad (6)$$

From equation (6), the time domain equations has been derived as fallows by using $j\omega f(\bar{r},\omega) \rightarrow \left(\frac{\partial}{\partial t}\right) f(\bar{r},t)$.

$$\begin{bmatrix} \frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} \\ \frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} \\ \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} \end{bmatrix} = \frac{\partial}{\partial t}\begin{bmatrix} K_y & 0 & 0 \\ 0 & K_z & 0 \\ 0 & 0 & K_y \end{bmatrix}\begin{bmatrix} D_x \\ D_y \\ D_z \end{bmatrix} + \frac{1}{\varepsilon_0}\begin{bmatrix} \sigma_y & 0 & 0 \\ 0 & \sigma_z & 0 \\ 0 & 0 & \sigma_y \end{bmatrix}\begin{bmatrix} D_x \\ D_y \\ D_z \end{bmatrix} \quad (7)$$

$$\begin{bmatrix} \frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} \\ \frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} \\ \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} \end{bmatrix} = \frac{\partial}{\partial t}\begin{bmatrix} K_y & 0 & 0 \\ 0 & K_z & 0 \\ 0 & 0 & K_y \end{bmatrix}\begin{bmatrix} B_x \\ B_y \\ B_z \end{bmatrix} + \frac{1}{\varepsilon_0}\begin{bmatrix} \sigma_y & 0 & 0 \\ 0 & \sigma_z & 0 \\ 0 & 0 & \sigma_y \end{bmatrix}\begin{bmatrix} B_x \\ B_y \\ B_z \end{bmatrix} \quad (8)$$

Time-domain PDEs relating magnetic field intensity to electric flux density and electric field intensity to magnetic flux density respectively

The flowchart of proposed GE-CFDTD as depicted in Algorithm 1. Initially H_x and H_y are updated and H_z is updated twice in iteration using group explicit algorithm. The input pulse is applied at any point in the computational domain. The output is analyzed at any point of interest in the domain.

Algorithm 1

1. Compute $H_x^{n+\frac{1}{2}}$.
2. Compute $H_y^{n+\frac{1}{2}}$.
3. Compute $H_z^{n+\frac{1}{2}}(i,j)$ and $H_x^{n+\frac{1}{2}}(i,j+\frac{1}{2})$ in a single iteration.
4. Apply PML boundary conditions.
5. Apply the input in any grid location in the designed object.
6. Observe the output at any grid location.

I. Antenna modeling and Simulation of Proposed Algorithm

In order to validate the proposed GE-CFDTD algorithm, a conformal antenna is considered as an object. A 1x2 hexagonal antenna array is chosen as computational domain. The FR-4 type dielectric material is used for the antenna array with dielectric constant 4.3. This antenna array is designed in MATLAB platform to apply the proposed GE-CFDTD algorithm is shown in Fig. 3.

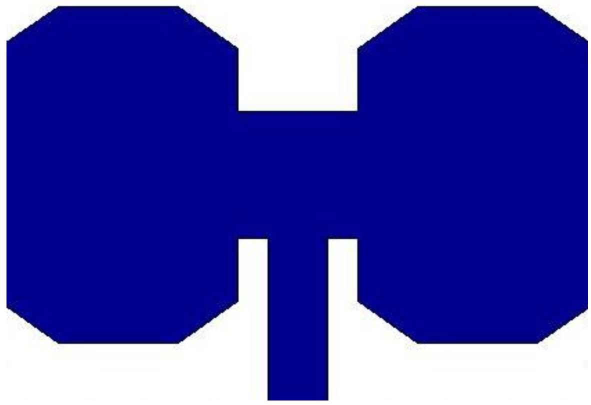

Fig. 3 Hexagonal Shaped 1X2 antenna array

The length and width of the antenna are considered as 3.2cm in x and 2.5cm in y-directions respectively. The thickness of the antenna is 1.6mm.To analyze the wave propagation, the antenna array is converted into grid format. The grid size of the antenna is 100x120 in x and y-direction as shown in Fig. 4.

