

Limit of localization of Gaussian pulse shaping filters in signal demodulation in DGT based future (5G) wireless multicarrier communication

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Abstract— Waveform design, for future wireless (5G) multicarrier (MC) communication systems such as generalized frequency division multiplexing (GFDM), pulse shaped OFDM (P-OFDM), and bi-orthogonal frequency division multiplexing (BFDM), is proposed to be based on time-frequency shift transmitter and receiver pulse shaping filters. These filters are aimed at, since these are shown to be robust against ISI and ICI interference effects, which are imposed by doubly dispersive channels, when high speed (5G) data is transmitted over these channels. Also it is shown that this robustness of the filters can be achieved, when the filters are well localized in time and frequency. Thus localization of filters plays a major role in minimization of interference effects. One problem for consideration is, if filters are designed for well localization, then is it possible to demodulate/ reconstruct the original signal at the receiver at this localization. This problem is addressed in this paper by simulations of data transmission in discrete Gabor transform (DGT) based MC system and the results are presented. Preliminary analysis of simulations reveal that there is a limit of localization, beyond which, original signal can't be demodulated. Or in other words faithful reproduction of the original signal can't be achieved. In the analysis it is identified that this limit is increasing in proportion to the increase in length of the signal. Thus the contribution of this paper is that this limit is a notable factor in the design of Gaussian pulse shaping filters, which are applicable in any multicarrier communication system.

Index Terms— Gaussian pulse shaping filters, localization of filters, time-frequency shift filters, DGT based MC system, GFDM, 5G Waveform design, BFDM

I. INTRODUCTION

Generalized frequency division multiplexing (GFDM), pulse shaped OFDM, and biorthogonal frequency division multiplexing (BFDM), are some of the proposed MC systems for future generation (5G) wireless (mobile) communication systems, since these systems are having several advantages when compared to conventional cyclic prefix (CP)-OFDM systems. One of the advantages is the minimization of inter symbol interference (ISI) and inter channel/carrier interference (ICI). This minimum interference is obtained from the well localization of pulse shaping filters. Many results are reported for design of localization of filters in MC

systems under time-frequency domains [1][2][3]. Recently a theoretical work is reported considering time-frequency shift pulses applicable to OFDM/ BFDM systems and the analysis showed that minimum ISI/ ICI is obtained when the pulses are well localized and optimized [4].

Eventhough much literature is available aiming at the design of pulse shaping filters, very meagre information is available on limit of localization of the filters in the demodulation of the signal at the receiver. Also as high speed data transmission over doubly dispersive channel is inevitable in future wireless communication systems, these systems are to be designed with time-frequency shift pulse shaping filters for minimization of interference effects. Even pulse shaping filters are well designed and optimized, minimum bit error rate (BER) is not achievable, if proper demodulation is not obtained at the receiver. For well localization or maximum energy concentration, if it is tried for possible localization, then one problem for consideration is, whether this localization is guaranteed for demodulation producing faithful reproduction of the original signal or not. Keeping in view to address this problem, a discrete Gabor transform (DGT) based MC transmission system is considered for the intended problem and is analysed in this paper. In the recent times it is shown that DGT based system is applicable as an equivalent of proposed future wireless (5G) communication systems, for example Generalized Frequency Division Multiplexing (GFDM) [5][6][7]. By developing computer simulations of the MC system, an analysis is performed for demodulation of the original signal at various localizations of the filters and the results are presented.

The rest of the paper is organized as follows: DGT based MC system and data transmission through the system are presented in section-II. Design of transmitting and receiving pulse shaping filters for the cases of biorthogonal condition as given by Wexler-Raz, and orthogonal (like) condition, is described in section-III. By showing computer simulation examples, limit of localization of filters is demonstrated in Section-IV and conclusions are drawn in section-V.

II. DGT BASED MC SYSTEM

A block diagram of DGT based MC system proposed for transmission of data, is shown in Figure-1.a. In this system, a

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pair of DGT and its inverse, IDGT, are used as transmitter/receiver. Applicability of DGT/IDGT transceivers has already been shown in [5] [6] for future wireless systems, for example, GFDM. Gabor expansion of a signal, originally proposed by Gabor [10], is later modified by others [11][9][12], for definitions of DGT and IDGT. DGT can be defined for a discrete periodic signal, $x(k)$, in length, L , as

$$a_{mn} = \sum_{k=1}^L x(k) w((k - m\bar{N}) \bmod L) \exp(-j2\pi.nk/N) \text{ -----(1)}$$

a_{mn} are Gabor coefficients for the signal $x(k)$. Now Gabor expansion, also called as IDGT, is given as

$$x(k) = \sum_1^M \sum_1^N a_{mn} g((k - m\bar{N}) \bmod L) \exp(-j2\pi.nk/N) \text{ -----(2)}$$

where $m= 1$ to M , and $n=1$ to N .

In the above equations, $g(k)$ and $w(k)$ are called as pulse shaping filters (or synthesis/ analysis window functions). M and N are number of samples in time and frequency and \bar{M} and \bar{N} are spacing in frequency and time respectively.

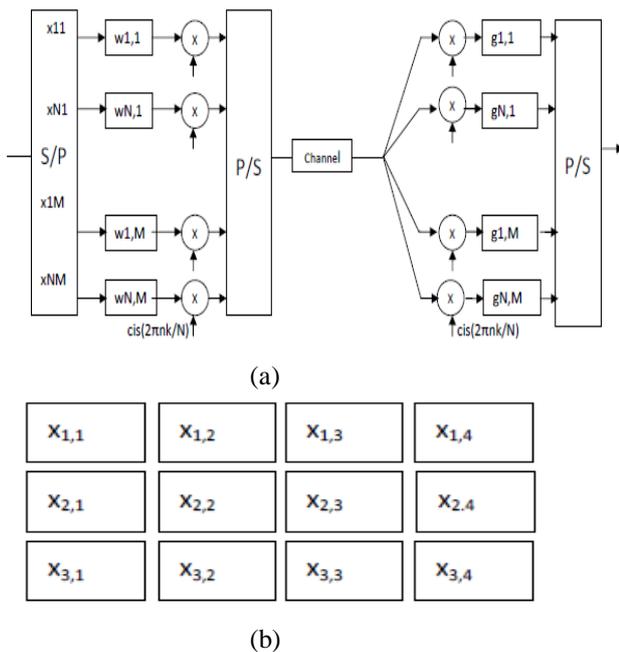


Figure-1: a) DGT based multicarrier (MC) system
b) Input MN data samples distribution over N=3, subcarriers, and in M=4, subsymbols, for a block of length L=MN

For existence of Gabor Transform, $g(k)$ and $w(k)$ should satisfy Wexler and Raz [9] biorthogonality condition, given as,

$$\sum_{k=1}^L g(k + mN) w(k) \exp(-j2\pi.nk/\bar{N}) = \delta_m \delta_n \text{ -----(3)}$$

where $m=1$ to \bar{M} and $n=1$ to \bar{N} . The transform defined for critical sampling with the condition, $L = \bar{M}\bar{N} = MN$, facilitates transmission of L , input samples of data, divided into parallel

streams to be transmitted over N subcarriers, such that each subcarrier data is passed through pulse shaping filters shifted in M time slots. Thus the transmitting signal is a block or symbol of MN samples, and is equivalent to a GFDM transmitting signal as detailed in [6][5]. For example, for a transmitting symbol (or a block) consisting of MN samples, input data distribution over $N=3$, subcarriers, and $M=4$, subsymbols, is shown in Figure-1.b. In this figure, each row corresponds to one subcarrier data in M subsymbols, and each column, represents N subcarrier data of one subsymbol.

Also, as it has been shown in [8], that MC system, in which, modulations are defined with discrete Hartley transform (DHT), is equal in performance to system defined with discrete Fourier transform, DFT. As such DHT modulations/demodulations are used in the present system, instead of DFT modulations used in other systems. One convenience with DHT, is since modulations/demodulations are defined with the same DHT, transmitter and receiver are of same function. Moreover, as suggested in [5], that when pulse shaping filters satisfies, Wexler-Raz condition, then the signal $x(k)$ and the coefficients, a_{mn} are same. Or in other words, one can be obtained from merely as summations of the other's data transmitted through parallel time-frequency shift filters, as represented in equations-1 and equation-2. Thus in the present system, DGT is used as transmitter and IDGT is used as receiver.