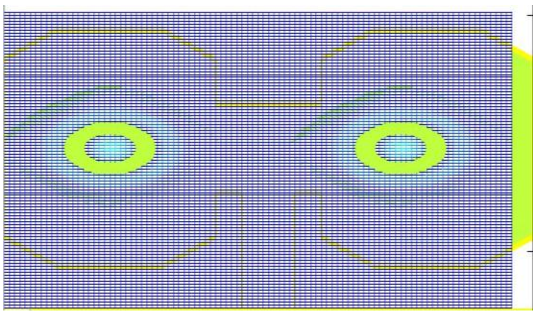

Fig. 4 Grid version of 1X2 Hexagonal Antenna

The sinusoidal wave is generated and applied as input pulse to the computational domain. The input pulse is propagating through the computational domain with respect to update H_x and H_y magnetic field update equations are reported in algorithm. The input pulse is applied at (55,20) grid location of the antenna. The wave propagating through the antenna at 400th iteration as depicted in the Fig. 5.

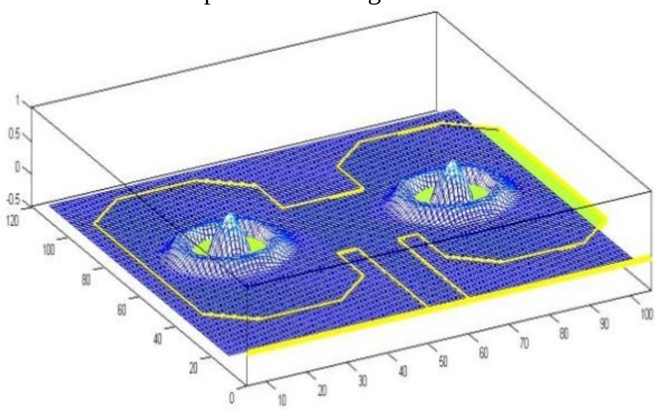

Fig. 5 Wave Propagation at time step 400 iterations

The resonant frequency of the antenna generated is 2.8GHz. In order to remove the reflections at the boundaries of the antenna, the Berenger's PML is applied. The output pulse is analyzed by computing the magnetic field component at the grid location (20,80) of the antenna. To validate the proposed GE-CFDTD method, the magnetic field response is compared with the conventional CFDTD method and corresponding results are shown in Fig. 6.

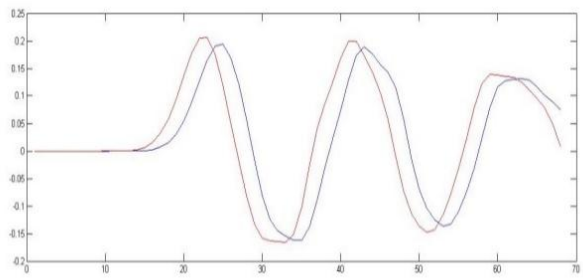

Fig. 6 Comparison results of Conventional CFDTD and GE-CFDTD

It is observed from the Figure 6 that the output response of our proposed GE-CFDTD method is nearly matched with the conventional CFDTD method. To verify the computational efficiency of our proposed method, the comparison made with the conventional CFDTD method. The comparison of computational time required for both methods are reported in table 1.

TABLE I

Comparison of computational time

	Proposed Method	Conventional CFDTD Method
Computational Time per one iteration	0.19 ms	0.28 ms
Number of Iterations	400	400

Simulations of proposed GE-CFDTD and Conventional CFDTD methods are carried out on Intel Core-i5 2.30 GHz machine. From the Table 1 the proposed GE-CFDTD method has proven to be computationally more efficient.

II. Conclusion

In this paper, an integrated GE-CFDTD algorithm is proposed. The conformal antenna analysis is highly time consuming. This limitation is overcome by using our accelerating method. The update equations are derived by grouping two neighboring cells of the computational domain. A sinusoidal wave is applied at feed location of computational domain. Second ordered PML is applied at the boundaries to eliminate the reflections. Numerical simulations are carried out, to analyze the hexagonal antenna array using conventional CFDTD and our method. It is concluded from the results, our proposed method is 1.47 times faster than conventional FDTD technique for 400 iteration.

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