A block of data samples of input signal, $x(k)$, of length, L , is divided and transmitted over N parallel subcarriers (or paths), such that each subcarrier data is passed through time shifted pulse shaping filter, $w(k)$. Filters output data of each path is modulated with DHT functions $\{ \cos(2\pi.nk/N) + \sin(2\pi.nk/N) \}$, and then data from these parallel paths are converted into serial form, which is taken as transmitting signal, a_{mn} . This is applied to a channel as shown in Figure-1a. With a focus on effects of localization of filters rather than the effects of the channel, channel here is assumed as an ideal. It implies no noise and no distortion effects. With this assumption, inclusion and deletion of CP is ineffective. Also by assuming perfect synchronization and D/A (A/D) at channel input (output) are as applied, the output of the (ideal) channel is taken as the received signal. This signal is processed in the receiver in just reverse to that used in the transmitter. Thus the received signal is converted into parallel streams to be processed with DHT demodulations before passing through time shifts of pulse shaping filter, $g(k)$, as shown in Figure-1. These filtered data streams are converted into serial form, which can be identified as a replica of original signal, $x(k)$. Zero forcing (ZF) receiver is used in this system, and the output of the receiver is obtained as demodulated original signal $x(k)$. Also since stable demodulation and well localization are achievable in oversampling, analysis is performed for oversampling, by taking parameters, $M=N=128$, $\bar{M} = \bar{N} = 16$, such that they satisfy $MN > L$ and $L = \bar{M}\bar{N} = \bar{N}M$. An example for Matlab simulation of an input signal, $x(k)$ (or $x(n)$), to the transmitter, and output signal from the receiver, illustrating the validity of the system, is shown in Figure-2. (This validation is also confirmed from several runs of simulations for different signals and filters for finite and infinite lengths.

These results are not shown here).

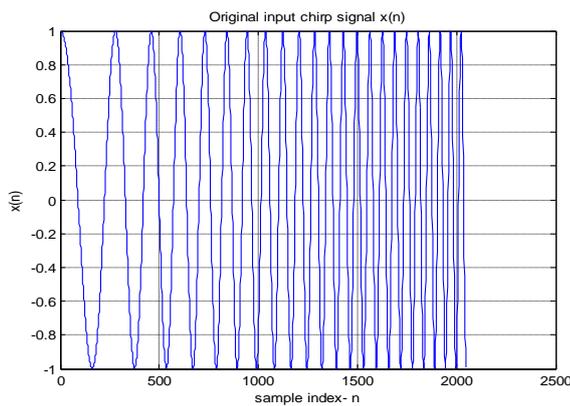


Figure-2: a) Input chirp signal to transmitter

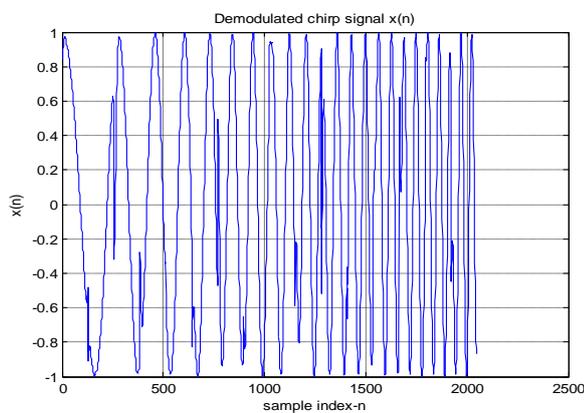


Figure-2: b) Demodulated chirp signal from the receiver

III. DESIGN OF PULSE SHAPING FILTERS BIORTHOGONAL FILTERS

In the proposed system, well localization of transmitting and receiving filters, $g(k)$ and $w(k)$, is crucial for minimization of ISI/ICI. Also for the existence of the DGT in the system, these filters should satisfy Wexler and Raz [9] bi-orthogonality condition,

$$\sum_{k=1}^L g(k + mN) w(k) \exp(j2\pi.nk/N) = \delta_m \delta_n \quad \text{-----(4)}$$

for $m = 1$ to M , $n=1$ to N , and $L=MN$.

where δ is dirac delta function. Equation-4, can be represented in matrix form as

$$H w = v \quad \text{-----(5)}$$

From equation-5, $w(k)$, can be obtained as

$$w = H^T (H H^T)^{-1} v \quad \text{-----(6)}$$

where $v = [1, 0, 0, \dots, 0]$ is a vector of length L , and matrix, H is obtained from the filter, $g(k)$. These filters are designed using Wexler and Raz [9] method. Initially a Gaussian filter is selected as, $g(k)$, then its biorthogonal filter, $w(k)$ is estimated from the samples of $g(k)$ by arranging H as block Hankel type matrix, which is organized as given in the following form

$$H = \begin{bmatrix} H^1 & \dots & H^M \\ \vdots & \ddots & \vdots \\ H^M & \dots & H^M - 1 \end{bmatrix} \quad \text{-----(7)}$$

where each block is given in terms of diagonal $g(k)$ functions as

$$H^l = \begin{bmatrix} W^1 & W^1 & W^1 \\ W^1 & W^2 & W^N \\ W^1 & W^N & W^2 \end{bmatrix} x$$

$$[g(lN + 1) \dots g(lN + N)] \quad \text{-----(8)}$$

where $W = \exp(j 2\pi/N)$ and $l = 1$ to \bar{M} -----(9)

Block Hankel type matrix is guaranteed for inversion, and hence, this matrix inversion facilitates stable reconstruction

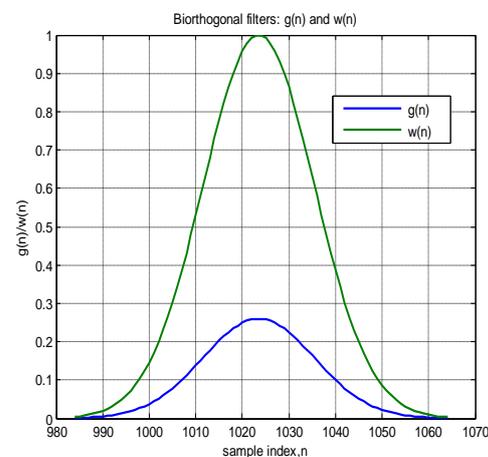


Figure-3: a) Biorthogonal Pulse shaping filters

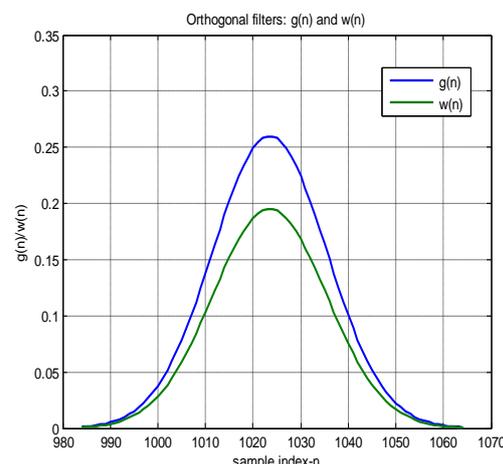


Figure-3: b) Orthogonal Pulse shaping filters

of the signal by avoiding the possibility of singularity of H matrix, which may lead to non existence of Gabor transform. As stable demodulation is achievable in oversampling, filters are designed for oversampling, by following the method given by Wexler and Raz [9]. An example for Matlab simulation of $g(k)$ and $w(k)$ is shown in Figure-3a.

ORTHOGONAL (LIKE) FILTERS

In the conventional OFDM, the filters satisfies orthogonality, when $g(k) = w(k)$. In order to assess the effects of localization, an orthogonal (like) condition is also considered in the analysis. This condition is achieved taking $\alpha = 0.75$ in the equation

$$w(k) = \alpha g(k) \quad \text{-----(10)}$$

An example for Matlab simulation of orthogonal filters is shown in Figure-3b.

IV. SIMULATION EXAMPLES

A gaussian pulse shaping filter, $g(k)$ is chosen, as it can be shown that it is the pulse, which is optimum simultaneously in time and frequency localizations[10]. $g(k)$, (of length,L) is generated from

$$g(k) = \sqrt{\frac{\sqrt{2}}{D}} \exp\left(-\pi \left(\frac{k - \frac{L}{2} - 0.5}{D^2}\right)^2\right), \quad \text{-----(11)}$$

where D is taken as width of the filter, and the filter satisfies unit energy. $w(k)$, a biorthogonal filter to $g(k)$, is designed using Wexler and Raz [9] identity as explained in section-III. An example for biorthogonal filters is shown in Figure-3.a.

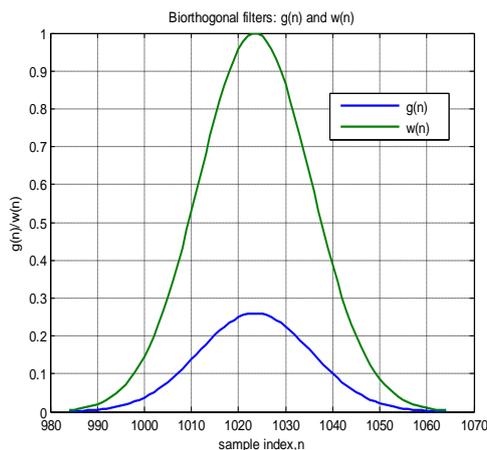


Figure-4: a) Biorthogonal Filters (width-1)

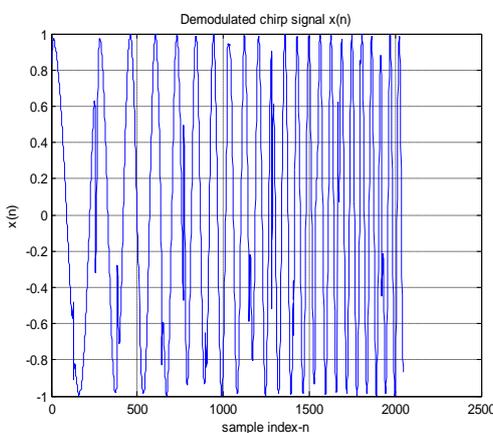


Figure-4: b) Well demodulated signal

It can be noted that, D is a key role parameter in the localization of the filter, such that increase in localization (or resolution) in time can be obtained by decreasing, D, of the filters. Orthogonal filters, are obtained as explained in section-III, and are shown in Figure-3.b. For a block of input data, a chirp signal, $x(k)$ (or $x(n)$), of length $L=2048$ samples, is taken as shown in Figure-2a. When this block is

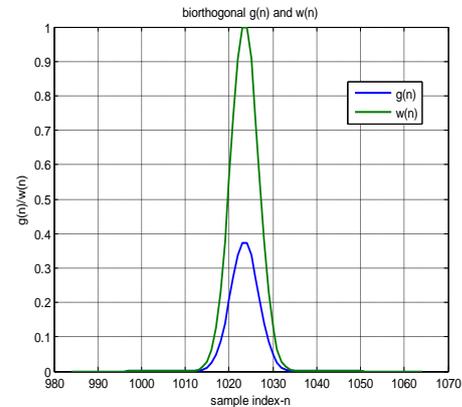


Figure-5: a) Biorthogonal Filters (width-2)

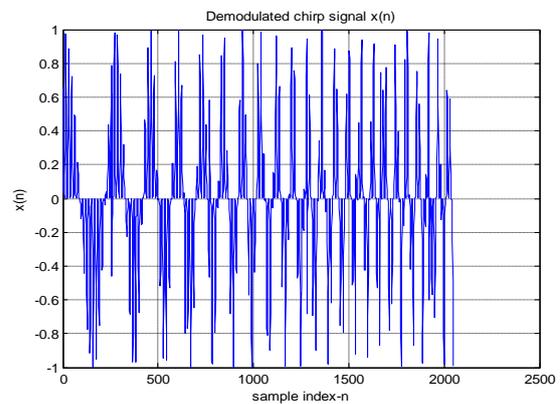


Figure-5: b) Failed Demodulated signal

transmitted through the system, demodulation of the signal at receiver is analyzed by changing the widths (width-1 and width-2) of the filters for different localizations.

In the analysis, limit of localization is illustrated, in Figures-4 to 7. When the filters are with width-1, i.e. $D=30$, there is a well demodulation for faithful reproduction of the original signal, both in biorthogonal condition, as shown in Figure-4, and in orthogonal condition, as shown in Figure-6.

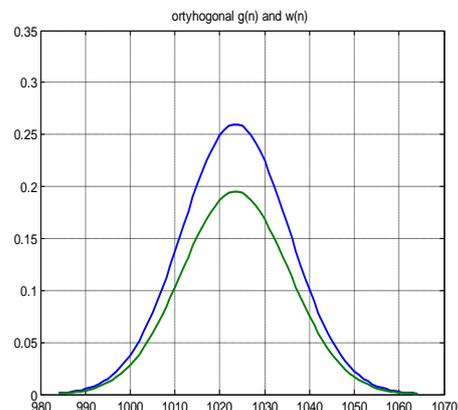


Figure-6: a) Orthogonal Filters (width-1)

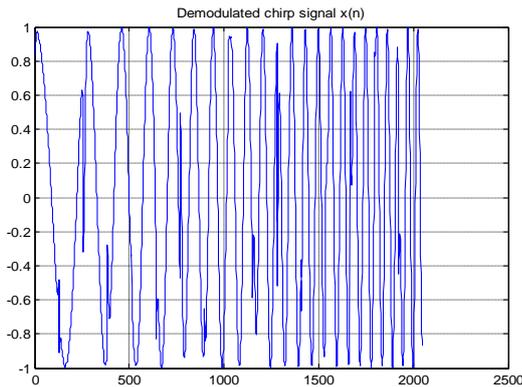


Figure-6: b) Well demodulated signal

When the width of filters are reduced to width-2, i.e, $D=8$, there is a failure of demodulation for original signal, both in biorthogonal condition, as shown in Figure-5, and in orthogonal condition, as shown in Figure-7. Since failure of demodulation is obtained, width-2 is taken as a limit of localization of the filters. It is to be noted that the set of figures from 4 to 7, is corresponding to infinite (long length) sequences.

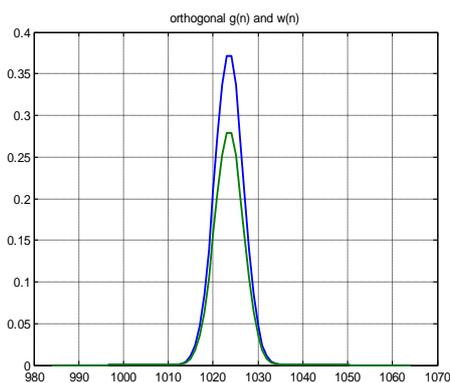


Figure-7: a) Orthogonal Filters (width-2)

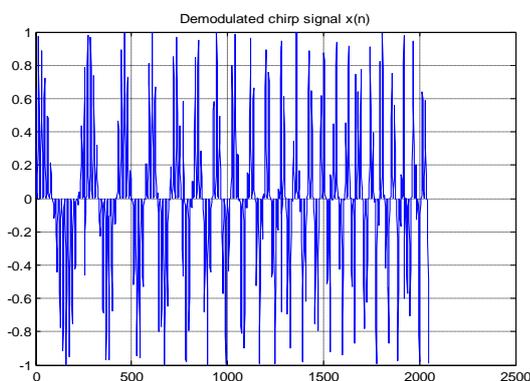


Figure-7: b) Failed demodulated signal

Similar to this set, when simulations are run with limit of localization i.e, with width-2, for sequences of finite (short length) and infinite lengths, then the result of these sets is shown in Figure-8. From this figure, it can be observed that the limit of localization is changing proportional to length of the signals.

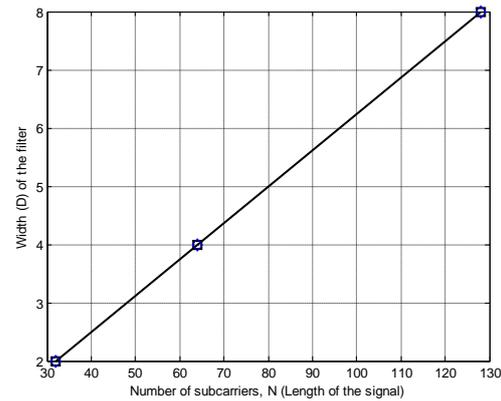


Figure-8: Width (D) of the filters versus Number of subcarriers (N) for finite and infinite length of the signals

V. CONCLUSIONS

It is demonstrated in this paper that there is a limit of localization of Gaussian time-frequency shift pulse shaping filters, designed for high speed data transmission through MC systems. The analysis of simulations reveal that, when the width of the filter is reduced to a limit i.e. width-2, resulted increase in time localization (or resolution), then the demodulation is failed in the sense that faithful reproduction of the original signal is not achieved. This failure can't be noticeable in terms of bit error rate (BER), which is an usual error estimation parameter in the communication systems. In other words, it can be said that, eventhough, received signal can be demodulated with higher BER with width-2, but actually, it is not the original baseband signal intended for transmission. Thus with an intention to achieve minimum of ISI/ICI effects, if the localization crosses a limit, then it is not possible to extract or demodulate the baseband signal. This may be due to increase in time resolution beyond a limit, results in decrease in frequency resolution, which may not be sufficient to produce all the frequency components of the original signal. It implies reliable communication is not possible beyond this limit, even under the ideal channel conditions. Hence, it can be concluded that, this limit is a noticeable factor in the design of the Gaussian pulse shaping filters, which are applied in any multicarrier communication system. However, as it is a preliminary analysis with a focus to determine the limit, further analysis is required with a consideration of measures of localization and limits in both time and frequency, for reliable multi carrier communication.

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